Journal of Computational and Applied Mechanics, Vol. 1., No. 2., (2000) pp. 137–147

INSTABILITY DUE TO INTERNAL DAMPING OF SYMMETRICAL ROTOR-BEARING SYSTEMS

László Forrai
Department of Mechanics, University of Miskolc
3515 Miskolc – Egyetemváros, Hungary
mechfl@gold.uni-miskolc.hu

[Received: April 30, 2000]

Abstract. This paper deals with the stability analysis of self-excited bending vibrations of linear symmetrical rotor-bearing systems with internal damping using the finite element method. The rotor system consists of uniform circular Rayleigh shafts with both internal viscous and hysteretic damping, symmetric rigid disks, and discrete isotropic damped bearings. The effect of rotatory inertia and gyroscopic moment are also included in the mathematical model. By combining the sensitivity analysis and the eigenvalue problem of the rotor dynamics equations presented in complex form, it is proved theoretically that the whirling motion of the rotor system becomes unstable at all speeds beyond the threshold speed of instability. Furthermore, it is found that the rotor stability is improved by increasing the damping provided by the bearings, whereas increasing internal hysteretic damping will result in a reduction in the threshold speed of instability. It is also shown that the corresponding whirling speed of the rotor is always higher than the first forward bending critical speed. Numerical examples are given to confirm the validity of the theoretical results.

Keywords: rotating machinery, stability, hysteretic damping, sensitivity analysis, threshold speed, finite elements

1. Introduction

Many authors have discussed the stability problems of rotors with internal damping [1-10]. The author of this paper investigated the stability of symmetric rotor-bearing systems using the finite element method in the work [13], in which only the viscous internal damping was incorporated into the mathematical model of the rotor system.

This paper generalizes the main results of the work [13] for similar symmetric rotor systems by using a more realistic model of internal damping, where both internal viscous and hysteretic damping are included into the finite element model of the rotor. By using the sensitivity analysis and the matrix representation of the rotor dynamics equations in complex form to evaulate stability, it is also proved theoretically that the whirling motion of the rotor-bearing system becomes unstable at all speeds above the threshold speed of instability. It is also shown that the rotor stability is improved by increasing the viscous damping provided by the isotropic bearings, whereas increasing internal hysteretic damping always destabilizes the rotor system. Furthermore, it is also found that corresponding whirling speed (frequency) is higher than the first forward bending critical speed.

Numerical examples are given to show the validity of the theoretical results of the present work. The threshold speed and the whirling speed of the rotor model are calculated using a computer program written in real form, which utilizes a standard QR-algorithm and an iterative technique developed by the author [10].

2. Equations of motion in complex form

2.1. Preliminaries and notations. For completeness and to make the paper self-explanatory, in what follows we repeat the main steps necessary for both the modelling and the stability investigation of the rotor system. In this section, the equations of motion for a rigid disk, finite shaft element with both internal viscous and hysteretic damping, isotropic damped bearing, and the complete rotor system are written in complex form by making use of a recent note by Nelson [11] and the paper by Zorzi and Nelson [8]. Note that the equation of motion for the shaft element in complex form [11] does not contain internal damping, whereas the effects of both viscous and hysteretic internal damping are included into the finite element model in the work by Zorzi and Nelson [8].

Consider a symmetric rotor system as shown in Figure 1. The rotor system consists of symmetrical rigid disks with negligible thicknesses, uniform circular Rayleigh shafts with viscous internal damping, and n isotropic damped bearings with stiffnesses k_i and damping coefficients $c_i\,(i = 1, 2, ..., n)$. The rotor is balanced, and rotates at a constant speed $\Omega(\Omega > 0)$. The reference system $Oxyz$ is fixed in space with the horizontal x-axis coinciding with the undeformed rotor centerline. The external damping, axial load and gravity are neglected.

