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ECONOMY OF WELDED STIFFENED STEEL PLATES AND CYLINDRICAL SHELLS

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Dedicated to István Páczelt on the occasion of his 65th birthday

Abstract. The main requirements for modern load-carrying structures are safety, producibility and economy. Economy is characterized by cost. We have developed a structural optimization system which assures the safety and producibility by fulfillment of design and fabrication constraints, and economy is achieved by minimization of a cost function. Using this system it is possible to make realistic comparisons between different structural versions. This is based on a cost calculation method which is developed mainly for welded structures. The thickness of plates and shells can be decreased by stiffening. Stiffened structures are economic, when the cost difference caused by thickness reduction is higher than the additional cost of stiffening material and welding. It is shown by numerical problems that stiffening is economic only in case when the sensitivity of plates and shells to buckling or transverse deformation can effectively be eliminated by it.

The economy of stiffened structures depends on the type of load and stiffening. Since plates are very sensitive to buckling and transverse deformation, their stiffening is always effective and economic. Cylindrical shells are sensitive to buckling in the case of external pressure thus ring-stiffening is economic in this case. On the other hand, they are not very sensitive to buckling in the case of axial compression or bending, thus ring-stiffening is uneconomic for these loads and stringer-stiffening is economic only in cases when the transverse deformation of the whole shell is limited.

Mathematical Subject Classification: 74K20, 74K25, 74P10
Keywords: stiffened plates and shells, welded structures, buckling, minimum cost design, structural optimization, fabrication cost calculation, economy

1. **Introduction**

One of the most important characteristics of welded structures is cost for welding is an expensive fabrication process. Cost is a realistic basis for the comparison of different structural versions. Optimization is needed to decrease the cost in order to develop competitive structures.

The most effective method to decrease the structural mass is to decrease thickness. Thin-walled structures show the following problems: large deformations, large residual welding stresses and distortions, buckling, vibration, sensitivity to torsion, fatigue due to large stress concentrations. To avoid these problems, stiffening can be used. Stiffening enables thickness to be decreased, but it incurs the additional cost of stiffener material and welding. Therefore stiffening is economic only in cases when the cost difference caused by thickness reduction is higher than the additional cost of stiffening material and welding.

In our research we have worked out several minimum mass or cost design problems related to stiffened plates and shells [1]. Our experience is that cost savings due to stiffening are different, in some cases the cost of an unstiffened structure is lower than that of the stiffened one. The aim of the present study is to give an overview of these optimization results.

In order to give realistic comparisons between stiffened and unstiffened structures, both structural versions should be optimized. Since optimization processes are complicated, only numerical treatments are possible. Therefore the conclusions of these comparisons cannot be general. In spite of this disadvantage, the results of this study can be useful for designers in selecting the most suitable structural versions.

The main components of an optimum design process are as follows: design constraints, fabrication aspects, cost function and mathematical algorithms of constrained function minimization. These components affect the comparisons, thus, it is important to use realistic aspects mainly in cost calculation. Therefore we try to formulate the cost function as realistically as possible.

For the design constraints the rules of API [2], ECCS [3] or DNV [4] can be used.

2. A welded stiffened plate subject to uniaxial compression

The plate is stiffened by ribs of three types as follows: flat, L-shape and trapezoidal shapes – see Figure 1. For the buckling stress constraint the Mikami-Niwa formulae [5] are used which consider the effect of residual welding stresses and initial imperfections. The unknowns are the number of stiffeners, thickness of the base plate (t_F) and stiffener dimensions (in the case of trapezoidal stiffeners the thickness is t_3, Figure 2).

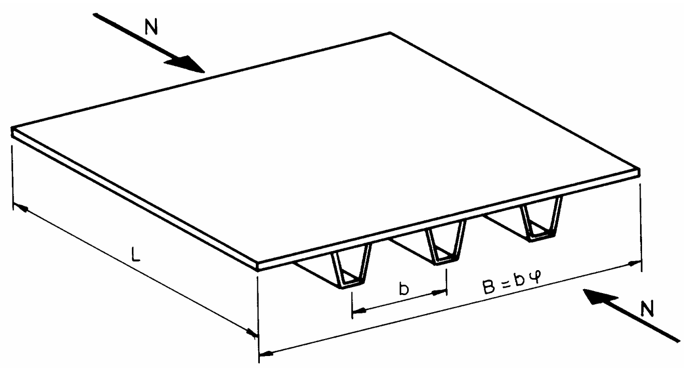

Figure 1. Uniaxially compressed plate with trapezoidal stiffeners

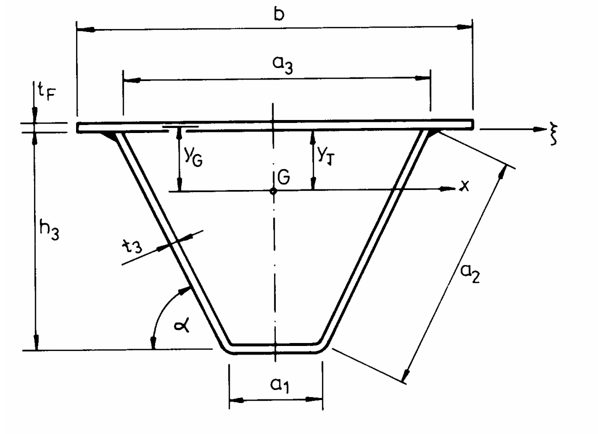

Figure 2. Dimensions of a trapezoidal stiffener

The overall buckling constraint is expressed in function of the reduced slenderness

$$\lambda = (f_y/\sigma_{cr})^{1/2} \tag{1}$$

where σ_{cr} is the classical critical buckling stress, which does not contain the above mentioned effects and, f_y is the yield stress.

