

THROUGH THE EYES OF A FELLOW WORKER

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Abstract. The paper aims to present the most important results of Tibor Czibere.

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1. Years in the Works Ganz

I learned about the vivid professional public life of experienced colleagues as well as of promising young fellow workers at the Engineers' Club of the Ganz Locomotive and Wagon Manufacturers and Mechanical Engineers in the second half of the 1950s. *Ernő Trenka*, head of the Hydraulic Department - earlier the Design Division of Pumps and Turbine - attached me to the Pump Division, though my intention was to join the turbine designers. His aim was to introduce me into the pump design that had been taught in the school of professors *Pattantyús*¹ and *Gruber*². The lectures in the Club by assistant professor *Fűzy* offered a very good opportunity to ask him a lot of questions concerning the problems I had trouble in what today is called numerical computational methods. But before he was able to respond another beginner gave surprisingly very competent and thoroughgoing answers to my questions. The name of the other young man was *Tibor Czibere*, who was working on the design of the blade rows of torque converters at another division of Ganz. Since the turbine blades of a converter were made of highly curved profiles, he necessarily had to deal with the cascades. In this way he must have gone over the task of designing hydrofoils for pump blades having straight camber lines. A short time after that we worked at desks opposite each other in the newly established Research Division of the Department. *Czibere's* task was the design of water turbine runners and mine involved pump impellers.

Czibere was able to exceed the old and still applied concept of solving engineering tasks on an empirical basis because he represented a different way of approach. *Kármán* had been a pioneer of the concept of using mathematics as the language of problem solving in engineering. Professor *Samu Borbély* founded a school in Miskolc which belonged to *Rothe's* famous school of applied mathematics in Berlin.

¹Géza Á. Pattantyús (1885-1956) Professor of Hydraulic Machines

²József Gruber (1915-1972) Professor of Fluid Mechanics

2. A nonlinear heat conduction problem

Before coming to the Ganz *Czibere* worked as an assistant of Professor *Borbély* for two years.

Metallurgy flourished in those years so the heat transfer problems arose related to the heating of blocks [1,2]. The non-linear partial differential equation (PDE) of heat conduction is as follows:

$$\nabla \cdot [k(\vartheta) \nabla(\vartheta)] = \rho(\vartheta) c(\vartheta) \frac{\partial \vartheta}{\partial t}$$

where the density ρ , the specific heat c , the heat conductivity k depend on the temperature ϑ and t is time. Without entering into details I refer only to the excellent technique of integral transformation by which the task of solving the PDE becomes a linear PDE of potential theory:

$$U(\mathbf{r}, t) = \int_0^{\vartheta(\mathbf{r}, t)} k(\lambda) d\lambda$$

where $U(\mathbf{r}, t)$ depends on the radius vector \mathbf{r} and the time t . Transforming the PDE we get the equation:

$$\Delta U = A(U) \frac{\partial U}{\partial t}, \quad \text{where} \quad A(U) = \frac{\rho c}{k}.$$

Applying *Green's* formulas one arrives at an integro-differential equation. The solution of a complicated problem should contain the solutions valid for its special case. As is well known, the solution of the heat conduction problem for physical parameters independent of the temperature can be given in terms of *Bessel* functions. *Czibere's* solution coincides with the solution valid for that special case.

3. Computational method for the design of a straight cascade of airfoils

The paper *Computational method for the design of straight cascade of airfoils with highly curved profile blades* by *Czibere* was published in the *Acta Technica Academiae Scientiarum Hungaricae* in 1960 [3]. In this profound article a method of computation to determine a straight cascade with a prescribed deflection angle of flow was presented. In another approach the deflection means the lift force arising on a profile, or putting it another way again, the change of the energy content of the flow passing through the blades. The treatise was based on the determination of the velocity field induced by vortices and sources on the plane of complex numbers. The vortex and source-sink distributions along a curve mean tangential and normal velocity jumps across it. These distributions are the so-called hydrodynamic singularities. The integral of the vortices along the curve gives the circulation around the profile, i.e., the deflection and the source-sink distribution determine the shape of the foil. Because the task is to find the very shape of the foil, i.e., the carrier curve of the hydrodynamic singularities, the solution can only be obtained by iterative computations.