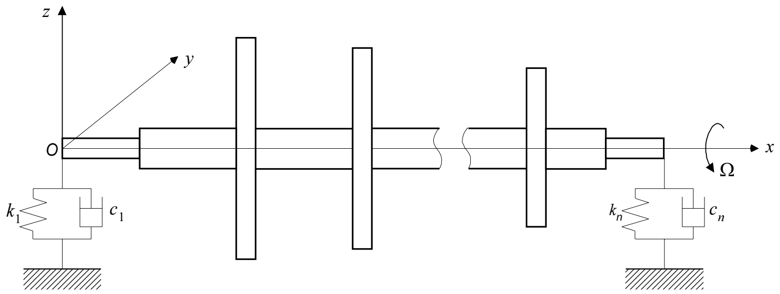

Figure 1. Symmetric rotor in isotropic damped bearings

Any node i of the rotor system has four degrees of freedom: two translations (v_i, w_i) in the (y, z) directions, and two rotations $(\varphi_{yi}, \varphi_{zi})$ about the (y, z) axes, respectively. The complex displacement vector of the ith node is defined by complex coordinates [11] as

$$\mathbf{p}_i = \begin{bmatrix} r_i \\ \varphi_i \end{bmatrix} = \begin{bmatrix} v_i + iw_i \\ \varphi_{yi} + i\varphi_{zi} \end{bmatrix}, \qquad i = \sqrt{-1}\,. \tag{2.1}$$

The component equations and the system equation in complex form may be written as presented below.

2.2. Rigid disk. The equation of motion for a rigid disk in complex form is given by

$$\left(\mathbf{M}_t^d + \mathbf{M}_r^d\right)\ddot{\mathbf{p}}^d - \Omega\mathbf{G}^d\dot{\mathbf{p}}^d = \mathbf{F}^d\ , \tag{2.2}$$

where $\mathbf{p}^d$ is the complex displacement vector corresponding to the four degrees of freedom $\left(v^d, w^d, \varphi_y^d, \varphi_z^d\right)$ of the node at which the disk is attached. The translational and rotational mass matrices $\left(\mathbf{M}_t^d, \mathbf{M}_r^d\right)$, and the gyroscopic matrix $\mathbf{G}^d$ are defined as

$$\mathbf{M}_t^d = \begin{bmatrix} m^d & 0 \\ 0 & 0 \end{bmatrix}, \tag{2.3}$$

$$\mathbf{M}_r^d = \begin{bmatrix} 0 & 0 \\ 0 & J_D \end{bmatrix}, \tag{2.4}$$

$$\mathbf{G}^d = \begin{bmatrix} 0 & 0 \\ 0 & iJ_P \end{bmatrix}, \tag{2.5}$$

where m^d , J_D and J_P are the mass, the diametral and polar moments of inertia of the disk, respectively.

2.3 Finite shaft element. By making use of the Lagrangian equations of motion for the damped finite element presented in real form in the work by Zorzi and Nelson [8], the equations of motion for the finite rotating shaft element with both hysteretic and viscous forms of internal damping can be rewritten in complex form as

$$\left(\mathbf{M}_t^e + \mathbf{M}_r^e\right)\ddot{\mathbf{p}}^e + \left(\eta_V\mathbf{K}_b^e - \Omega\mathbf{G}^e\right)\dot{\mathbf{p}}^e + \left[\frac{1+\eta_H}{\sqrt{1+\eta_H^2}}\mathbf{K}_b^e + \left(\eta_V\Omega + \frac{\eta_H}{\sqrt{1+\eta_H^2}}\right)\mathbf{K}_c^e\right]\mathbf{p}^e = \mathbf{F}^e\ , \tag{2.6}$$

where

$$\mathbf{p}^e = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_j \end{bmatrix} \tag{2.7}$$

is the $(4x1)$ complex nodal displacement vector of the shaft element with nodes i and j, η_V is the internal viscous damping coefficient, η_H is the hysteretic damping loss factor,

$$\mathbf{K}_c^e = -i\mathbf{K}_b^e \tag{2.8}$$

is the complex circulation matrix of the shaft element.

