Journal of Computational and Applied Mechanics, Vol. 4., No. 2., (2003), pp. 147–158

COMPUTATION OF TURBULENT FLOW IN AN S-SHAPED CHANNEL

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[Received: December 20, 2002]

Abstract. The application of a new stochastic turbulence model for curved channel flow is presented. The numerical computation was performed using the finite volume method on colocated variable arrangements and SIMPLE based pressure-correction method was used to treat the velocity-pressure coupling. The widely used approach for computation of laminar flow was extended by the discretization of the turbulent Reynolds stress tensor modelled by the new stochastic turbulence model of Czibere [1]. In this paper an application of this turbulence model is shown for curved channel flow. The computed velocity field was compared with the experimental data measured by LDV.

Mathematical Subject Classification: 76D06
Keywords: turbulent flow, curved channel, turbulence model

1. Introduction

Many engineering turbulence models use the Boussinesq-hypothesis to close the system of governing equations, but in some situations some of them fail to produce acceptable results [2, 3, 4]. A new stochastic turbulence model and related numerical computations are presented in this paper. The discretization methods are also shown after having introduced the governing equations.

Computations were performed in an S-shaped confuser and comparison was made with experimental data measured by LDV. Similar geometry can be found in Leoffler [5], but he made computations in an S-shaped diffuser. In this paper two-dimensional, incompressible flow is assumed.

In a statistically steady turbulent flow the Reynolds-averaged Navier-Stokes (RANS) equations read:

$$\frac{\partial\left(\rho\mathbf{v}\right)}{\partial t}+\operatorname{Div}\left(\rho\mathbf{v}\circ\mathbf{v}\right)=\mathbf{f}\rho-\nabla p+\operatorname{Div}\boldsymbol{\tau}+\operatorname{Div}\mathbf{F}_R\ , \tag{1.1}$$

where $\mathbf{F}_R=-\rho\left(\overline{\mathbf{v}'\circ\mathbf{v}'}\right)$ is the Reynolds stress tensor. This tensor introduces new unknown variables so the system of equations is not closed, and we have to use a turbulence model to close the system.

2. Application of the stochastic turbulence model

2.1. **Description of the stochastic turbulence model.** This model is the three-dimensional extension of the well-known Kármán similarity hypothesis for two-dimensional turbulent flow. The turbulent stress tensor $\mathbf{F}_R$ is defined in the so-called natural coordinate system according to the model of Czibere [1]:

$$\mathbf{F}_R = \rho\kappa^2 l^2 \mathbf{H} \left|\Omega\right| \Omega \ ,$$

where $\kappa = 0.41$ is the Kármán constant, l is the length scale and $\mathbf{H}$ is the similarity tensor:

$$\mathbf{H} = \begin{pmatrix} \alpha & 1 & \mu \\ 1 & \beta & \vartheta \\ \mu & \vartheta & \gamma \end{pmatrix} \ ,$$

where α, β, γ, μ and ϑ are the constants of the model [1]. The basis vectors of the natural coordinate system are defined with the time-mean velocity $\mathbf{v}$ and the vorticity vector $\nabla \times \mathbf{v}$:

$$\mathbf{e}'_3 = -\frac{\nabla \times \mathbf{v}}{|\nabla \times \mathbf{v}|}, \quad \mathbf{e}'_2 = \frac{\mathbf{v} \times (\nabla \times \mathbf{v})}{|\mathbf{v} \times (\nabla \times \mathbf{v})|}, \quad \mathbf{e}'_1 = \mathbf{e}'_2 \times \mathbf{e}'_3 \ .$$

The advantage of this definition is that two components of the vorticity vector are zero in this system.

Let us introduce the following notation:

$$\Theta\left(q'_1, q'_2, q'_3, t\right) = \rho\kappa^2 l^2 \left|\Omega\right| \Omega \ , \tag{2.1}$$

where Θ is the dominant turbulent stress in turbulent shear flow.