In the middle of 1950s *Scholtz* and *Schlichting* published their work on straight cascade but the camber line was supposed to be a straight line and the thickness of the profile was also restricted size. *Czibere* extended the method in his paper on 69 pages including not only the theory but the detailed algorithm of computations as well. The sequence of calculations was supported with appropriate tables applicable by anyone. Furthermore several elaborated examples showing blade shapes, velocity and pressure distributions proved the practical use of the method. The value and importance of the algorithm and the pattern of computation can really be appreciated if we remember the tools of calculation available forty years ago. In the age of mechanical calculators the numerical evaluation of improper integrals with acceptable accuracy was really of a high value. I remember when after having returned from a conference held in London in 1963, I was given a possibility to acquire a computing machine from abroad but the western company refused to sell it to Hungary. I scarcely believe that one could find anybody willing to undertake a task like that having such rudimentary tools nowadays.

Achieving such a theoretical result could justly be praised any time. But it should not be forgotten that we worked for an industrial company where the goal is always to serve customers with products, therefore a pure theoretical method can satisfy just a few people but not the company. Research activities must be determined accordingly.

In the year of publication, in 1960 the 10th Congress of International Applied Mechanics was held at Stresa, Italy, where *Czibere* presented a paper entitled *Iterative method for the determination of straight and radial cascades* [4]. *Theodore von Kármán* also attended the congress and noticed the young lecturer for two reasons. First the name *Czibere* had to be a Hungarian one, and secondly, the idea of vortex had made *Kármán's* name famous all over the world. The appreciation by *von Kármán* put *Czibere's* name onto the list of the world-famous engineers of the Ganz Works. There was a common saying: an engineer either belonged or belongs or will belong to the Ganz. Unfortunately this a saying of the past now.

In 1963 the Hungarian Academy of Sciences awarded *Czibere* the Ph. D. degree for the thesis in which he worked out a method for the determination of highly cambered straight cascade of foils [5,6]. His method was worth mentioning in the book *Vorlesungen über Theoretische Mechanik* by Professor *István Szabó* of the Technische Universität Charlottenburg, West-Berlin. Professor *Szabó* invited *Czibere* as visiting professor to deliver lectures about his method.

4. Two main tasks concerning the cascade of airfoils

The design method of straight cascades can only be applied to runners and impellers of axial flow machines having a constant meridional width. The blade channels of mixed-flow machines with a variable channel width needed the extension of the method. The functions of complex variables can only be applied to plane flow. The conformal mapping provides a possibility to establish the relation between the flow around straight and radial cascades. A cascade cut off a mixed flow impeller is also a two dimensional one but it is on a surface of revolution. One can obtain an integral transformation

between a plane flow and the flow on a surface of revolution having the same properties like that of a conformal mapping by solving an ordinary differential equation. Let us denote the arc length of the meridional curve of the surface of revolution by σ , the distance from the axis of rotation by $r(\sigma)$, and the angle of a point by φ . The angle and ratio preserving the transformation between the points on the surface and the points (σ, φ) on the plane of Descartes coordinates (x, y) are as follows:

$$x = K \left[-\frac{1}{2} + \frac{1}{a} \int_0^\sigma \frac{d\sigma}{r(\sigma)} \right], \quad y = K \frac{\varphi}{a},$$

where K and a are constants. The connection between the velocity components on the plane and on the surface F is:

$$c_{x,y} = \frac{a}{K} r(\sigma) c_{F\sigma,\varphi}.$$

This transformation brought a decisive change in the determination of the velocity field in a part channel with variable width b and density of fluid ρ of a mixed-flow machine.

The PDE for the velocity potential function ϕ on the plane is of the form

$$\Delta\phi = -\frac{\partial\phi}{\partial x} \frac{1}{\rho b} \frac{d(\rho b)}{dx}.$$

One has to write another equation between the velocity and the density of the flow. The solution of these equations relates not only to the hydraulic machines but to the compressors and turbines working with compressible fluid as well.

I must make a remark concerning the applicability of the method. While the theoretical work was being made by *Czibere*, I designed the first double suction pump with a mixed-flow impeller of the type DST.

There are two main tasks concerning the cascade of airfoils: 1.) to design the geometry of the foils apt to deflect the flow as required, 2.) to determine the velocity field around the given cascade of foils. In the first case the boundary curve is sought for on which the distributions of hydrodynamic singularities are prescribed and in the domain around the PDE has to be satisfied. The second task could be solved utilizing the potential theory directly, i.e., the PDE can be reduced to an integral equation. By solving the integral equation a potential density is obtained. Substituting the latter into the *Green* formula, the solution of the PDE, i.e., the velocity potential function will be determined. However, we do not need the velocity potential but its derivatives only. The velocity jump across the boundary is either prescribed or sought for. On the right hand side (RHS) of the PDE the through flow velocity component and the varying flow density are included. Because the PDE is *Poisson* like, which includes the unknown velocity distribution on the RHS, the solution can only be obtained by iterative computations. Consequently, the canonical way is not advantageous, instead, the extension of the method worked out for the straight cascade with constant width and density - for which the same PDE stands with zero RHS - offers a more suitable computational procedure. Without showing the details, I must make a remark that

in the early 1960s we could not know that the applied method would later be called as the boundary element method.