The translational and rotational mass matrices $(\mathbf{M}_t^e, \mathbf{M}_r^e)$, the gyroscopic matrix $\mathbf{G}^e$, and the bending stiffness matrix $\mathbf{K}_b^e$ of the shaft element are defined as

$$\mathbf{M}_t^e = \frac{\mu l}{420}\begin{bmatrix} 156 & & Hermitian & \\ i22l & 4l^2 & & \\ 54 & -i13l & 156 & \\ -i13l & -3l^2 & -i22l & 4l^2 \end{bmatrix}, \quad (2.9)$$

$$\mathbf{M}_r^e = \frac{\mu r^2}{120l}\begin{bmatrix} 36 & & Hermitian & \\ i3l & 4l^2 & & \\ -36 & -i3l & 36 & \\ i3l & -l^2 & -i3l & 4l^2 \end{bmatrix}, \quad (2.10)$$

$$\mathbf{G}^e = i2\mathbf{M}_r^e\,, \quad (2.11)$$

$$\mathbf{K}_b^e = \frac{EI}{l^3}\begin{bmatrix} 12 & & Hermitian & \\ i6l & 4l^2 & & \\ -12 & i6l & 12 & \\ i6l & 2l^2 & -i6l & 4l^2 \end{bmatrix}, \quad (2.12)$$

where μ is the mass per unit length, l is the length of shaft element, r is the element radius, EI is the bending rigidity of the shaft element. Since, in practice, $\eta_H^2 \ll 1$, we may write equation (2.6) in an approximate form as

$$(\mathbf{M}_t^e + \mathbf{M}_r^e)\,\ddot{\mathbf{p}}^e + (\eta_V \mathbf{K}_b^e - \Omega\mathbf{G}^e)\,\dot{\mathbf{p}}^e + [(1+\eta_H)\mathbf{K}_b^e + (\eta_V\Omega + \eta_H)\mathbf{K}_c^e]\mathbf{p}^e = \mathbf{F}^e\,. \quad (2.13)$$

Note that the above equation is an extended form of equation (2.6) presented in the work [13].

2.4. Linear isotropic damped bearings. The linear isotropic damped bearings can be modeled by the equation:

$$\mathbf{C}^b\dot{\mathbf{p}}^b + \mathbf{K}^b\mathbf{p}^b = \mathbf{F}^b\,, \quad (2.14)$$

where $\mathbf{p}^b$ is the complex displacement vector at the bearing location (node), $\mathbf{F}^b$ is the complex bearing force vector. The damping and stiffness matrices $\left(\mathbf{C}^b, \mathbf{K}^b\right)$ of the isotropic bearings are defined as

$$\mathbf{C}^b = \begin{bmatrix} c^b & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathbf{K}^b = \begin{bmatrix} k^b & 0 \\ 0 & 0 \end{bmatrix}, \quad (2.15)$$

where c^b and k^b are the direct damping and stiffness coefficients for the translational displacements, respectively.

2.5. System equations. The equations of motion of the complete rotor-bearing system can be obtained by assembling all component equations of the form equations (2.2), (2.13) and (2.14). The resulting equation is of the form

$$\mathbf{M}\ddot{\mathbf{p}} + (\eta_V\mathbf{K}_b + \mathbf{C} - \Omega\mathbf{G})\dot{\mathbf{p}} + [1 + \eta_H - i(\eta_V\Omega + \eta_H)]\mathbf{K}_b + \mathbf{K}]\mathbf{p} = 0\,, \quad (2.16)$$

where

$$\mathbf{p}^T = \left[\mathbf{p}_1^T\mathbf{p}_2^T...\mathbf{p}_N^T\right] \quad (2.17)$$

is the $(2\,Nx1)$ complex nodal displacement vector of the rotor system (N equals the number of nodes), the letter “T“ denotes the transpose.

