RNCET
"TRAIAN VUIA" POLYTECHNIC INSTITUTE OF TIMIŞOARA
STEEL STRUCTURES DEPARTMENT

4th CONFERENCE ON STEEL STRUCTURES

ROMANIA- TIMIŞOARA 11...13 OCTOBER 1985

Table 2.
(continued)

$M_3^{I} = H\cdot\ell + P\cdot V_1^{I} + \psi\cdot P\cdot V_2^{I} = 161{,}6$ kNm	162,6 kNm
$R^{I} = \frac{3EI_1}{\ell^3}\cdot\frac{C}{B^2} = 2{,}361$ kN	2,335 kN
$R^{II} = \frac{3EI_1}{\ell^3}\cdot\frac{C-\frac{P}{P_E}(A+\psi\cdot\frac{\ell}{\ell_1})}{B^2} = 1{,}472$ kN	1,453 kN
$P_{cr} = \frac{\pi^2\cdot EI_1}{4\ell^2}\cdot\frac{C}{A+\psi\cdot\frac{\ell}{\ell_1}} = 1593$ kN	4553 kN

CONCLUSIONS

1. A very simple method to investigate the P – Δ effect is presented in this paper.

2. The results obtained by the energy method are in good agreement with those obtained by a rigorous method of calculus .

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Optimal design of plane trusses, regarding the strength of nodes

by

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In order to economically optimize a structure, the method employed should exhibit a reasonable degree of efficiency of operation as well as some assurance of obtaining the optimum design.

The aim of this work is to advance arguments in favour of the mathematical programming approach by illustrating that, in the case of a limited class of structure and failure modes, acceptable efficiency can be achieved.

Description of the problem

The problem selected is the design of a truss to yield the minimum total mass of the entire structural roof system. The roof system is composed of parallel chord steel trusses. All joints are welded. The following loads are considered: uniform live load on the span, plus an approximate dead load /25 kN/m and 1 kN/m respectively/. The truss is symmetrical about the center line, the chords are continuous over the panel points and the panels are of equal lenght.

Preassigned parameters. – The parameters determined by the designer are as follows: /1/ Limiting tensile stresses for chords and web members /R_u = 190 MPa and 165 MPa respectively/; /2/ truss span /L/ ; /3/ the geometry of plane truss, number of members and nodes; /4/ magnitude and type of loading /partial or total/; /5/ deflection limit of a choosen node.

Design variables. – The variables to be determined by the automated scheme are as follows /1/ dimensions of the square hollow cross – section bars; /2/ the number of various type of bars to be applied.

Objective function. - The truss is constructed from bars made of steel 37, and we considered the material cost of the structure

$$K\,/A_i/ = \sum_{i=1}^{n} k_i\, L_i\, A_i \qquad /1/$$

where k_i the material cost per unit volume

L_i member length

A_i cross - sectional area of the ith member

Stress constraints are as follows:

$$|N_i| \,/A_i \leq R_{ui} \qquad /2/$$

where R_{ui} are the ultimate /allowable/ stresses for compressed bars which depend on the slendeness ratio.

Displacement constraints in this case have the form

$$\sum_{i=1}^{n} \frac{N_i\, n_i\, L_i}{A_i\, E_i} \leq w^{\alpha} \qquad /3/$$

where N_i and n_i are internal forces in the i th member due to applied and virtual load, respectively.

Size constraints are

$$A_i \geq A_{i\,min} \qquad /4/$$

when the stress constraints are considered only, in statically determinate structures, the minimum cost may be achieved by

the fully stressed design.

Stress and displacement analysis by finite element method

Typical bar elements are used in the finite element program with two nodes in the plane. The axial displacement is assumed to be linear and the axial force will be constant throughout the length of the element. The bar elements do not carry bending or torsion.

If the employer has no displacement constraint, the fully stressed design /FSD/ generally coincides with the minimum weight design, excluding the case of statically indeterminate structures, but the differences in weight are small in most cases.