For the thesis that contained the solutions of the two main tasks of the hydrodynamic cascade theory the Hungarian Academy of Sciences awarded Czibere the degree Doctor of Technical Sciences in 1967, see [7,8] for details.

The application of the method in engineering practice proved to be fruitful in the hydraulic design of several Francis and Kaplan turbines. *Czibere* not only directed but also took part in the work.

I would also like to mention a detail from his career here. He was appointed Minister of Culture and Education in 1988. I think it was because of his scientific, industrial and academic activities and his successful work as the head of the University. Being a minister is first and foremost a political position, where – I think – the assertion of the professional's intention, his engineering expertise, integrity, scientific talents and personal excellence may have to yield priority to other issues.

5. Further investigations in incompressible and compressible flows

Czibere was chosen to be a corresponding member of the Hungarian Academy of Sciences (1976). His inaugural speech was *The determination of the boundary layer plane flow based on vortex discontinues* [9]. The basic idea is that a boundary layer around the surface of a solid body in the stream is simultaneously a vortex layer. The vortex density function is either discontinuous or has a pole on the surface. After having determined the vector potential of the vortex layer, an integral expression will serve to compute the velocity field around the body. The proper choice of the vector potential ensured that the computed theoretical results had good agreement with the experiments. This study proved the generally accepted concept that the flow can be dealt with as an ideal fluid apart from the immediate vicinity of the body first stated by *Prandtl*.

Czibere was promoted to be an ordinary member of the Hungarian Academy of Sciences in 1985. The theme of his inaugural lecture was quite different from the previous one and had the title *Shock waves in a supersonic gas ejector* [10]. The nature of the supersonic flow is quite different from the subsonic one. Shock waves always occur if the speed of the compressible fluid exceeds the speed of sound. This is the case when the gas is flowing around a bullet or an aeroplane and if the gas flows in a channel. The properties of the gas - like velocity, pressure, density, temperature, and entropy - passing over a shock wave suddenly change therefore the basic equations of motion are not applicable. These jumps take place in a narrow strip. Depending on the angle of crossing we speak about normal or oblique shock waves. Since entropy always increases, the shock wave can only be an expansion wave. Two fluids are mixed in a gas ejector flowing in the same direction but with a different speed: one below, the other above the speed of sound. The driving gas arrives from a *Laval* nozzle, which accelerates the gas over the speed of sound. First the possible flow patterns in the *Laval* nozzle are examined. Then mixing procedures are dealt with depending

on the pressure, density, mass ratios of the gases and the ratio of cross sections the gases flow through. This subject requires different mathematical tools as the tasks mentioned earlier. The process must be followed step by step. The practical goal of this study was to decrease the vacuum in order to avoid cavitation in the water ring vacuum pump coupled with the ejector. The comparison of the calculated and the measured pressure distributions proved the theoretical approach.

6. Research for the industry

The forces exerted on the rotor of a turbomachine are of hydrodynamic origin. Due to the variation of the eccentricity in circumferential direction between the staying and rotating sleeves of a multistage pump unsteady forces arise like in a slide bearing. The impellers are subjected to axial thrust because the area opposite the impeller eye is under less pressure than the pressure on its back shroud. The thrust can be balanced automatically by a rotating disk mounted onto the shaft. The axial gap between the rotating and staying disk also varies because of the flexible deformation of the shaft. This causes a non-symmetrical pressure distribution around the shaft which leads to a pulsing bending moment in the shaft. The study *Determination of the dynamic interacting between solid and fluid continua* suggests a more comprehensive method to compute the unsteady hydrodynamic forces and momenta. The special cases, e.g., the theory of lubrication of slide bearings can be deduced from the general method.