2.6. Positive definite matrices. Since kinetic energy and strain energy cannot be negative, the system matrices $(\mathbf{M}, \mathbf{K}_b)$ are positive definite Hermitian matrices [9]. Thus the following relations hold:

$$\bar{\mathbf{p}}^T \mathbf{M} \mathbf{p} > 0 \,, \qquad \bar{\mathbf{p}}^T \mathbf{K}_b \mathbf{p} > 0 \,, \qquad (\mathbf{p} \neq 0) \,, \tag{2.18}$$

where the bar denotes the complex conjugate operator.

Note that the system gyroscopic matrix $\mathbf{G}$ is not Hermitian, however, by using the definitions of the component gyroscopic matrices presented by equations (2.5) and (2.11) it, can be expressed as

$$\mathbf{G} = i\mathbf{M}_g \,, \tag{2.19}$$

where

$$\bar{\mathbf{p}}^T \mathbf{M}_g \mathbf{p} > 0, \quad (\mathbf{p} \neq 0) \,. \tag{2.20}$$

Evidently $\mathbf{C}$ and $\mathbf{K}$ are positive definite diagonal matrices, the nonzero elements of which are the damping coefficients and the stiffnesses of the isotropic bearings, respectively.

3. Stability analysis

3.1. Stability threshold speed determination On seeking a solution to equation (2.16) of the form

$$\mathbf{p} = \mathbf{P} e^{\lambda t} \,, \tag{3.1}$$

we obtain the eigenvalue problem

$$\{\lambda^2 \mathbf{M} + \lambda\,(\eta_V \mathbf{K}_b + \mathbf{C} - \Omega \mathbf{G}) + [1 + \eta_H - i(\eta_V \Omega + \eta_H)]\mathbf{K}_b + \mathbf{K}\}\mathbf{P} = 0 \tag{3.2}$$

with $4N$ eigenvalues λ_j and the corresponding eigenvectors $\mathbf{P}_j$ $(j = 1, 2, ..., 4N)$. The eigenvalues λ are of the form

$$\lambda = \alpha + i\omega \,, \tag{3.3}$$

where α is the damping coefficient or decay rate, ω is the damped natural frequency or whirl speed.

For later use, the eigenvalue problem will be given in a modified form. To this end, we premultiply equation (3.2) by the complex conjugate eigenvector $\bar{\mathbf{P}}^T$. Then we obtain the following scalar equation:

$$\bar{\mathbf{P}}^T \{\lambda^2 \mathbf{M} + \lambda(\eta_V \mathbf{K}_b + \mathbf{C} - \Omega \mathbf{G}) + [1 + \eta_H - i(\eta_V \Omega + \eta_H)]\mathbf{K}_b + \mathbf{K}\}\mathbf{P} = 0 \,, \tag{3.4}$$

which can be rewritten as

$$m\lambda^2 + (\eta k_b + c - ig\Omega)\lambda + [1 + \eta_H - i(\eta_V \Omega + \eta_H)]\, k_b + k = 0 \,, \tag{3.5}$$

where the scalars m, k_b, c, g and k_B are in all positive real quantities [9] defined by

$$\bar{\mathbf{P}}^T\mathbf{M}\mathbf{P} = m > 0\,, \tag{3.6}$$
$$\bar{\mathbf{P}}^T\mathbf{K}_b\mathbf{P} = k_b > 0\,, \tag{3.7}$$
$$\bar{\mathbf{P}}^T\mathbf{C}\mathbf{P} = c > 0\,, \tag{3.8}$$
$$\bar{\mathbf{P}}^T\mathbf{G}\mathbf{P} = ig(g > 0)\,, \tag{3.9}$$
$$\bar{\mathbf{P}}^T\mathbf{K}\mathbf{P} = k > 0\,. \tag{3.10}$$

Note that the inequalities (3.6) - (3.10) hold on account of the positive definite matrices of the rotor system (see Section 2.5.).