The directions of the coordinate axes change from point to point, as the velocity and the curl of the velocity change. In order to perform the numerical computation it is easier to use the physical coordinate system, since in this way transformation from the natural coordinate system q'_1, q'_2, q'_3 can be performed. The Reynolds stress tensor $\mathbf{F}_R$ in the physical coordinate system reads:

$$\mathbf{F}_R = \Theta\left(q_1, q_2, q_3, t\right) \mathbf{G} \ , \tag{2.2}$$

where tensor $\mathbf{G}$ is the transformation of the tensor $\mathbf{H}$ defined in the natural coordinate system:

$$\mathbf{G} = \mathbf{E}\,\mathbf{H}\,\mathbf{E}^T \ . \tag{2.3}$$

$\mathbf{E}^T$ is the transpose tensor of the tensor $\mathbf{E}$. The scalar elements of the tensor $\mathbf{E}$ are defined by the time-mean velocity $\mathbf{v}$ and the vorticity vector $\nabla \times \mathbf{v}$. For numerical computation the Cartesian coordinate system x, y, z was used instead of the

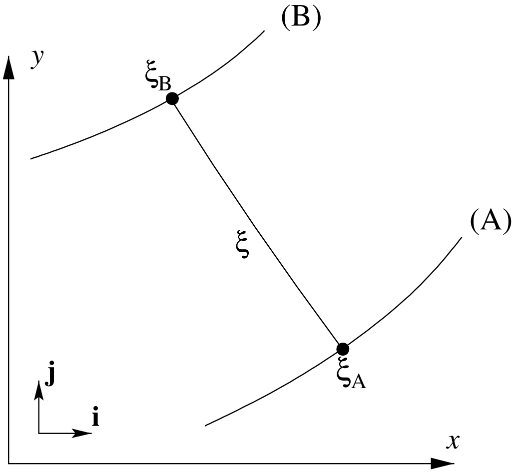

Figure 1. Definition of the scale function along a trajectory

curvilinear orthogonal coordinate system q_1, q_2, q_3:

$$E_{xx} = \frac{1}{\sqrt{1-\lambda^2}}\left(\frac{v_x}{|\mathbf{v}|} - \lambda\frac{\Omega_x}{|\mathbf{\Omega}|}\right) \quad , \quad E_{xy} = \frac{1}{\sqrt{1-\lambda^2}}\frac{v_y\Omega_z - v_z\Omega_y}{|\mathbf{v\Omega}|} \quad , \quad E_{xz} = -\frac{\Omega_x}{|\mathbf{\Omega}|} \quad ,$$

$$E_{yx} = \frac{1}{\sqrt{1-\lambda^2}}\left(\frac{v_y}{|\mathbf{v}|} - \lambda\frac{\Omega_y}{|\mathbf{\Omega}|}\right) \quad , \quad E_{yy} = \frac{1}{\sqrt{1-\lambda^2}}\frac{v_z\Omega_x - v_x\Omega_z}{|\mathbf{v\Omega}|} \quad , \quad E_{yz} = -\frac{\Omega_y}{|\mathbf{\Omega}|} \quad ,$$

$$E_{zx} = \frac{1}{\sqrt{1-\lambda^2}}\left(\frac{v_z}{|\mathbf{v}|} - \lambda\frac{\Omega_z}{|\mathbf{\Omega}|}\right) \quad , \quad E_{zy} = \frac{1}{\sqrt{1-\lambda^2}}\frac{v_x\Omega_y - v_y\Omega_x}{|\mathbf{v\Omega}|} \quad , \quad E_{zz} = -\frac{\Omega_z}{|\mathbf{\Omega}|} \quad ,$$

where $\lambda = (\mathbf{v\Omega}) \,/\, (|\mathbf{v}|\,|\mathbf{\Omega}|)$.

The (1.1) momentum equation can be reshaped according to the stochastic turbulence model:

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \mathrm{Div}\,(\rho\mathbf{v}\circ\mathbf{v}) = \mathbf{f}\rho - \nabla p + \mathrm{Div}\,\boldsymbol{\tau} + \mathrm{Div}\,(\Theta\mathbf{G}) \ . \tag{2.4}$$

The length scale l is always zero on the wall. According to the Kármán-Prandtl similarity hypothesis, the mixing length $l_{mix} = \kappa l$ is a linear function when approaching the wall. In the numerical computation the length scale l appearing in the definition of Reynolds stress tensor is approximated by a fourth-order polynomial:

$$l(\xi) = 4S\left(1 - \frac{(4S-T)}{S}\left(\frac{\xi-\xi_0}{\xi_B-\xi_A}\right)^2\right)\frac{(\xi-\xi_A)\cdot(\xi_B-\xi)}{(\xi_B-\xi_A)} \ ,$$

$$\text{where} \quad \xi_A \le \xi \le \xi_B \quad , \quad \xi_0 = \frac{\xi_A+\xi_B}{2} \quad \text{and} \quad S,\, T > 0 \ .$$

The $(\xi_B - \xi_A)$ means the distance between the two points, and ξ is the parameter of a trajectory (see Figure 1).

