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STRENGTH ANALYSIS OF HIGH PRESSURE STEEL PIPES REINFORCED BY COMPOSITE LAYERS

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Abstract. Due to certain civil engineering activities, e.g. road construction, the safety factor of already working high pressure oil and gas transmitting pipes in the vicinity should be previously increased. For doing so the relatively new, but well proved, pipe guard technique is applied many times, by which several layers of glass fiber reinforced epoxy material is applied to the external cylindrical surface. For the knowledge of the mechanical behaviour and for the possibility of the standardisation of this new anisotropic and heterogeneous structure an analytical procedure and a computer program has been worked out in the frame of linear elasticity. In parallel, a number of experiments were also carried out for example for the determination of the pressure-volume change characteristics and for that of the bursting pressure. Results of the analytical calculations and experiments were compared and a good correlation was found.

Keywords: High pressure steel pipes, composite materials, reinforcement

1. Introduction

In the practice of oil and gas transmission some sections of the high pressure steel pipes should be reinforced to avoid local damages due to the construction of civil engineering establishments like roads, railway lines etc nearby. The so-called clock spring technique is a highly recommended procedure for doing that by which several fiber reinforced epoxy layers are applied to the outer cylinder-jacket of the pipe. An analytical procedure has been worked out for predicting the expectable strength behaviour of this new complex, anisotropic and heterogeneous tube. On this base a simple computer program for the possibility of standardisation has also been developed. In this respect the basic question is the necessary number of the layers if both the initial and the attainable safety factors are known for a given tube geometry, steel material, working pressure and composite parameters. In addition to the analytical and computer analysis experimental investigations have been also carried out for the determination of the pressure – volume-change characteristics both for the original (pure steel) tube and for the reinforced tube structures. These experiments have been performed up to the ultimate bursting pressure, approx. 170-200 bar.

In the frame of the analytical procedure the model was considered as an infinitely long, complex tube constructed from inside by the original steel pipe and reinforced by an arbitrary number of epoxy-glass fiber layers from outside. The layers have the same thickness and they were reinforced by fibers with cross-bonded orientation per layer in such a way that the filling fibres were placed in circumferential and the chain fibers were placed in axial directions. Special measurements have been carried out to determine the chain and the filling direction elasticity moduli and the Poisson ratios. For controlling certain transformation formulae the elasticity modulus of the degree of 45 positioned composite layer was also investigated. The tube was assumed to be linearly elastic. Sections 2 and 3 present the closed form solutions for the radial displacements and the characteristic stress components and also the equivalent stresses in the steel tube and epoxy layer(s). Section 4 is devoted to the experimental determination of the safety factor and to the comparison of the experimental and computed results. Conclusions are presented in the last section.

2. The mechanical model and the computation strategy

By application of the clock spring technique we get a heterogeneous tube which is made up of the original steel pipe from inside and a multilayered glass fiber reinforced by a structural constituent made of epoxy material from outside. Mechanically the tube is considered to be infinitely long, linearly elastic, and the load is a constant pressure exerted on the internal surface. Consequently, in a cylindrical co-ordinate system the displacements in the axial direction z are free and the corresponding stresses are equal to zero. On this basis, for the computation of the characteristic displacement, strain and stress components along the thickness of the steel tube a boundary value problem should be established and analyzed making use of the data

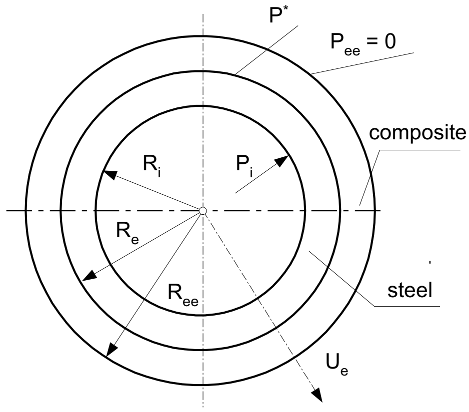

Figure 1. Boundary problem of a complex tube

shown in Figure 1. The main point of this activity is the determination of the internal pressure p^* between the steel and the epoxy as we shall see later.