Instability occurs if one of the eigenvalues has a positive real part. Thus, the problem of determining the limit of stability of the rotor system is reduced to finding the shaft speed Ω_s (threshold speed of instability), at which the greatest real part of all eigenvalues λ_j equals zero. The corresponding imaginary part ω_s is the *whirling speed.*

For the possible limit ω , the substitution of the eigenvalue of the form

$$\lambda = i\omega \tag{3.11}$$

into equation (3.5) yields

$$-m\omega^2 + g\omega\Omega + (1+\eta_H)k_b + k + i\left[\omega(\eta_V k_b + c) - (\eta_V\Omega + \eta_H)k_b\right] = 0\,. \tag{3.12}$$

After seperating equation (3.12) into real and imaginary parts, we obtain

$$-m\omega^2 + g\omega\Omega + (1+\eta_H)k_b + k = 0\,, \tag{3.13}$$
$$\omega(\eta_V k_b + c) = (\eta_V\Omega + \eta_H)k_b\,. \tag{3.14}$$

It is clear from equation (3.14) and inequalities (3.7) and (3.8) that

$$\Omega = \omega\left(1 + \frac{c}{\eta_V k_b}\right) - \frac{\eta_H}{\eta_V} > 0(\omega > 0)\,. \tag{3.15}$$

Thus the particular undamped whirl mode induced at the stability threshold speed is forward ($\omega > 0$) and asynchronous ($\Omega \neq \omega$) It is noteworthy that all backward precessional modes of the rotor are stable for any rotational speed.

Now we shall prove that the rotor loses its stability at all speeds above the possible stability limit. Here, we apply the eigenvalue sensitivity analysis. Let us suppose that the shaft speed Ω is an independent parameter, and differentiate equation (3.5) with respect to Ω :

$$\begin{aligned}&\lambda'(2m\lambda + \eta_V k_b + c - ig\Omega) - ig\lambda - i\eta_V k_b + m'\lambda^2 + \\ &+(\eta_V k_b' + c' - ig'\Omega)\lambda + [1 + \eta_H - i(\eta_V\Omega + \eta_H)]k_b' + k' = 0\,.\end{aligned} \tag{3.16}$$

where primes denote differentiation with respect to Ω . The quantity $\lambda' = \partial\lambda/\partial\Omega$ is referred to as an eigenvalue sensitivity coefficient [12], which can be written, with the aid of equation (3.3), in the form:

$$\frac{\partial\lambda}{\partial\Omega} = \frac{\partial\alpha}{\partial\Omega} + i\frac{\partial\omega}{\partial\Omega}\,. \tag{3.17}$$

To calculate $\partial\lambda/\partial\Omega$ from equation (3.16) at the possible limit Ω , we substitute again equation (3.11) into equation (3.16):

$$\frac{\partial\lambda}{\partial\Omega}[\eta k_b + c + i(2m\omega - g\Omega)] + g\omega - i\eta k_b + \underline{(-m'\omega^2 + g'\omega\Omega + (1+\eta_H)k_b' + k')} \\ \underline{+i[\omega(\eta_V k_b' + c') - (\eta_V\Omega + \eta_V)k_b']} = 0. \quad . \tag{3.18}$$

Since the eigenvalue derivative $\partial\lambda/\partial\Omega$ represents the unique solution of equation (3.18) at the possible limit of stability, and hence its value is not influenced by any normalization criterion for the eigenvector $\mathbf{P}$, therefore the underlined terms will vanish:

$$-m'\omega^2 + \omega\Omega g' + (1+\eta_H)k_b' + k' = 0, \tag{3.19}$$

$$\omega(\eta_V k_b' + c') = (\eta_V\Omega + \eta_V)k_b' \, . \tag{3.20}$$

We then obtain the following expression for the damping sensitivity coefficient $\partial\alpha/\partial\Omega$

$$\frac{\partial\alpha}{\partial\Omega} = \frac{A}{(\eta_V k_b + c)^2 + (2m\omega - g\Omega)^2} \, , \tag{3.21}$$

where

$$A = [\eta_V(2m\omega - g\Omega) - g(\eta_V\Omega + \eta_H)]k_b \, . \tag{3.22}$$

