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OPTIMUM DESIGN OF A COMPOSITE MULTICELLULAR PLATE STRUCTURE

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Dedicated to Professor József FARKAS on the occasion of his seventy-fifth birthday

Abstract. This study presents the optimal design of a new complex structural model [laminated carbon fiber reinforced plastic (CFRP) deck plates with aluminium (Al) stiffeners] which is depicted in Figure 1. The structure was designed both for minimal cost and minimal weight. Design constraints on maximum deflection of the total structure, buckling of the composite plates, buckling of the Al webs, stress in the composite plates and stress in the Al stiffeners are considered in the calculation. The Rosenbrock's Hillclimb algorithm is used in the optimization process.

Mathematical Subject Classification: 74P05
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1. **Introduction**

Sandwich structures utilize the advantages of different structural components. These components can have different structural configurations (e.g. plates or beams) or different material properties (e.g. density or damping coefficients). In the design of layered beams, plates and shells, one can exploit the different beneficial characteristics of these components. Prime examples are orthotropic sandwich structures, which have a high ratio of bending stiffness to density. Hence they are often used in light-weight structures.

Recent literature reviews [1, 2] highlight the significant effort directed at the design, analysis, and application of sandwich structures. Examples include a bending theory for sandwich beams with thick faces in [3]. Notable work is reflected in [4, 5], as well as the proceedings of international conferences on sandwich constructions [6, 7, 8, 9]. There is also a report on marine applications of sandwich construction [10].

The optimum design of specialized welded sandwich panels for ship floors was treated in [11], while a five layer beam was analysed and optimized in [12, 13]. This beam consists of a rubber layer, two aluminium profile beams and two CFRP deck layers.

In the present study a new structural model is investigated. Sandwich plates have deck layers made of metal or FRP (fiber reinforced plastic) plates, and their inner layer is usually made of foam or honeycomb. On the contrary, cellular plates consist of metal deck plates and metal stiffeners welded into the deck plates. Our new structural model combines sandwich and cellular plates, since it has FRP deck plates and two or more aluminium square hollow section stiffeners riveted into the deck plates. So it is a new combination of materials, stiffeners and fabrication technology.

The multicellular sandwich plate is constructed from a number of longitudinal Al (aluminium) square hollow section beams and two laminated CFRP deck plates (Figure 1). The connection between the beams and deck plates is effected through riveting. This type of sandwich plate can be applied in many engineering load carrying structures such as ship floors, bridges, airplanes, building floors, etc.

The main aim of the present study is to work out an optimum design procedure for such a structural model. In doing so, design constraints are formulated on the buckling strength of the compressed deck plate, the local buckling of the aluminium square hollow section plate elements, stress in the composite plates and in the Al stiffeners as well as the deflection of the simply supported beams subjected to distributed pressure acting on the total surface.

In order to achieve cost savings in the design stage, a cost function is formulated on the basis of material and fabrication cost analysis. The mass function used in the optimization process includes the sum of the mass of CFRP plates and beams.

Mathematical programming methods for constrained function minimization are an integral part of the procedure. The Rosenbrock's Hillclimb algorithm [14] is used for the determination of the optimal dimensions of the structural model.

2. A new multicellular sandwich plate model

The sandwich plate model under consideration is depicted in Figure 1. The CFRP plates are constructed from laminated layers. The fiber volume fraction is 61% and

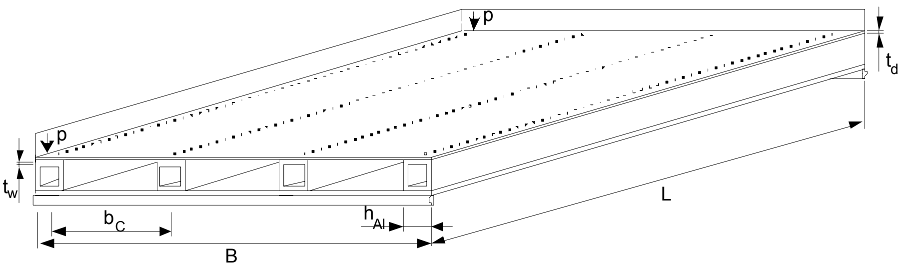

Figure 1. Multicellular sandwich plate structure

the matrix volume fraction is 39%. All of the fibers of a layer and laminate are arranged in the longitudinal direction. The plates are riveted to the upper and lower flanges of the aluminium square hollow section (SHS) profiles. The calculated required distance between the rivets is 31 mm.