According to the transformation from the natural coordinate system q_1', q_2', q_3' to the Cartesian coordinate system x, y, z, every (q_1', q_2', q_3') triad corresponds to triad (x, y, z) according to the following equations:

$$x = x\left(q_1', q_2', q_3'\right), \quad y = y\left(q_1', q_2', q_3'\right), \quad z = z\left(q_1', q_2', q_3'\right) \ .$$

2.2. **Transformation to the computational coordinate system.** An arbitrary scalar value (such as the function Θ) is not changed during the transformation:

$$\Theta\left(x, y, z, t\right) \equiv \Theta'\left(q_1', q_2', q_3', t\right) \ .$$

A vector (for example the velocity vector $\mathbf{v}$) can be transformed from the q_1', q_2', q_3' natural coordinate system to the x, y, z Cartesian coordinate system with the following equation:

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{pmatrix} \cdot \begin{pmatrix} v_{1'} \\ v_{2'} \\ v_{3'} \end{pmatrix} \ . \tag{2.5}$$

The $\mathbf{G}$ tensor in the x, y, z Cartesian coordinate system can be calculated using the similarity tensor $\mathbf{H}$ defined in the q_1', q_2', q_3' natural coordinate system:

$$\begin{pmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{pmatrix} = \begin{pmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{pmatrix} \cdot \begin{pmatrix} \alpha & 1 & \mu \\ 1 & \beta & \vartheta \\ \mu & \vartheta & \gamma \end{pmatrix} \cdot \begin{pmatrix} E_{xx} & E_{yx} & E_{zx} \\ E_{xy} & E_{yy} & E_{zy} \\ E_{xz} & E_{yz} & E_{zz} \end{pmatrix} \ .$$

This tensor equation can be written by the following six scalar equations:

$$\begin{aligned}
G_{xx} &= \alpha E_{xx}^2 + \beta E_{xy}^2 + \gamma E_{xz}^2 + 2\left(E_{xx}E_{xy} + \mu E_{xx}E_{xz} + \vartheta E_{xy}E_{xz}\right), \\
G_{yy} &= \alpha E_{yx}^2 + \beta E_{yy}^2 + \gamma E_{yz}^2 + 2\left(E_{yx}E_{yy} + \mu E_{yx}E_{yz} + \vartheta E_{yy}E_{yz}\right), \\
G_{zz} &= \alpha E_{zx}^2 + \beta E_{zy}^2 + \gamma E_{zz}^2 + 2\left(E_{zx}E_{zy} + \mu E_{zx}E_{zz} + \vartheta E_{zy}E_{zz}\right), \\
G_{xy} &= G_{yx} = \alpha E_{xx}E_{yx} + \beta E_{xy}E_{yy} + \gamma E_{xz}E_{yz} + \\
&+ \left(E_{xx}E_{yy} + E_{xy}E_{yx}\right) + \mu\left(E_{xx}E_{yz} + E_{xz}E_{yx}\right) + \vartheta\left(E_{xy}E_{yz} + E_{xz}E_{yy}\right), \\
G_{xz} &= G_{zx} = \alpha E_{xx}E_{zx} + \beta E_{xy}E_{zy} + \gamma E_{xz}E_{zz} + \\
&+ \left(E_{xx}E_{zy} + E_{xy}E_{zx}\right) + \mu\left(E_{xx}E_{zz} + E_{xz}E_{zx}\right) + \vartheta\left(E_{xy}E_{zz} + E_{xz}E_{zy}\right), \\
G_{yz} &= G_{zy} = \alpha E_{yx}E_{zx} + \beta E_{yy}E_{zy} + \gamma E_{yz}E_{zz} + \\
&+ \left(E_{yx}E_{zy} + E_{yy}E_{zx}\right) + \mu\left(E_{yx}E_{zz} + E_{yz}E_{zx}\right) + \vartheta\left(E_{yy}E_{zz} + E_{yz}E_{zy}\right) \ .
\end{aligned}$$

3. Governing equations for two-dimensional flow

The RANS equations for steady flow can be written in the following way, as the continuity equation:

$$\frac{\partial\left(\rho u\right)}{\partial x} + \frac{\partial\left(\rho v\right)}{\partial y} = 0 \ , \tag{3.1}$$

and as the momentum equations for viscous fluid according to the model of Czibere:

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y} = -\frac{\partial p}{\partial x}+\eta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)+\eta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) +\frac{\partial(\Theta G_{xx})}{\partial x}+\frac{\partial(\Theta G_{xy})}{\partial y}\ , \tag{3.2}$$

$$\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y} = -\frac{\partial p}{\partial y}+\eta\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)+\eta\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right) +\frac{\partial(\Theta G_{yx})}{\partial x}+\frac{\partial(\Theta G_{yy})}{\partial y}\ . \tag{3.3}$$

Here ρ is the density, u, v and x, y are the Cartesian velocity components and coordinate directions, respectively, and η is the dynamic viscosity.