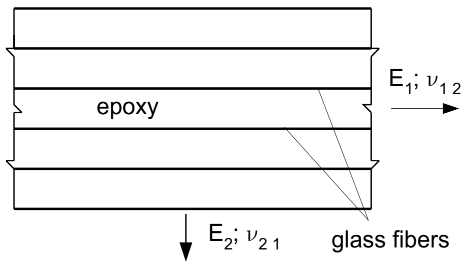

Figure 2. Composite layer reinforced in one direction

According to Figure 1 we have the following boundary conditions:

$$\begin{aligned} \sigma_R(R = R_i) &= -p_i\,, \\ \sigma_R(R = R_{ee}) &= 0\,. \end{aligned} \tag{2.1}$$

At the common radii $R = R_e$ the radial displacement co-ordinate is denoted by u_e.

The material constants for the steel are the usual ones, i.e., the Young modulus (denoted by E), and the Poisson ratio (denoted by ν). According to the literature of composite materials – see e.g.[1,2] – the material parameters for the substituting anisotropic thin layer are (see Figure 2.) as follows:

– E_1 is the elastic modulus in the direction of the fibers,
– E_2 is the elastic modulus in the direction perpendicular to the fibers,
– ν_{12} and ν_{21} are the anisotropic Poisson ratios,
– G is the shearing modulus of elasticity.

The well-known reciprocal relation reads

$$E_1\nu_{21} = E_2\nu_{12} \tag{2.2}$$

which means that only four material parameters are independent of each other. In our case the fibers are placed parallel either to the circumferential direction or to the axial direction, therefore the value of the shearing modulus has no importance.

If the material parameters are known, the geometrical data and the yield stress of the steel the computation can be based on the following ideas:

– first the function that relates the radial displacement at an arbitrary point of the external steel tube surface u_{e1} to p^* , as yet unknown contact pressure, should be determined in terms of the given internal pressure p_i,
– secondly the function that relates the radial displacement at an arbitrary point on the internal surface of the epoxy tube u_{e2} to p^*, which can be considered as an internal pressure for this constituent, should be determined (the material of the epoxy tube is assumed to be orthogonally anisotropic),
– taking the equality $u_e = u_{e1} = u_{e2}$ into account, we obtain a simple linear equation which can easily be solved for the unknown p^*,
– in the following calculations we are concerned with the steel tube subjected to the internal pressure p_i and the contact pressure p^* as external pressure, then

the circumferential stress and the von Mises equivalent stress are calculated at the internal diameter which is the critical region of the tube,
- the last step is the determination of the safety factor n.

It is obvious that the above calculations can be repeated for one, two etc. clock spring layered structures in order to find the number of necessary layers for a given and improved safety factor.

3. Basic formulae for the computation

3.1. The steel tube. Following the steps of the above strategies first we have to deal with the well-known Euler type second order differential equation of a homogeneous and isotropic cylinder. Its closed form solution has been published in a number of textbooks. (See e.g. [3].) Recalling the usual notations we can write that:
- u is the radial displacement,
- $\epsilon_R, \epsilon_\varphi, \epsilon_z$ are the strains in radial, tangential and axial directions, respectively,
- $\sigma_R, \sigma_\varphi, \sigma_z$ are the normal stresses in the same directions,
- $\gamma_{R\varphi}$ is the shear strain along the meridian,
- $\tau_{R\varphi}$ is the shear stresses along the meridian.

With these notations the stress and the strain vectors and also Hook's law can be constructed

$$\boldsymbol{\sigma}^T = [\ \sigma_R, \quad \sigma_\varphi, \quad \sigma_z, \quad \tau_{R\varphi}\ ] \qquad \boldsymbol{\epsilon}^T = [\ \epsilon_R, \quad \epsilon_\varphi, \quad \epsilon_z, \quad \gamma_{R\varphi}\ ]$$