Now let us show that A is positive. To this end we substitute (3.15) into equation (3.22):

$$A = k_b\eta_V\{2\omega\left[m - g\left(1 + \frac{c}{\eta_V k_b}\right)\right] + g\frac{\eta_H}{\eta_V}\} \, . \tag{3.23}$$

The bracketed term in equation (3.23) is positive:

$$m - g\left(1 + \frac{c}{\eta_V k_b}\right) > 0 \, , \tag{3.24}$$

since in this case equation (3.13) has only one positive root ω . The latter statement comes from the following equivalent form of equation (3.13):

$$\omega^2\left[m - g\left(1 + \frac{c}{\eta_V k_b}\right)\right] + \omega g\frac{\eta_H}{\eta_V} - [(1+\eta_H)k_b + k]\} = 0 \, . \tag{3.25}$$

We have thus proved that the damping sensitivity coefficient $\partial\alpha/\partial\Omega$ is positive at any possible limit of stability. Thus, the lowest value of the above stability limits for the particular forward whirl modes is considered as the *threshold speed of instability* of the rotor-bearing system. Consequently, the whirling motion of the rotor becomes unstable at all speeds above the threshold speed of instability.

3.2. Effect of bearing damping on rotor stability Now we shall prove that an increase in the bearing damping coefficients results in an increase in the threshold speed of instability, thus the rotor stability will be improved.

Let us consider the bearing damping coefficient $c_i (i = 1, 2, ..., n)$ of the i-th isotropic damped bearing as an independent parameter, and differentiate equations (3.13) and

(3.14) with respect to c_i :

$$\omega'(g\Omega - 2m\omega) + \omega g\Omega' + \underline{(-m'\omega^2 + \omega\Omega g' + (1+\eta_H)k_b' + k')} = 0 \ , \tag{3.26}$$

$$\omega'(\eta_V k_b + c) - \eta_V k_b \Omega' + \underline{\omega(\eta_V k_b' + \tilde{c}) - (\eta_V\Omega + \eta_H)k_b'} = -\omega c^* \ , \tag{3.27}$$

where primes denote differentiation with respect to c_i ,

$$\tilde{c} = \frac{\partial \bar{\mathbf{P}}^T}{\partial c_i}\mathbf{CP} + \bar{\mathbf{P}}^T\mathbf{C}\frac{\partial \mathbf{P}}{\partial c_i} \ , \tag{3.28}$$

and

$$c^* = \bar{\mathbf{P}}^T \frac{\partial \mathbf{C}}{\partial c_i}\mathbf{P} > 0 \ . \tag{3.29}$$

By using the same reasoning that we have applied in connection with equation (3.18), it is clear that the underlined terms in equations (3.26) and (3.27) will vanish at the threshold speed Ω . The whirling speed sensitivity coefficient ω' and the threshold speed sensitivity coefficient Ω' can now be obtained from the above two equations as

$$\frac{d\omega}{dc_i} = \frac{gc^*\omega^2}{A} \ , \tag{3.30}$$

$$\frac{d\Omega}{dc_i} = \frac{\omega(2m\omega - g\Omega)c^*}{A} \ . \tag{3.31}$$

By using equations (3.22) and (3.29) as well as the inequality (3.9), it is easy to see that the above sensitivity coefficients are positive. Thus, the addition of bearing damping improves the rotor stability. It is also clear that the whirling speed is always greater than the first forward bending critical speed of the rotor system. The latter statement follows from the fact that the threshold speed of symmetrical rotors with viscous internal damping, supported by undamped isotropic bearing coincides with the first forward critical speed [9]. It can further be concluded from equation (3.30) that when the gyroscopic moments of the rotor are neglected ($g = 0$), then the whirling speed remains constant (the first critical speed of the rotor) regardless of the magnitude of the bearing damping coefficients.