In two-dimensional problems the vorticity vector $\mathbf{\Omega}$ is perpendicular to the flow plane, so the x and y components of this vector are cancelled in the transformed tensor. For two-dimensional flow the scalar elements of the transformation tensor $\mathbf{E}$ can be simplified:

$$E_{xx}=\frac{u}{\sqrt{u^2+v^2}}\ ,\quad E_{xy}=-\frac{v}{\sqrt{u^2+v^2}}\ ,\quad E_{xz}=0\ ,$$

$$E_{yx}=\frac{v}{\sqrt{u^2+v^2}}\ ,\quad E_{yy}=\frac{u}{\sqrt{u^2+v^2}}\ ,\quad E_{yz}=0\ ,$$

$$E_{zx}=0\ ,\quad E_{zy}=0\ ,\quad E_{zz}=1\ .$$

The elements of the tensor $\mathbf{G}$ can be calculated from transformation (2.3):

$$G_{xx}=\alpha\frac{u^2}{u^2+v^2}+\beta\frac{v^2}{u^2+v^2}-2\frac{uv}{u^2+v^2}\ , \tag{3.4}$$

$$G_{yy}=\alpha\frac{v^2}{u^2+v^2}+\beta\frac{u^2}{u^2+v^2}+2\frac{uv}{u^2+v^2}\ , \tag{3.5}$$

$$G_{xy}=G_{yx}=(\alpha-\beta)\frac{uv}{u^2+v^2}+\left(\frac{u^2}{u^2+v^2}-\frac{v^2}{u^2+v^2}\right)\ . \tag{3.6}$$

According to the stochastic model of Czibere [1] and the measured values of Laufer [6] the constant elements of the tensor $\mathbf{H}$ are: $\alpha=3.9714$ and $\beta=1.5734$.

The turbulent dominant shear stress Θ can be written in the physical coordinate system using the relations $\Omega_x=0$, $\Omega_y=0$ and $\Omega_z=\Omega_{3'}$. The vorticity vector for two-dimensional flow (where the velocity vector contains only two components in the directions $\mathbf{i}$ and $\mathbf{j}$):

$$\mathbf{\Omega}=\nabla\times\mathbf{v}=\Omega_z\mathbf{k}=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\mathbf{k}\ , \tag{3.7}$$

and the function Θ can be obtained from (2.1) as:

$$\Theta(x,y,t)=\rho\kappa^2 l^2\left|\Omega_z\right|\Omega_z\ . \tag{3.8}$$

The turbulent terms contain the velocity components and the coefficients of the turbulence model. Using the length scale l, the system of the algebraic equations is closed. We have four equations: the continuity equation (3.1), the two momentum equations (3.2)-(3.3) and the definition of function Θ (3.8), and four unknowns: velocities u and v, the pressure and the turbulent dominant shear stress Θ.

4. Discretization method

4.1. **Basic equations.** The conservation equations for mass and momentum in integral form serve as the starting point for finite-volume solution methods:

$$\int_{\Delta V} \operatorname{div}(\rho \mathbf{v})\, dV = \int_{(\delta A)} \rho \mathbf{v} \cdot d\mathbf{A} = 0 \,, \tag{4.1}$$

the momentum equation using the turbulence model of Czibere:

$$\int_{\Delta V} \operatorname{Div}(\rho \mathbf{v} \circ \mathbf{v})\, dV = -\int_{\Delta V} \nabla p \, dV + \int_{\Delta V} \operatorname{Div} \boldsymbol{\tau} \, dV + \int_{\Delta V} \operatorname{Div}(\Theta \mathbf{G})\, dV \,,$$

and applying the Gauss divergence theorem the volume integrals can be transformed into surface integrals:

$$\int_{(\delta A)} \mathbf{v}\,(\rho \mathbf{v} \cdot d\mathbf{A}) = -\int_{\Delta V} \nabla p \, dV + \int_{(\delta A)} \boldsymbol{\tau} \cdot d\mathbf{A} + \int_{(\delta A)} \Theta \mathbf{G} \cdot d\mathbf{A} \,. \tag{4.2}$$

Here, ΔV is the volume and (δA) is the surface of an arbitrary rectangular control volume. Steady flow, Newtonian fluid, constant density ρ and dynamic viscosity η are assumed here; also, gravitational body force is included in the pressure p. The computation was performed using the finite volume method on a curvilinear orthogonal rectangular coordinate system (Figure 2) with collocated variable arrangement, and Cartesian velocity components were used.