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\epsilon} \tag{3.1}$$

where the constitutive matrix is of the form

$$\mathbf{D} = \frac{E\,(1-v)}{(1+v)\,(1-2v)} \begin{bmatrix} 1 & \dfrac{v}{1-v} & \dfrac{v}{1-v} & 0 \\ \dfrac{v}{1-v} & 1 & \dfrac{v}{1-v} & 0 \\ \dfrac{v}{1-v} & \dfrac{v}{1-v} & 1 & 0 \\ 0 & 0 & 0 & \dfrac{1-2v}{2\,(1-v)} \end{bmatrix} \tag{3.2}$$

It follows from (3.1) and (3.2) that

$$\sigma_R = \frac{E\,(1-v)}{(1+v)\,(1-2v)} \left[\epsilon_R + \epsilon_\varphi \frac{v}{1-v}\right], \tag{3.3a}$$

$$\sigma_\varphi = \frac{E\,(1-v)}{(1+v)\,(1-2v)} \left[\epsilon_\varphi + \epsilon_R \frac{v}{1-v}\right]. \tag{3.3b}$$

Substituting the above stress components into the geometrical formulae

$$\epsilon_R = \frac{\mathrm{d}u}{\mathrm{d}R} \qquad \text{and} \qquad \epsilon_\varphi = \frac{u}{R} \tag{3.4}$$

and the results into the equilibrium equation

$$\frac{\mathrm{d}\sigma_R}{\mathrm{d}R} + \frac{\sigma_R - \sigma_\varphi}{R} = 0\,. \tag{3.5}$$

the well-known tube equation is obtained

$$\frac{\mathrm{d}^2 u}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}u}{\mathrm{d}R} - \frac{u}{R^2} = 0\,. \tag{3.6}$$

Its general solution takes the form

$$u = K_1 R + \frac{K_2}{R} \qquad\qquad \frac{\mathrm{d}u}{\mathrm{d}R} = K_1 - \frac{K_2}{R^2} \tag{3.7}$$

where K_1 and K_2 are constants of integration which depend on the boundary conditions. Making use of the above relations but omitting the details we shall find that

$$u_{e1} = p^* I + J \tag{3.8}$$

in which the constants I and J can be given in terms of other constants $A, \ldots, H$:

$$I = AR_e + \frac{D}{R_e} \qquad\qquad J = -\left(BR_e + \frac{F}{R_e}\right) \tag{3.9a}$$

$$A = \frac{1}{C}\frac{R_e^2}{(R_i^2 - R_e^2)} \qquad\qquad B = \frac{1}{H}\frac{p_i R_e^2}{(R_i^2 - R_e^2)} \tag{3.9b}$$

$$D = \frac{1}{C}\frac{R_e^2}{(R_i^2 - R_e^2)} \qquad\qquad F = \frac{1}{H}\frac{p_i R_e^2 R_i^2}{(R_i^2 - R_e^2)} \tag{3.9c}$$

$$C = \frac{E\,(1-v)}{(1+v)\,(1-2v)} \qquad\qquad H = \frac{E}{1+v} \tag{3.9d}$$

3.2. The epoxy tube. For the anisotropic epoxy tube produced by the clock spring technique the constitutive matrix can be taken from the book [4] by Lekhnitsky:

$$\mathbf{D} = \hat{H}\begin{bmatrix} n\left(1-\nu_2^2\right) & \nu_1 + n\nu_2^2 & n\nu_2\,(1+\nu_1) & 0 \\ \nu_1 + n\nu_2^2 & n\left(1-n\nu_2^2\right) & n\nu_2\,(1+\nu_1) & 0 \\ n\nu_2\,(1+\nu_1) & n\nu_2\,(1+\nu_1) & 1-\nu_1^2 & 0 \\ 0 & 0 & 0 & n(1-\nu_1^2)(1-2n\nu_2^2) \end{bmatrix} \tag{3.10}$$

where

$$\hat{H} = \frac{E_2}{(1+v_1)\,(1-v_1+2n\nu_2^2)}$$

and

$$v_1 = v_{12};\;\; v_2 = v_{21}; \qquad m = \frac{G_2}{E_2}, \qquad n = \frac{E_1}{E_2}\;.$$

Here the indexes 1 and 2 denote the principal material directions. These are parallel with the circumferential and axial directions.