The material parameters of a pre-impregnated CFRP layer are given as follows: the thickness of a layer $t = 0.2$ mm, longitudinal Young's modulus $E_x = E_c = 120$ GPa and the transverse modulus $E_y = 9$ GPa. The mass of the CFRP plate is $\rho^* = 180$ g/m^2, and Poisson's ratios are $\nu_{xy} = 0.25$ and $\nu_{yx} = 0.019$.

3. **Optimization**

3.1. **Cost function.** The structure is optimized with respect to minimum cost K, which can be formulated as the sum of the material and manufacturing costs [15],

$$\begin{aligned} f(x) &= K = K_{CFRP} + K_{Al} + K_{\text{heat treatment}} + K_{\text{manufacturing}} \\ K(Euro) &= 2(31.047n) + k_{Al}[n_s(\rho_{Al} 4h_{Al} t_w L)] + \\ &\quad + 2n\frac{525}{528} + k_f\left[n14_{\min} + n_s 26_{\min} + 110_{\min}\right] , \end{aligned} \tag{1}$$

where n represents the number of CFRP layers, n_s the number of stiffeners, ρ_{Al} the density of the Al profile, h the height and t_w thickness of the SHS Al profiles.

The main contribution to the material cost arises from the raw material for the composite plates. In our case this cost reached 31.047 Euro/layer. The cost of the Al profile is 4.94 Euro/kg. The specific fabrication cost is k_f=0.6 Euro/min.

The cost of heat treatment depends on the volume of deck plates to be heat treated and the type of the resin matrix. In our case these cost components can be calculated as a function of layer number and plate dimension. The heat treatment cost of a manufactured 220x1200x2mm CFRP plate is known, so compared to it the cost of the examined plates based on volume can be calculated. The resulting ratio can be seen in eq. (1).

The total fabrication cost (as the function of time [min]) is the sum of the cost required for the manufacturing of the CFRP plates ($n14_{\min} + 110_{\min}$), the cutting cost of the Al profiles ($n_s 6_{\min}$) and the total assembly costs ($n_s 20_{\min}$). The time associated with manufacturing of the CFRP plates consists of the time lost in press form preparation, layer cutting, layer sequencing and final working. Final assembly consists of drilling of the CFRP plates and the Al profiles, and also riveting. Drilling of the holes is an implicit function of the number of layers.

The design variables are the height h and thickness t_w of the SHS Al profiles, the number of layers n of the CFRP plates and the number of stiffeners n_s. The fiber orientation is fixed for all layers (0^o) as described above.

3.2. **Mass function.** The total cost of the structure is the sum of the CFRP and Al components:

$$m = 2\rho_c\left[BL(nt^*)\right] + n_s\rho_{Al}\left[L(4h_{Al}t_w - 4t_w^2)\right] , \tag{2}$$

where t^* is the thickness of a laminate.

3.3. **Constraints.**

3.3.1. *Deflection of the total structure.*

$$w_{\max} = \frac{5pL^4}{384(E_c I_c + E_{Al} n_s I_{Al})} + \frac{5\Delta M L^2}{48(E_c I_c + E_{Al} n_s I_{Al})} \leq \frac{L}{200}\,, \tag{3}$$

where:

I_c, I_{Al}: moment of inertia of the CFRP plate and Al profile,
E_c, E_{Al}: reduced modulus of elasticity of the CFRP lamina and Young's modulus of Al profile.

There is the effect of the relative movement between the components, and is expressed as a function of the differences in predicted stresses in the middle of Al profile and CFRP plate. Due to difference in stress ($\Delta\sigma$) there is a corresponding difference in the equivalent applied moment (ΔM). So the second part of the equation is the additional deflection due to the sliding.

3.3.2. *Composite plate buckling [15].*

$$\left(\frac{b_c}{nt^*}\right) \leq \sqrt{\frac{\pi^2}{6\sigma_{\max}(1-\nu_{xy}\nu_{yx})}\left[\sqrt{E_x E_y} + E_x \nu_{xy} + 2G_{xy}(1-\nu_{xy}\nu_{yx})\right]}\,, \tag{4}$$

where:

b_c: plate width between stiffeners,
$\sigma_{\max}$: maximum stress in the CFRP lamina,
E_x, E_y, G_{xy}: laminate moduli,
ν_{xy}, ν_{yx}: Poisson's ratios.