The solution domain is subdivided into a finite number of control volumes (CV), and a computational node is placed at the center of each CV. The integral expressions are applied to each CV, and the integrals are numerically evaluated.

4.2. **Turbulent terms.** Only the turbulent terms are considered here; for a more detailed description of the discretization methods for laminar flow, see e. g. [7].

The function Θ contains the length scale l, which only depends on the geometry of the computational domain. In the case of non-moving boundaries this function has constant values, but the vorticity vectors must be updated in every iteration step.

The turbulent terms according to the turbulence model of Czibere can be written by surface integrals in the u-momentum equation:

$$F_u^t = \int_{(A_\mathrm{e})} (\Theta G_{xx})\, dA_\mathrm{e} - \int_{(A_\mathrm{w})} (\Theta G_{xx})\, dA_\mathrm{w} + \int_{(A_\mathrm{n})} (\Theta G_{xy})\, dA_\mathrm{n} - \int_{(A_\mathrm{s})} (\Theta G_{xy})\, dA_\mathrm{s} \,,$$

where e, w, n, s denote the east, west, north and south boundary of the control volume, respectively. In the v-momentum equation:

$$F_v^t = \int_{(A_\mathrm{e})} (\Theta G_{yx})\, dA_\mathrm{e} - \int_{(A_\mathrm{w})} (\Theta G_{yx})\, dA_\mathrm{w} + \int_{(A_\mathrm{n})} (\Theta G_{yy})\, dA_\mathrm{n} - \int_{(A_\mathrm{s})} (\Theta G_{yy})\, dA_\mathrm{s}\ .$$

The elements of the tensor **G** are defined with the relations (3.4)-(3.6). In the discretization these elements can be divided into two parts, where the coefficients of u^{m+1} and v^{m+1} in G_{xx} and G_{xy} are identical as are those of u^{m+1} and v^{m+1} in G_{yx} and G_{yy}:

$$G_{xx} = u^{m+1}\left(\alpha\frac{u}{u^2+v^2} - \frac{v}{u^2+v^2}\right)^m + v^m\left(\beta\frac{v}{u^2+v^2} - \frac{u}{u^2+v^2}\right)^m , \qquad (4.3)$$

$$G_{xy} = u^{m+1}\left(\frac{u}{u^2+v^2} + \alpha\frac{v}{u^2+v^2}\right)^m - v^m\left(\frac{v}{u^2+v^2} + \beta\frac{u}{u^2+v^2}\right)^m , \qquad (4.4)$$

$$G_{yx} = v^{m+1}\left(\alpha\frac{u}{u^2+v^2} - \frac{v}{u^2+v^2}\right)^m - u^m\left(\beta\frac{v}{u^2+v^2} - \frac{u}{u^2+v^2}\right)^m , \qquad (4.5)$$

$$G_{yy} = v^{m+1}\left(\frac{u}{u^2+v^2} + \alpha\frac{v}{u^2+v^2}\right)^m + u^m\left(\frac{v}{u^2+v^2} + \beta\frac{u}{u^2+v^2}\right)^m . \qquad (4.6)$$

Terms G_{xx}, G_{xy}, G_{yx} and G_{yy} are linearized in a way that u^{m+1} and v^{m+1} are considered unknown velocity variables, while u^m and v^m are known from the previous iteration. Due to this division of the turbulent terms, the coefficients of the two momentum equations are equal for the colocated variable arrangement.

From the previous iteration step the following can be written using the second term on the right-hand side (RHS) in equations (4.3)-(4.6):

$$Q_u^t = v_\mathrm{e}\Theta_\mathrm{e}A_\mathrm{e}\left(\beta\frac{v}{u^2+v^2} - \frac{u}{u^2+v^2}\right)_\mathrm{e} - v_\mathrm{w}\Theta_\mathrm{w}A_\mathrm{w}\left(\beta\frac{v}{u^2+v^2} - \frac{u}{u^2+v^2}\right)_\mathrm{w}$$
$$-v_\mathrm{n}\Theta_\mathrm{n}A_\mathrm{n}\left(\frac{v}{u^2+v^2} + \beta\frac{u}{u^2+v^2}\right)_\mathrm{n} + v_\mathrm{s}\Theta_\mathrm{s}A_\mathrm{s}\left(\frac{v}{u^2+v^2} + \beta\frac{u}{u^2+v^2}\right)_\mathrm{s} ,$$