3.3. Influence of internal hysteretic damping on rotor stability We shall now prove that internal hysteretic damping is always a destabilizing influence on rotor systems. Let us assume that η_H is an independent system parameter. By differentiatiating equations (3.13) and (3.14) with respect to η_H, we get

$$\omega'(-2m\omega + g\Omega) + \omega g\Omega' \underline{- m'\omega^2 + g'\omega\Omega + (1+\eta_H)k_b' + k'} = -k_b \ , \tag{3.32}$$

$$\omega'(\eta_V k_b + c) - \eta_V k_b\Omega' + \underline{\omega(\eta_V k_b' + c') - (\eta_V\Omega + \eta_H)k_b'} = k_b \ , \tag{3.33}$$

where prime denotes differentiation with respect to η_H . Since the underlined expressions vanish at the stability threshold, the whirling speed sensitivity coefficient ω' and threshold speed sensitivity coefficient Ω' are determined by

$$\frac{d\omega}{d\eta_H} = \frac{k_b\eta_V - g\omega}{A}k_b \ , \qquad \frac{d\Omega}{d\eta_H} = \frac{-(2m\omega - g\Omega) + (\eta_V k_b + c)}{A}k_b \ . \tag{3.34}$$

By using inequalities (3.6) - (3.10), and equations (3.13) and (3.14), simple reasoning will show that the whirling speed sensitivity coefficient is positive, whereas the threshold speed sensitivity coefficient becomes negative at the stability threshold speed. As can be seen, the introduction of internal hysteretic damping η_H causes a reduction in the threshold speed of instability, thus hysteretic damping has always a destabilizing influence on rotor systems. It is noteworthy that the corresponding whirling frequency will be raised. From equation (3.30) and the work by the author [9] it clearly follows that the latter is always higher than the first forward bending critical speed.

4. Numerical examples

4.1. To demonstrate the validity of the above theoretical results, two numerical examples are provided. In both examples, the simply supported uniform shaft studied by Zorzi [8] is considered. The rotor model consists of a 10.16 cm diameter and 127 cm long steel shaft supported by two identical isotropic damped bearings at both ends. The stiffnesses of the bearings are: $k_1 = 1.75 \times 10^{11} N/mT$. The material properties of shaft are: Young's modulus $E = 2.06 \times 10^{11} N/m^2$, and density $\rho = 7800\, kg/m^3$. The rotor is modeled as an assembly of four finite elements of equal length. In the calculations, the damping coefficients c_1 of the bearings and the hysteretic damping loss factor η_H for the shaft are considered to be parameters.

4.2. As a first example, we shall examine the influence of the damping coefficient c_1 on the rotor stability for $\eta_V = 0.0002\, s$.and $\eta_H = 0.0002$. Table 1 shows the numerical

Table 1. Effect of bearing damping (c_1) on rotor stability

bearing damping (Ns/m)	threshold speed (rad/s)	whirling speed (rad/s)
0	520.410	521.412
100	543.115	521.417
200	565.835	521.430
300	588.563	521.450
400	611.315	521.478
500	634.083	521.513

values of the threshold speeds Ω_s and the whirling speeds ω_s of the rotor for different values of c_1. The first forward critical speed of the rotor was found to be $\Omega_{F1} = 521.392\, rad/s$.

As can be seen from Table 1, the introduction of bearing damping will raise the threshold speed of instability, thus the stability of the rotor system will be improved. It should be noted that increasing the bearing damping may cause only small increases in the whirling speeds, which are greater than the first forward bending critical speed of the rotor. Clearly the numerical results of Table 1 are in quite good agreement with the theoretical results obtained in Section 3.2.