Displacement constraints can be divided into two groups: if the equality sign governs the optimal design /see eq. 3/, the constraint is said to be active; if the inequality occurs, the constraint is passive. This division is not known, of course, until the final design is reached.

The optimality - criteria method [1]

In this case the optimality - criteria method is very useful, since only few iteration steps are needed.

The optimality criteria are as follows:

$$k_i\, L_i + \sum_{j=1}^{p} \lambda_j \frac{\partial g_j}{\partial A_i} = 0 \qquad /5/$$

$$\lambda_j \geq 0 \qquad /6/$$

$$\lambda_j\, g_j = 0 \qquad /7/$$

where the nonlinear inequality constraint are

$$g_j /A_i/ \leqq 0 \qquad j = 1, \ldots, p \qquad /8/$$

λ_j are the Lagrange multipliers.

For a single displacement constraint the following expression is valid for the determination of the cross-sectional areas:

$$A_k = \frac{\sqrt{N_k\, n_k}}{w^{*}\, E} \sum_{i=1}^{n} L_i \sqrt{N_i\, n_i} \qquad /9/$$

The members can be active /i $\in m_a$/, or passive /the member sizes have no effect on the structure displacement/. Considering passive members as well, /9/ is modified as follows.

$$A_k = \frac{\sqrt{N_k\, n_k}}{E\, /w^{*}-w_{pass}/} \sum_{ma} L_i \sqrt{N_i\, n_i} \qquad /10/$$

Numerical example of a planar truss with parallel chords shown in Fig. 1 .

We have considered only the stress constraints in the first part of the calculation.

The computer program determined the "fully stressed design" solutions which were obtained for structures of different heights. Height 1,25 . a gives the minimum volume, where "a" is the lenght of a panel.

The members are divided into 4 groups according to the numbering shown in Fig. 1 .

The profiles have been suboptimized with regard to buckling of different bar lengths and forces according to [2] . The effects of initial imperfections and residual welding stresses

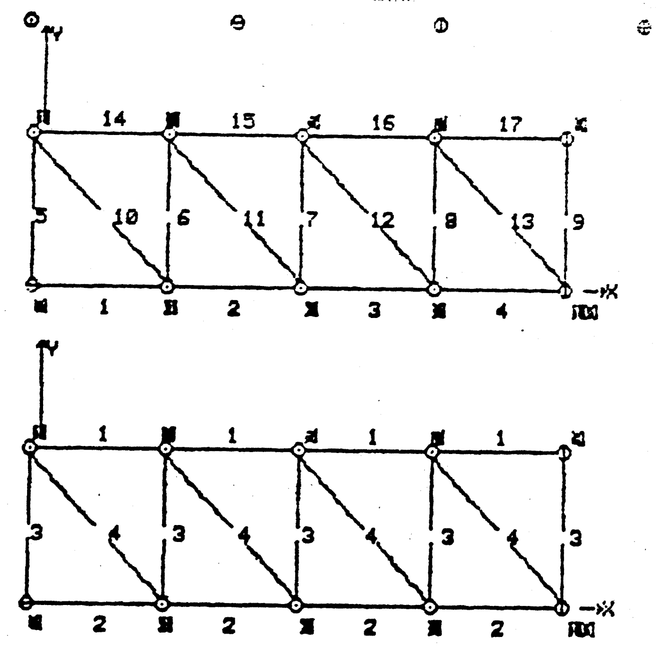

Fig. 1

were considered, but the interaction of buckling modes was neglected. A number of optimum sections were computed by using a Fortran program developed for the backtrack method.

Let the allowable deflection be $w^{*} = 24$ mm.

Considering this displacement constraint, the optimum results are as follows.