3.3.3. *Web buckling in the Al profiles [16].*

$$\frac{h_{Al}}{t_w} \leq 42\sqrt{\frac{235 E_{Al}}{240 E_{Steel}}}\,, \tag{5}$$

where: E_{Al}, E_{Steel}: Young's modulus of elasticity of Al and Steel.

3.3.4. *Stress in the composite plates.* The moment acting on the total structure is distributed on the components of the structure. So it can be calculated as the sum of the distributed moment components acting on the composite plates and Al profiles. $M = X_c M + X_{Al} M$

$$\frac{X_c M}{I_c}\frac{h_{Al} + nt^*}{2} \leq \sigma_{Call}\,, \tag{6}$$

where:

$$X_c = \frac{E_c I_c}{E_{Al} n_s I_{Al} + E_c I_c}; \quad M = \frac{pL^2}{8}; \quad \sigma_{\text{Call}} = \frac{\sigma_T}{\gamma_c}$$

σ_{Call} : allowable stress,
$X_c M$: moment acting on composite plate,

σ_T: tensile strength of composite lamina,
γ_c: safety factor (=2).

Because of the high number of stiffeners in the case of optimum design, the stress due to the transversal bending moment can be neglected.

3.3.5. *Stress in the Al stiffeners.*

$$\frac{X_{Al}M}{n_s I_{Al}}\frac{h_{Al}}{2} \leq \sigma_{Alall}\,, \tag{7}$$

where:

$$X_{Al} = \frac{E_{Al}n_s I_{Al}}{E_{Al}n_s I_{Al} + E_c I_c}; \quad \sigma_{Alall} = \frac{f_y}{\gamma_{Al}}\,,$$

σ_{Alall} : allowable stress,

$X_{Al}M$: moment acting on Al tube,

f_y: yield stress of Al,

γ_{Al}: safety factor (=1.5).

3.3.6. *Size constraints for the design variables.*

$$\begin{aligned} &10 \leq h_{Al} \leq 100\,,\\ &2 \leq t_w \leq 6\,,\\ &2 \leq n \leq 32\,,\\ &7 \leq n_s \leq 20\,. \end{aligned} \tag{8}$$

These represent physical limitations on the design variables [mm], taking economic and manufacturing aspects into consideration.

3.4. **Problem formulation.** The optimum design problem under consideration is mathematically stated as:
Find

$$x^* = (x_1, x_2, \ldots x_n) \in R^n \tag{9}$$

that minimizes a cost function $f(x)$ subject to the constraints

$$\begin{aligned} &g_j(x) \leq 0, \quad j = 1, 2, \ldots m\,,\\ &h_j(x) \leq 0, \quad j = 1, 2, \ldots r\,, \end{aligned} \tag{10}$$

where $f(x), g_j(x), h_j(x)$ are scalar functions of the design variables x. The optimum solution is denoted by x^*.

3.5. **Rosenbrock's non-linear mathematical programming method.** The Rosenbrock's direct search non-linear mathematical programming method is used to determine the optimal geometric values, required number of Al stiffeners and the values of the objective functions.

Rosenbrock's method [14] is a simple but efficient mathematical programming method, which uses derivative-free direct searches. Instead of continuous line searches, the algorithm takes discrete steps during searches in orthogonal search directions. In each iteration, the procedure searches successively along n linearly independent and orthogonal directions. When a new point is reached at the end of an iteration, a new set of orthogonal search vectors is constructed. Boundary zones are introduced to slow down the algorithm when it approaches the constraint boundaries.

A modified objective function, using penalty functions, is used to accommodate the constraints. Instead of continually searching in the coordinate space corresponding to the directions of the independent variables, the method achieves an improvement after one cycle of coordinate searches through alignment of the search directions in an orthogonal system. Here, the overall step of a previous stage is used as the first building block for the new set of orthogonal directions. After iteration k^{th}, Rosenbrock's method locates $x^{(k+1)}$ after completing unidimensional searches from the previous point $x^{(k)}$ along a set of orthonormal directions. The method is easy to implement, and attractive for many problems in engineering, even though the method may converge to local minima instead of the global minimum.