4.3. As a second example, we consider the influence of the hysteretic damping loss factor η_H on the rotor stability for $c_1 = 500\,Ns/m$ and $\eta_V = 0.0002$. Table 2 presents the calculated values of the threshold speeds and whirling speeds for different values

Table 2. Effect of hysteretic damping (η_H) on rotor stability

hysteretic loss factor	threshold speed (rad/s)	whirling speed (rad/s)
0.0002	634.083	521.513
0.0003	633.622	521.523
0.0004	633.152	521.533
0.0005	632.691	521.544

of η_H . The numerical results show clearly that the stability of the rotor is reduced by increasing internal hysteretic damping. For example, for the hysteretic damping loss factor of $\eta_H = 0.0002$ the rotor becomes unstable at the threshold speed $\Omega_s = 634.083\,rad/s$. By increasing the hysteretic loss factor to $\eta_H = 0.0003$, the stability threshold speed of instability of the rotor will be reduced to $\Omega_s = 633.622\,rad/s$. It should be noted that increasing hysteretic damping causes only small increases in the whirling speed ω_s. Evidently the numerical results summarized in Table 2 are in good agreement with the theoretical results presented in Section 3.3.

5. Summary and conclusions

In this paper a finite element stability analysis of self-excited bending vibrations of symmetric rotors with both internal viscous and hysteretic damping, supported by isotropic damped bearings has been presented. By combining the sensitivity method and the eigenvalue problem of the rotor dynamics equations in complex form, it is proved theoretically that the whirling motion of the rotor becomes unstable at all speeds above the stability threshold speed. Furthermore, the rotor stability is improved by increasing the damping provided by the bearings, whereas internal hysteretic damping destabilizes the whirling motion of the rotor. It is also shown that the corresponding whirling speed of the rotor system is higher than the first forward bending critical speed. Numerical examples are provided to confirm the validity of the above theoretical results.

Acknowledgement. This research has been partially supported by the National Fund for Scientific Research of Hungary (OTKA) under grant T 030096.

REFERENCES

1. Kimball, A. L.: *Internal friction theory of shaft whirling*, General Electric Review, **27**, (1924), 244-251.
2. Newkirk, B.L.: *Shaft Whipping*, General Electric Review, 27 (1924), 169-178.
3. Smith, D.M.: *The motion of a rotor carried by a flexible shaft in flexible bearings*, Proceedings of the Royal Society of London, Series A, 142 (1933), 92-118.

4. Dimentberg, F.M.: *Flexural Vibrations of Rotating Shafts*, Butterworths, London 1961.

5. Ehrich, F.F.: *Shaft whirl induced by rotor internal damping*, ASME Journal of Applied Mechanics 31, (1964), 279-282.

6. Tondl, A.: *Some Problems of Rotor Dynamics*, Publishing House of the Czechoslovak Academy of Sciences, Prague 1965.

7. Gunter, E.J. and Trumpler, P.R.: *The influence of internal friction on the stability of high speed rotors with anisotropic supports*, ASME Journal of Engineering for Industry 91 (1969), 1105-1113.

8. Zorzi, E.S. and Nelson, H.D.: *Finite element simulation of rotor-bearing systems with internal damping*, ASME Journal of Engineering for Power 99 (1977) 71-76.

9. Forrai, L.: *Stability analysis of symmetrical rotor-bearing systems with internal damping using finite element method*, Presented at the International Gas Turbine and Aeroengine Congress and Exhibition, Birmingham, UK (1996), ASME Paper No. 96-GT-407.

10. Forrai, L.: *Stability of rotor-bearing systems with internal damping using finite elements*, Proc. 9th World Congress on the Theory of Machines and Mechanisms, Milano, Italy (1995), 1204-1208.

11. Nelson, H.D.: *Rotor dynamics equations in complex form*, ASME Journal of Vibration, Acoustics, Stress and Reliability in Design 107, (1985), 460-461.

12. Rajan, M., Nelson, H. D. and Chen, W. J.: *Parameter sensitivity in the dynamics of rotor-bearing systems*, Journal of Vibration, Acoustics, Stress, and Reliability in Design, **108**, (1986), 197-206.

13. Forrai, L.: *A finite element model for stability analysis of symmetrical rotor systems with internal damping*, Journal of Computational and Applied Mechanics, **1**(1), (2000), 37-47.