Type of bar	1	2	3	4
Number of active bars	17	15	7	2
Profile dimensions /mm/	210x7	180x5	120x3	160x3

Area /mm²/	5880	3600	1440	1920

Half volume $V/2 = 1{,}84418 \cdot 10^8\ mm^3$

Displacement is 23,1 mm

Results without displacement limit are as follows:

Type of bar	1	2	3	4
Number of active bars	17	15	7	2
Profile dimensions /mm/	150x5	150x4	120x3	140x2
Area /mm²/	3000	2400	1440	1120

Half volume $V/2 = 1{,}1822 \cdot 10^8\ mm^3$

Displacement is 37,7 mm

Strength of the welded nodes

The empirical equations of Wardenier [3] for the dimensions of bars, the material yield stresses and axial forces have been used.

For this node the following equations should be considered. For chord failure the critical forces are as follows:

$$N_{1k} = 9{,}8\ R_{yo}\ t_o^2 \sqrt{\frac{h_o}{2t_o}}\ \frac{\mathcal{A}\,\beta^x}{\sin\Theta_1}\ ;\quad N_{2k} = N_{1k}\ \frac{\sin\Theta_1}{\sin\Theta_2}$$

where

$$\mathcal{A} = 1{,}3 - \frac{0{,}4}{\beta}\left|\frac{\sigma_o}{R_{yo}}\right|$$

$$\beta = \frac{b_1+b_2}{2b_o}\ ;\quad \beta^x = \frac{b_1+h_1+b_2+h_2}{4\ b_o}$$

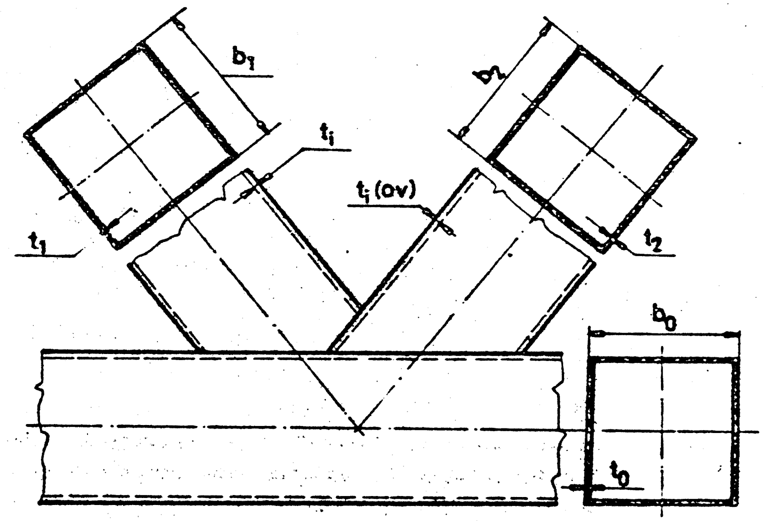

Fig. 2

the notation is shown in Fig. 2.

For the shear failures of chords the shear criteria are:

$$N_{ik} = R_{yi}\ t_i\ /2\ h_i - 4\ t_i + b_i + b_{eo}/$$

where

$$b_{eo} = \frac{c}{b_o/t_o} \cdot \frac{R_{yo}\ t_o}{R_{yi}\ t_i}\ b_i \qquad i = 1,2$$

$$c = 13{,}5 \qquad \text{for steel 37}$$

For the shear failures of webs the shear criteria are:

$$N_{ik} = \frac{R_{yo}\ AQ}{\sqrt{3}\ \sin\Theta_i}\ ;\quad N_{ok} = /A_o - A_Q/\ R_{yo}\sqrt{1 - /\frac{Q}{Q_p}/^2}$$

where $A_Q = 2\, h_o\, t_o + \alpha\, b_o\, t_o$

$$\alpha = \left[1 + 4\, g^2/3\, t_o^2\right]^{-0,5}$$

$$Q_p = \frac{A_Q\, R_{yo}}{\sqrt{3}}\ ; \quad Q = N_i \sin\theta_i \qquad i = 1,2$$

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