Following the chain of ideas leading to equation (3.6), we arrive at the Euler type differential equation

$$\frac{\mathrm{d}^2 u}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}u}{\mathrm{d}R} - \frac{1-nv_2^2}{1-v_2^2}\frac{u}{R^2} = 0 \tag{3.11}$$

which the radial displacement u within the anisotropic tube should meet. The general solution to this equation takes the form

$$u = C_1 e^{\sqrt{s}} R + C_1 e^{\sqrt{s}} \frac{1}{R} \tag{3.12}$$

where

$$s = \frac{1 - n v_2^2}{1 - v_2} \, . \tag{3.13}$$

As one can expect for $n = 1$ differential equation (3.11) is that of the isotropic tube. From the technical point of view just the positive value of s is taken into consideration.

The constants of integration C_1 and C_2 in (3.12) can be calculated from the boundary conditions

$$\begin{aligned} \sigma_R &= -p^* \qquad \text{if} \quad R = R_e \, , \\ \sigma_R &= 0 \qquad \text{if} \quad R = R_{ee} \, . \end{aligned} \tag{3.14}$$

Making use the above formulae and the boundary conditions (3.14) but omitting again the details we shall find that

$$u_{e2} = T p^* \tag{3.15}$$

where the constant T is given in terms of the constants P, Q, S and C:

$$\begin{aligned} T &= \frac{s}{(P - s)\, Q} e^{\sqrt{s}} R_i + \frac{1}{s - P} e^{\sqrt{s}} \frac{1}{R_i} \\ P &= \frac{1}{R_e^2} C e^{\sqrt{s}} \left(2 n v_2^2 - n + v_1 \right) \; ; \qquad Q = C e^{\sqrt{s}} \left(n + v_1 \right) \\ S &= \frac{1}{R_{ee}^2} C e^{\sqrt{s}} \left(2 n v_2^2 - n + v_1 \right) \; ; \qquad C = \frac{E_2}{(1 + v_1)(1 - v_2 - 2 n v_2)} \quad . \end{aligned} \tag{3.16}$$

3.3. The contact pressure and the safety factor. Recalling the third point of our computation strategy, i.e., using the equality (3.8) to (3.15) the contact pressure p^* can be determined as

$$p^* = \frac{J}{T - I} \, . \tag{3.17}$$

With this value the strain and stress components ϵ_R, ϵ_φ, σ_R and σ_φ can easily be determined at the external diameter D_e and the internal diameter D_i which is the critical surface of the steel tube.

Then the equivalent stress can also be computed

$$\bar{\sigma} = \sigma_\varphi - \sigma_R \, . \tag{3.18}$$

Taking the equivalent stress on D_i the safety factor can be obtained from the relation

$$n = \frac{\sigma_{allowable}}{\bar{\sigma}} \tag{3.19}$$

4. Parametric study

Making use of the formulae (3.3a,b), (3.7), (3.9a, ..., d), (3.16), (3.17) and (3.19) a simple program has been developed in order to compute all the mechanical quantities and the safety factors both for an 'original' tube and for a clock spring reinforced tube by superimposing one, two etc. layers. As a parametric study a 12" gas transmitting steel pipe was investigated provided that it is reinforced up to maximum six clock spring layers. The operational pressure $p_i = 6$ MPa. The other input and output data are summarized below.

Data for the original steel pipe:
The inside diameter is $D_i = 326$ mm and the outside diameter is $D_e = 338$ mm. The Young modulus is $E = 2.1 \cdot 10^5$ MPa, the Poisson ratio is $\nu = 0.33$, and the yield stress is $\sigma_Y = 300$ MPa. The following data were calculated:

ϵ_{Re}	$\epsilon_{\varphi e}$	σ_{Re}	$\sigma_{\varphi e}$
-0.000338	0.000678	0.	160.0

ϵ_{Ri}	$\epsilon_{\varphi i}$	σ_{Ri}	$\sigma_{\varphi i}$	$\bar{\sigma}$	n
-0.000376	0.000716	-6.0	166.0	172.1	1.74

Table 1.