4. **Numerical results**

4.1. **Mass optimization.** Table 1 shows the result of mass optimization of the examined structure according to the mass function (eq. 2) and design constraints (eq. 3-8). The obtained optimal number and standard geometries of the stiffeners for the case of different numbers of layers (12-32 pieces) of CFRP deck panels can be seen in Table 1.

Table 1. Result of mass optimization

Number of layers	Optimal discrete stiffener numbers and geometries			Mass
n [pieces]	h_{AL} [mm]	t_w [mm]	n_s [mm]	[kg]
12	60	3	19	98.391
14	60	3	17	93.32
16	60	3	15	88.25
18	60	3	13	83.179
20	60	3	13	86.419
22	60	3	10	77.193
24	60	3	9	76.278
26	50	2.5	9	68.091
28	50	2.5	8	68.445
30	50	2.5	7	68.799
32	**40**	**2.5**	**7**	**67.787**

The global mass optimum is obtained in case of a laminate of 32 layers and 7 pieces (limited in size constraints) of 40x40x2.5 mm stiffeners. This optimum is a global optimum only for the examined interval of n (Figure 2), but it is clear that the total stiffness of the examined structure can be increased by the continuous increase of the number of layers of the deck panel which causes a reduction in the number and geometry of stiffeners. So a lighter structure can be constructed in this way, but the cost will be extremely high.

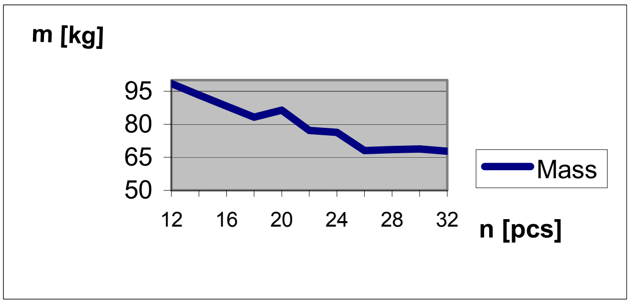

Figure 2. Mass of the structure versus number of CFRP layers

The obtained optimal mass structure of Table 1 was compared to the mass of a total steel multicellular plate structure (Figure 3) optimized in [11]. The dimension (B, L) of the steel structure and the applied pressure (p) are the same as in case of the sandwich plate structure described above.

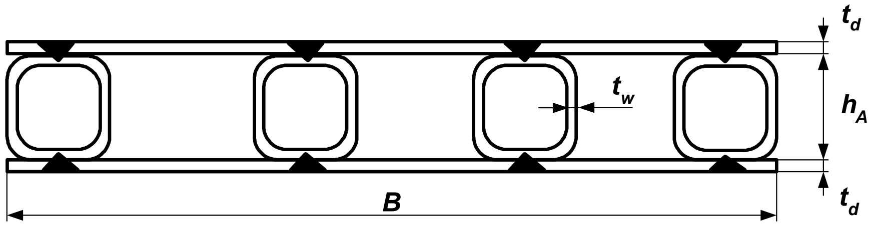

Figure 3. Steel multicellular plate structure

Table 2 summarizes the mass comparison of optimized plate structures made of steel or sandwich.

Table 2. Mass comparison of optimized plate structures made of steel or sandwich

	Optimal discrete stiffener numbers and geometries			Thickness of deck plates	Mass kg	Mass ratio [%]
	h_S; h_{AL} [mm]	t_w [mm]	n_s [mm]	t_d [mm]		
Steel structure (f_y=355 Mpa)	40	2	6	2	517	100
Composite structure	40	2.5	7	6.4	67.787	13.11

It can be seen that an extremely high mass reduction can be achieved by the application of a modern light-weight structure instead of a traditional steel structure. This numerical example proves that 86.89 % mass saving can be realized by the application of a CFRP-Al sandwich plate structure instead of a total steel structure.