and the implicit terms can be derived from equations (4.3)-(4.6) using the first term on the RHS:

$$F_u^t = -u_\mathrm{e}\Theta_\mathrm{e}A_\mathrm{e}\left(\alpha\frac{u}{u^2+v^2} - \frac{v}{u^2+v^2}\right)_\mathrm{e} + u_\mathrm{w}\Theta_\mathrm{w}A_\mathrm{w}\left(\alpha\frac{u}{u^2+v^2} - \frac{v}{u^2+v^2}\right)_\mathrm{w}$$
$$-u_\mathrm{n}\Theta_\mathrm{n}A_\mathrm{n}\left(\frac{u}{u^2+v^2} + \alpha\frac{v}{u^2+v^2}\right)_\mathrm{n} + u_\mathrm{s}\Theta_\mathrm{s}A_\mathrm{s}\left(\frac{u}{u^2+v^2} + \alpha\frac{v}{u^2+v^2}\right)_\mathrm{s} .$$

The convective and diffusive terms were discretized with the first-order upwind difference scheme (UDS) and the second-order central difference scheme (CDS) and deferred correction was used to connect them. The discretized algebraic equations were solved with Stone's strongly-implicit procedure (SIP) [8]. The solution of the coupled set of equations for u, v and p is based on the SIMPLE algorithm [9]. The coefficients of the discretized equations are updated and solved in turn and the process is repeated until convergence.

5. **Results of computations**

In order to show how to apply the stochastic turbulence model, turbulent flow was investigated in a curved-channel. Measurements were also performed with laser doppler velocimetry (LDV) at the University of Magdeburg [10]. The present computations requires a few minutes on today's personal computers. Using the present model we do not need any additional differential equation apart from the continuity and momentum equations, in contrast with many other engineering turbulence models, so the computational time per iteration can be reduced.

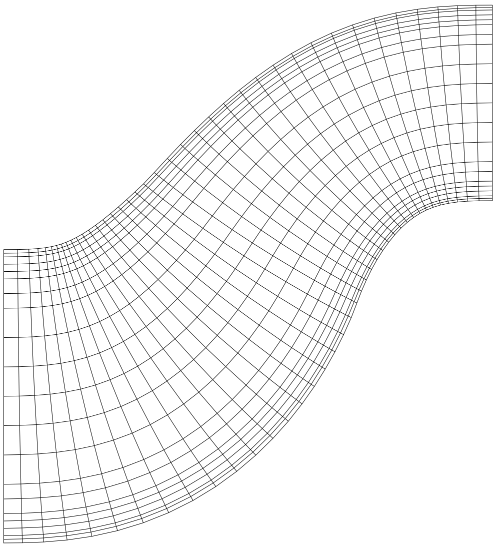

Figure 2. Computational grid

The contour plots of the computed and measured values are shown in dimension m/s in Figures 3-6. The colors and the scales of these figures mean the same values in pairs. Figures 7 and 8 represent the contour plots of the differences of the calculated and measured velocity components. As can be seen the agreement is quite good.

The extension of the present numerical methods for three-dimensional problems is planned in the near future.

6. **Conclusions**

Based on the results of computations presented above, the following conclusions can be drawn.

Using the stochastic turbulence model of Czibere the system of governing equations is closed, and the two unknown velocity components and the pressure can be determined from the two momentum equations and the continuity equation.