Data for the clock spring layers (1 and 2 identify the axial and tangential directions):
$E_1 = 1 \cdot 10^5$ MPa, $E_2 = 2,5 \cdot 10^4$ MPa, $v_{12} = 0,44$, $v_{21} = 0,11$, $G = 7,5 \cdot 10^3$MPa.
The maximum number of layers were 6 and the thickness of one layer was 1.0 mm.

The computational results are presented in Table 2:

No of layers	σ_{Ri}	$\sigma_{\varphi i}$	$\bar{\sigma}$	n
1	-6.0	157.0	157.0	1.91
2	-6.0	138.0	144.0	2.08
3	-6.0	128.0	134.0	2.25
4	-6.0	119.0	125.0	2.41
5	-6.0	111.0	117.0	2.57
6	-6.0	104.0	110.0	2.73

Table 2.

According to the expectations the more layers we have, the higher the safety factor is. The functional connection is linear.

5. Experiments, measurements and validation of the computation strategy

The authors had a chance to make experiments in lab circumstances with the so called cross-bonded plies of thickness 0.75 mm. The quality, diameter and density of the

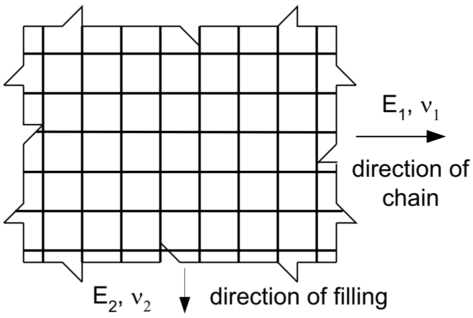

Figure 3. Composite layer reinforced in two direction

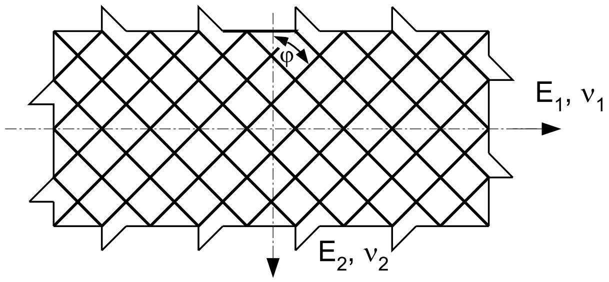

Figure 4. Composite layer reinforced in direction 45^o

fibers were equal both in the chain direction and in the filling direction along a ply as is shown in Figure 3. Previously, by using flat samples with 2, 3 and 4 reinforced layers, tests were carried out for the determination of the unidirectional tension strength applying the load in chain direction, then in filler direction and finally in a 45 degree oblique direction – see Figure 4. It is easy to see that in these cases the samples should behave like an isotropic material. The measurements are in good agreement with this expectation since the data measured are nearly equal to each other – see Table 3. For the practical determination of the volume – pressure characteristic and the elastic limit and bursting pressure an experimental investigation was carried out by using a compressor testing set at the Department of Mechanical Technology, University of Miskolc. The facility is characterized by a maximum pressure limit of 40 MPa and a maximum volume change of 2500 cm^3 per one stroke of the piston. Both an original tube and a six layer reinforced tube were measured. The length of the tube