4.2. **Cost optimization.** Cost saving can be a prime design aim of sandwich structures because composite materials are very expensive. Table 3 shows the result of cost optimization of the analyzed structure based on the cost function (eq. 1) and design constraints (eqs. 3-8). The optimal number and standard geometries of the stiffeners obtained and total costs for different numbers of layers (12-32 pieces) are as follows:

Table 3. Result of cost optimization

Number of layers	Optimal discrete stiffener numbers and geometries			Cost
n [pieces]	h_{AL} [mm]	t_w [mm]	n_s [mm]	[Euro]
22	**90**	**4**	**7**	**2072**
24	60	3	9	2140
26	50	2.5	9	2226
28	60	3	7	2356
30	60	3	6	2464
32	60	3	5	2572

Table 4. Cost components

Number of layers [pieces]	Cost of CFRP deck panels [Euro]	Cost of Al stiffeners [Euro]	Cost of heat treatment [Euro]	Cost of fabrication [Euro]	Total cost [Euro]
22	**1366**	**302.5**	**43.75**	**360**	**2072.25**
24	1490	194.5	48	408	2140.5
26	1614	135	52	425	2226
28	1739	151	56	410	2356
30	1863	130	60	411.6	2464
32	1987	108	64	413	2572

Table 4 and Figure 4 show the optimum values and cost components of the optimized structures. It can be seen that the biggest part of the total cost is the cost of the composite deck plates, mainly in case of a high number of layers, while the other components are much smaller compared to it. It can also be seen that increasing the number of deck layers will increase the plate cost to a large extent, so the material cost of CFRP plates is the most decisive cost component. Based on these facts the structure having the smallest layer number is considered to be the most economical.

Because of the high number of stiffeners in case of a small layer number of deck plates a limitation should be defined for the number of stiffeners ($n_s \leq 11$). According to this limitation stating that the minimal number of stiffeners is equal to or smaller than 11, the optimum is a laminated plate with 18 layers and 11 pieces of 70x70x4 mm stiffeners.

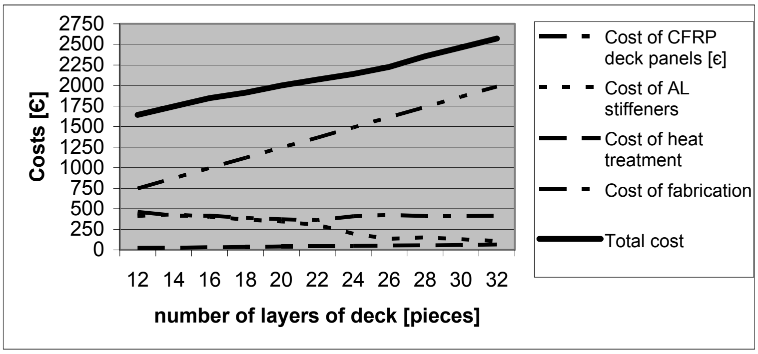

Figure 4. Cost components

The optimal cost obtained – see Table 1 – is also compared to the cost of a total steel multicellular plate structure (Figure 3).

Table 5. Cost comparison of the optimised plate structures made of steel or sandwich

	Optimal discrete stiffener numbers and geometries			Thickness of deck plates	Cost [Euro]	Cost ratio [%]
	h_S; h_{AL} [mm]	t_w [mm]	n_s [mm]	t_d [mm]		
Steel structure (f_y=355 Mpa)	40	2	5	2.5	1014	100
Composite structure	90	4	7	4.4	2072	204

The attainable significant mass saving achieved by the application of modern composite materials causes high additional costs. In our example this extra cost of the optimized composite structure is 104 % compared to the steel structure.

So it can be summarized as follows – based on the mass saving and the disadvantageous extra cost – the application of fibre reinforced laminates is recommended in applications where mass saving is the prime design aim and cost saving is only secondary (e.g.: space flight, air-, water- and land vehicles, building parts, etc.).

Additional advantageous characteristics of these composite structures include vibration damping and corrosion resistance. Due to corrosion resistance, surface treatment and painting can be neglected, which can result in significant cost saving.

5. Conclusion

A new structural model of a sandwich plate riveted from two aluminium square hollow section rods and two CFRP deck plates is investigated by an optimization procedure.

In an optimum design procedure the dimensions and number of stiffeners and number of layers of sandwich plates are determined, which fulfil the design constraints and minimize the cost and mass. It is shown that significant mass and cost savings can be achieved in the design stage through optimization.

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