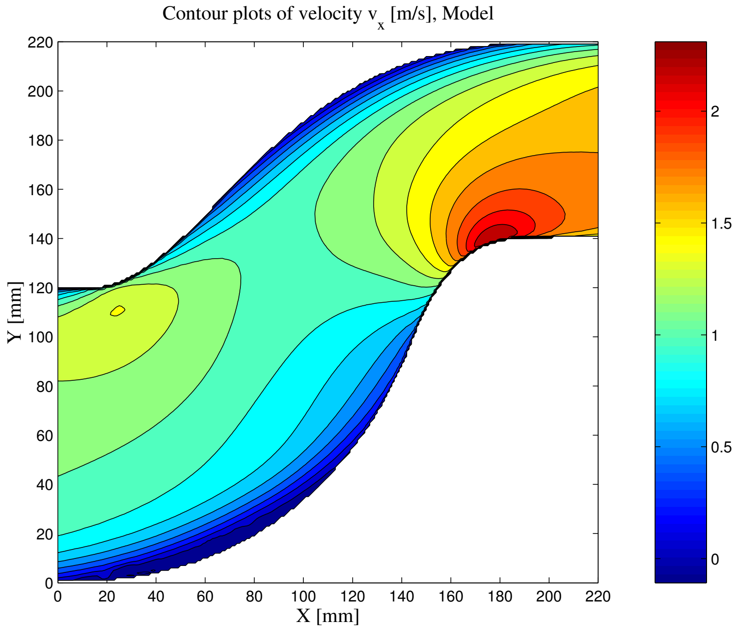

Figure 3. Calculated velocity components in x direction

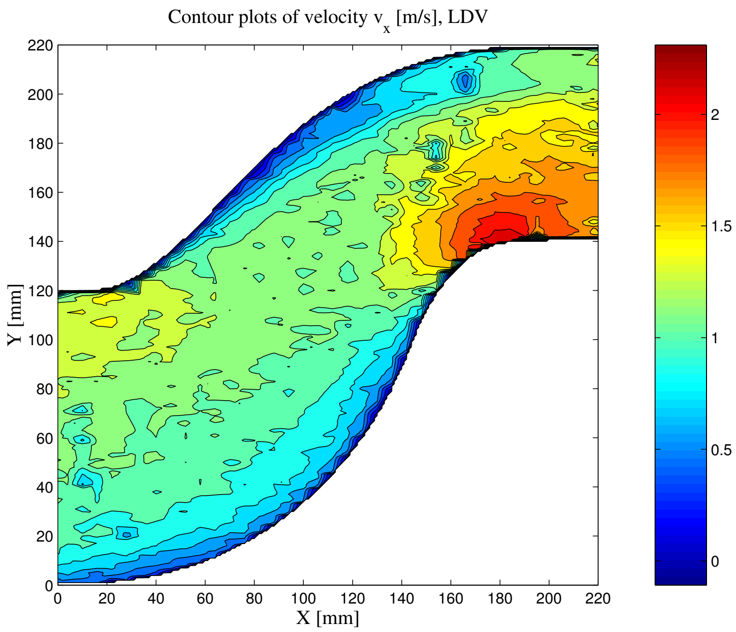

Figure 4. Measured velocity components in x direction

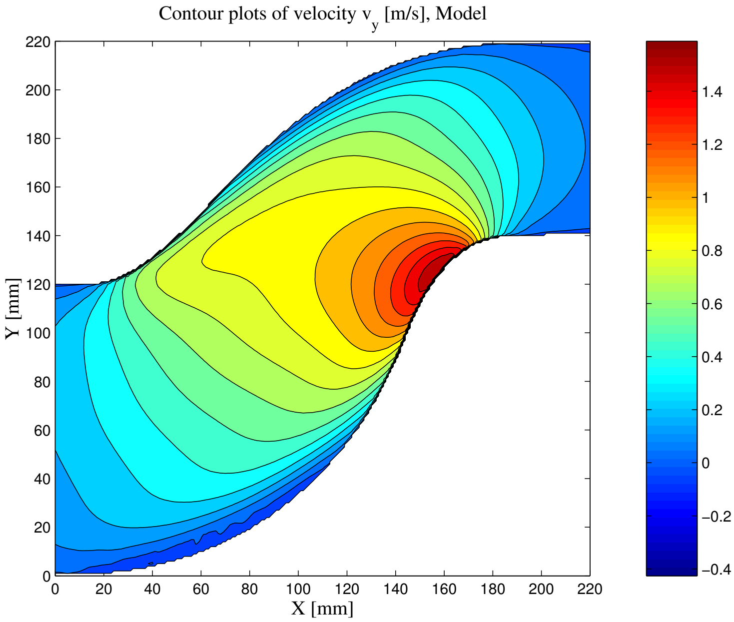

Figure 5. Calculated velocity components in y direction

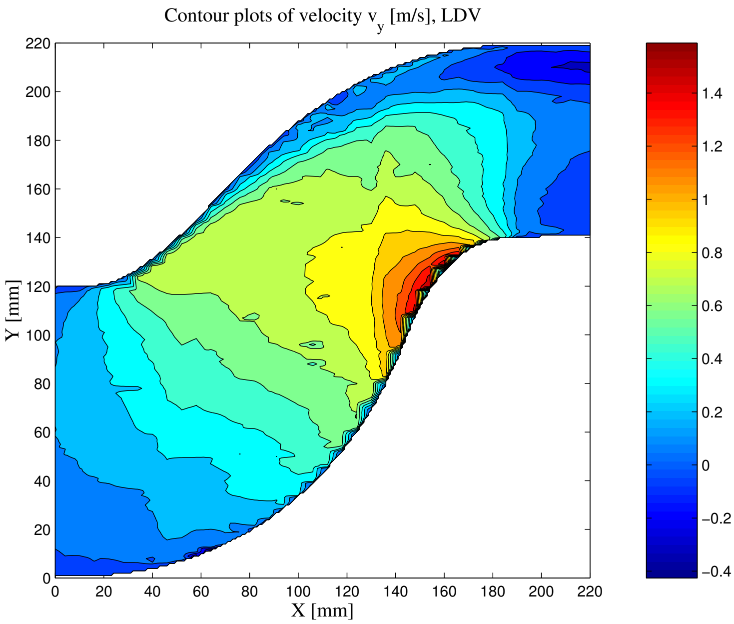

Figure 6. Measured velocity components in y direction

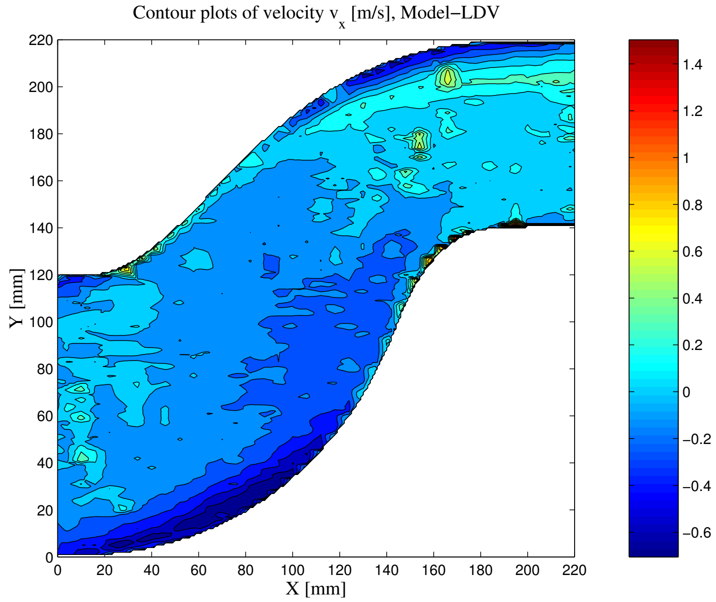

Figure 7. Differences of the calculated and measured velocity components in x direction

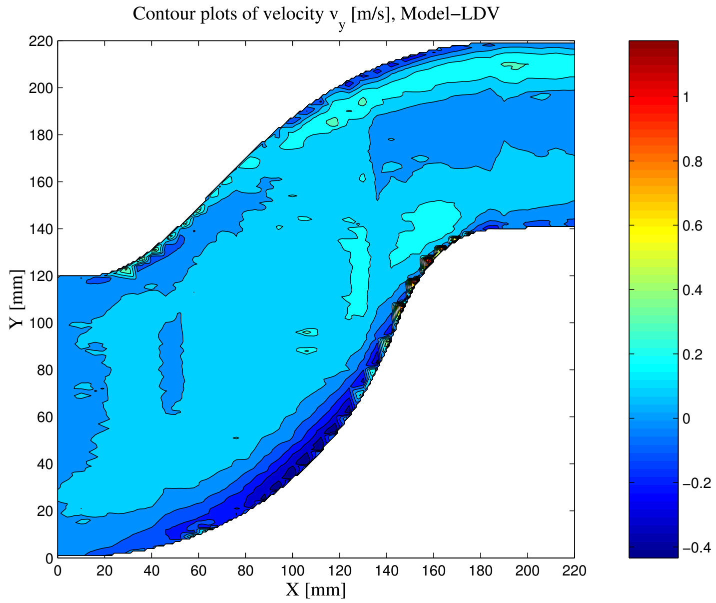

Figure 8. Differences of the calculated and measured velocity components in y direction

The application of this model does not require the solution of any additional differential equation apart from the continuity and momentum equations, in contrast with many other engineering turbulence models. Using this model the computational time per iteration step is less than that of most engineering models.

Computational results compare well with experimental results.

Acknowledgement. The author wishes to acknowledge the guidance of Professor Czibere. The support provided by the János Bolyai Research Scholarship is also greatly appreciated.

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