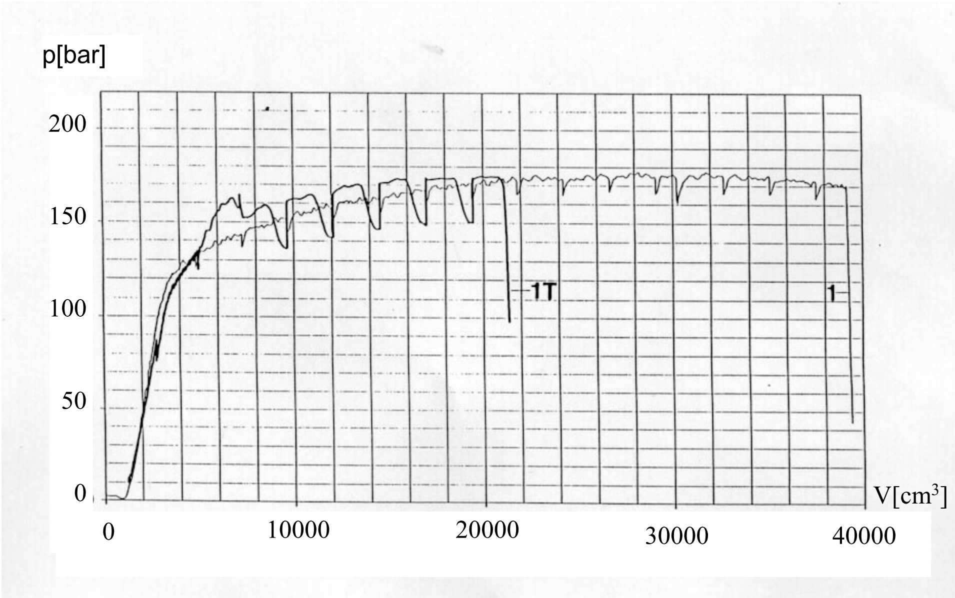

Figure 5. Pressure against volume for steel and a reinforced tube (curves 1 and 1T)

Direction of force	Strength in tension [MPa]		
	Layers		
	2	3	4
chain	238	241	249
filling	254	245	251
oblique	237	241	247

Table 3.

sample was 2115 mm, the external diameter 325 mm, and the thickness of the steel pipe was 5,5 mm. The normal operational pressure of this pipe is 6.5 MPa. The material parameters of the steel were $E_{steel} = 2.0 \cdot 10^5$ MPa, $v_{steel} = 0.33$, and $\sigma_{allowable} = 360\,\mathrm{MPa}$, while the parameters of the composite layers were $E = E_1 = E_2 = 5 \cdot 10^4 MPa$, $v = v_1 = v_2 = 0.44$, $G = 7.5 \cdot 10^3 MPa$, the number of layers was 6 and the thickness of one layer was 0.75 mm.

As can be seen in Figure 5. the pressure increase was continued to the total burst of the specimens. This value for the homogeneous pipe (curve 1) is 178 bar and for the heterogeneous pipe reinforced by six composite layers (curve 1T) is 175 bar. They are practically the same. The elastic limit for curve 1 is 134 bar. On the other hand the elastic limit is 163 bar for curve 1T, which clearly shows that there is an increase of 21.6%.

Numerical calculations for a maximum of six plies have been also performed with the results shown in Tables 4 and 5.

Tube made only of steel

ε_{Re}	$\varepsilon_{\varphi e}$	σ_{Re}	$\sigma_{\varphi e}$
-0.000380	0.000760	0.	180

ε_{Ri}	$\varepsilon_{\varphi i}$	σ_{Ri}	$\sigma_{\varphi i}$	σ	n
-0.000420	0.000801	-6.4	186	192.3	1.87

Table 4.

Number of layers	σ_{Ri}	$\sigma_{\varphi i}$	σ	n
1	−6.4	180	186	1.93
2	−6.4	174	180	2.00
3	−6.4	168	175	2.06
4	−6.4	163	170	2.12
5	−6.4	159	165	2.18
6	−6.4	154	161	2.24

Table 5.

According to the computation the improvement in the safety factor for the case of six layers is 19.7%, which is very close to the previous value.

6. Concluding remarks

Both the analytical and experimental investigations and results proved the applicability of the clock spring technique by which the safety factor of high pressure transporting steel pipes can be improved. In the algorithm an arbitrary number of layers and arbitrary fiber orientation can be taken into consideration, however, up to the moment only orthotropic and transversely isotropic problems have been solved. Making use of the computational procedure we have developed the necessary number of the composite layers for given geometrical and material data can also be determined. As far as the value of the pressure p^* is known from the analysis, a future task could be the determination of the strains and stresses in the composite plies. On the other hand, taking the non-linear behavior of the steel into consideration (for example by assuming an elastic-ideally plastic constitutive law) the bursting pressure could analytically be determined and the results could also be compared with experimental data. This work is in progress.

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