The next paper *Computation of the eigenfrequencies of multicomponent rotors loaded by hydrodynamic forces and momenta* employs the previous report. The conditions supposed in this paper are much less restrictive than in other investigations. The angle torsion of a cross section of the shaft revolving with constant angular velocity perpendicular to the axis of rotation is allowed. The rotating shaft will make precessional motion. After having determined the hydrodynamic and shearing forces and momenta, a fourth order PDE is set up for the neutral axis which will be deformed to a space curve. The function sought for is of a complex value with complex variables. The PDE can be reduced to an ordinary differential equation the eigenfunctions of which are complex functions depending on real variables. The method is for analyzing the bending vibration of a rotating shaft with finite length. The complex deformation and angular torsion of the center of gravity of a cross section and the bending moment as well as the shear forces can be computed. The rotor of a multistage machine can be divided to loaded and unloaded parts. All the cases which may occur to a shaft are treated: e.g. varying cross sections, rigid disk, clutch, fixed shaft end, flexibly supported bearing etc. Matrix equations describe these sections of the rod. These transfer matrices can be coupled in a rather simple way. The study ended by an example analyzing numerically the bending vibrations of a multistage boiler feed pump.

The computer simulation of the air and exhaust system including the boiler of the 200 MW block of the Mátra Power Plant was a new task. The operational parameters such as the temperature, pressure, etc. were to be determined for varying thermal loads of the boiler, different fuels and air inputs. The setting of the closing and

throttling valves and the unavoidable leaks and through flows were to be taken into consideration. The characteristics of the pulverizer fan, which is a special part of the system, had to be determined for different concentrations and grain sizes of the lignite. The balance equations and the continuity conditions at the nodal points resulted in a non-linear system of 45 equations. A great number of constant parameters of the block were also needed but their site measurements could not be carried out with acceptable accuracy or at all. The original design parameter values were not available. Had those been available, they would have been obsolete. After all the model of the system had to correspond with the measurable operational values. Finally the software developed had to be apt for the every day use by the staff running the block.

To determine the real characteristics of a turbomachine the viscous effects should be taken into account [11,12]. An extension of the cascade theory must involve frictions. A method was worked out for a cascade of foils bounded by two plane walls [13]. The effect of blading and friction on the flow were taken into consideration separately. The blade effect is represented hydrodynamically by a field of constraint forces which are determined by the change of the moment of momentum in the inviscid flow. The frictional effect on the fluid flowing through a cascade is taken into account based on the analogy between the channel flow and the flow in the bladed space. The results are the energy loss along the blades in the through flow direction and other quantities. Further steps aiming at the application to a mixed-flow channel made it possible to compute the real head-discharge characteristics of a pump for different flow rates and pre-whirl of the absolute flow. The calculated and measured characteristics showed a good agreement.

7. A heat transfer problem associated with phase change

One of the important courses of the Ph.D. programs at our University is thermodynamics for which Professor *Czibere* is responsible. Within this heat conduction is his favorite as he dealt with it in his younger age. The transport processes are treated in his book *Heat conduction* published in 1998 [14]. The global balance equations, the conservation laws, the main laws of thermodynamics and the similarity laws are dealt with in a much more comprehensive way than one could expect it upon the title of this book. The general PDE of heat conduction and its initial and boundary conditions are discussed including the heat transport in metals when phase change occurs. A great many steady and unsteady tasks in 1, 2, and 3 dimensions are solved on about 200 pages, several amongst them would be worth publishing as separate scientific papers. The mathematics applied in the book can be found in its appendix. Not only the Ph.D. students but researchers can use the book in their work as well.

8. A three dimensional stochastic turbulence model

The investigation of turbulence phenomena of flow has been the basic problem of fluid dynamics since the end of the 19th century, when the first concept was created by *Osborne Reynolds*. *Theodore von Kármán* stated in 1930 that the flow patterns at different points of the velocity field are mechanically similar in a fully developed

turbulent flow. His hypothesis was restricted only to two dimensional flow though the turbulence is always three dimensional. Recently *Czibere* extended the mechanical similarity to three dimensional flow. Details in connection with his investigations can be found in the first paper of the present issue. In spite of this I would like to present repeatedly the most important result of his paper in order to give a somewhat different interpretation to it and for the sake of completeness as well. According to *G. I. Taylor's* at that time (1935) quite new concept, the turbulent velocity fluctuation \mathbf{v}' is to be considered a random variable. \mathbf{v}' is the difference of the instantaneous velocity and the mean velocity \mathbf{v} . The assumptions were also made that the turbulence is homogenous and isotropic. These hypotheses cannot be maintained for a real turbulent flow today. The correlation tensors by which the phenomenon attempted to describe the turbulence led to more unknown variables than equations could be set up. Consequently, the determination of the *Reynolds* stress tensor \mathbf{F}_R could not be attained. There are tremendous models for turbulence but they are valid only in special particular cases. Attempts to obtain the *Reynolds* tensor assumed some connection between it and the strain rate tensor. As *Czibere's* theory proves it \mathbf{F}_R cannot be coupled with the strain rate tensor [15].

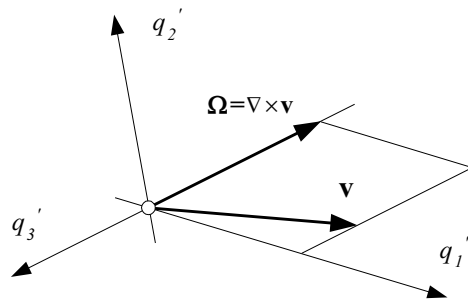


Figure 1.

Applying *Friedman's* theorem for conservation of vector lines to flow of fluids of constant density ρ , *Czibere's* theorem states, as analogy to *Helmholtz's* vortex theorem, that

$$\frac{\partial \Omega}{\partial t} + (\mathbf{v} \cdot \nabla) \Omega - (\Omega \cdot \nabla) \mathbf{v} = \nu \cdot \Delta \Omega + \nabla \times (\overline{\mathbf{v}' \times \Omega'}) ,$$

where $\Omega = \nabla \times \mathbf{v}$, $\Omega' = \nabla \times \mathbf{v}'$, ν is the kinematic viscosity coefficient, t is time and the time average is denoted by an overbar. The conclusion is that the vortex line will not persist either in fully viscous or in fully turbulent flow. The vortex theorem for the turbulent velocity fluctuation is:

$$\frac{\partial \Omega'}{\partial t} + (\mathbf{v}' \cdot \nabla) \Omega' - (\Omega' \cdot \nabla) \mathbf{v}' = (\Omega \cdot \nabla) \mathbf{v}' + \nu \Delta \Omega' .$$

It follows from this that the vortex diffusion occurs even in an inviscid fluid. One can conclude that the source of turbulence is the vorticity of the main flow Ω .

The velocity fluctuation vector can be expressed as the rotation of a vector potential function. After proper geometrical and physical similarity transformations, a PDE

for the dimensionless vector potential \mathbf{f} can be obtained. Applying the solution, the *Reynolds* stress tensor will have the following form:

$$\mathbf{F}_R = -\rho (\mathbf{v}' \circ \mathbf{v}') = -\rho \ell^2 |\Omega| \Omega (\overline{\nabla \times \mathbf{f} \circ \nabla \times \mathbf{f}}),$$

where ℓ is the scale factor of turbulence.

The representation of the stochastic process of the intrinsic mechanism of turbulence in the natural orthogonal coordinate system (q_1, q_2, q_3) with a coordinate plane spanned by the vectors $\Omega = \nabla \times \mathbf{v}$ and \mathbf{v} , - the direction of Ω being opposite to q_3 - looks like

$$\Omega = \frac{1}{H_1 H_2} \frac{\partial (v_1 H_1)}{\partial q_2}.$$

where H_i are the *Lamé* coefficients. The *Reynolds* stress tensor will have the form:

$$\mathbf{F}_R = \rho \kappa^2 \ell^2 \mathbf{H}_0 \left| \frac{1}{H_1 H_2} \frac{\partial (v_1 H_1)}{\partial q_2} \right| \frac{1}{H_1 H_2} \frac{\partial (v_1 H_1)}{\partial q_2},$$

here κ is the *Kármán's* constant and the similarity tensor is:

$$\mathbf{H}_0 = \begin{pmatrix} \alpha & 1 & \mu \\ 1 & \beta & \vartheta \\ \mu & \vartheta & \gamma \end{pmatrix}.$$

The elements of \mathbf{H}_0 are constant numbers. A very important circumstance is that the number of equations and the unknown variables of the turbulent flow are equal.

The shortly outlined concept of turbulence applied to the flow in tubes of circular cross-section resulted in velocity distributions that are in good agreement with the measurements carried out by *Nikuradse* about 70 years ago. *Czibere* is currently working on more complicated applications of his concept.

I ought not forget to mention that the computer codes are worked out and numerical computations are carried out by himself.

I suppose that this unavoidably short summing up the activities of *Tibor Czibere* would make any evaluation superfluous. What I may only do is to express my pleasure that I could be a witness of the thoughts arising and from time to time I was a fellow worker to discuss the problems. I am also very proud of my friendship with him. At the same time the fact that *Czibere's* work has not yet been utilized as completely as it would be required and possible - like the achievements of many other Hungarian Scientists - fills me with sorrow. One can only hope that time will make up for the delay.

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