The classical critical buckling stress for a uniaxially compressed longitudinally stiffened plate (Figure 1) is

$$\sigma_{cr} = \frac{\pi^2 D}{hB^2}\left(\frac{1+\gamma_S}{\alpha_R^2} + 2 + \alpha_R^2\right) \quad \text{for} \quad \alpha_R = L/B < \alpha_{R0} = (1+\gamma_S)^{1/4} \tag{2}$$

$$\sigma_{cr} = \frac{2\pi^2 D}{hB^2}\left[1 + (1+\gamma_S)^{1/2}\right] \quad \text{for} \quad \alpha_R \geqslant \alpha_{R0} \tag{3}$$

where, with

$$\nu = 0.3\ , \qquad D = \frac{Et_F^3}{12\,(1-\nu^2)} = \frac{Et_F^3}{10.92} \tag{4}$$

we have

$$h = t_F + \frac{A_S}{b}\ , \qquad b = \frac{B}{\varphi}\ , \tag{5a}$$

$$A_S = (a_1 + 2a_2)\,t_3\ , \qquad I_S = a_1 h_3^3 t_3 + \frac{2}{3} a_2^3 t_3 \sin^2\alpha\ . \tag{5b}$$

According to the Stahlbau Handbuch [6] $a_1 = 90$, $a_3 = 300\,\text{mm}$, thus

$$h_3 = \left(a_2^2 - 105^2\right)^{1/2}\ , \qquad \sin^2\alpha = 1 - \left(\frac{105}{a_2}\right)^2\ . \tag{5c}$$

Here A_S is the cross-sectional area of a stiffener, $\varphi - 1$ is the number of stiffeners,

$$\gamma_S = \frac{EI_S}{bD} \tag{6}$$

and I_S is the moment of inertia of a stiffener about the ξ axis.

Local buckling of a trapezoidal stiffener is defined as

$$a_2/t_3 \leqslant 38\varepsilon \ . \tag{7}$$

This constraint is treated as active.

Single panel buckling. This constraint eliminates the local buckling of the base plate parts between the stiffeners. From the classical buckling formula for a simply supported uniformly compressed bar we obtain

$$\sigma_{crP} = \frac{4\pi^2 E}{10.92}\left(\frac{t_F}{b}\right)^2 \tag{8}$$

the reduced slenderness is

$$\lambda_P = \left(\frac{4\pi^2 E}{10.92 f_y}\right)^{1/2}\frac{b}{t_F} = \frac{b/t_F}{56.8\varepsilon} \ , \qquad \varepsilon = \left(\frac{235}{f_y}\right)^{1/2} \tag{9}$$

and the actual local buckling stress considering the initial imperfections and residual welding stresses is

$$\sigma_{UP}/f_y = 1 \qquad \text{for } \lambda_P \leqslant 0.526 \ , \tag{10a}$$

$$\frac{\sigma_{UP}}{f_y} = \left(\frac{0.526}{\lambda_P}\right)^{0.7} \qquad \text{for } \lambda_P \geqslant 0.526 \ . \tag{10b}$$

Then the factor ρ_P is as follows:

$$\rho_P = \begin{cases} 1 & \text{if } \ \sigma_{UP} > \sigma_U \ , \\ \sigma_{UP}/f_y & \text{if } \ \sigma_{UP} \leqslant \sigma_U \ . \end{cases} \tag{11}$$

Knowing the reduced slenderness (1) the actual global buckling stress can be calculated as follows:

$$\sigma_U/f_y = \begin{cases} 1 & \text{for } \ \lambda \leqslant 0.3 \ , \\ 1 - 0.63\,(\lambda - 0.3) & \text{for } \ 0.3 \leqslant \lambda \leqslant 1 \ , \\ 1/\left(0.8 + \lambda^2\right) & \text{for } \ \lambda > 1 \ . \end{cases} \tag{12}$$

The global buckling constraint is defined by

$$\frac{N}{A} \leqslant \sigma_U^* = \sigma_U \frac{\rho_P + \delta_S}{1 + \delta_S} \tag{13}$$

where

$$A = B t_F + (\varphi - 1)\, A_S \tag{14}$$

and

$$\delta_S = \frac{A_S}{b t_F} \ . \tag{15}$$

Here ρ_P can be determined considering the single panel buckling of the base plate parts between the stiffeners. The factor $(\rho_P + \delta_S)/\,(1 + \delta_S)$ expresses the effect of the effective width of the base plate parts.

The cost function is given by

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left(\Theta_{dW}\sqrt{\kappa \rho V} + 1.3 T_2\right) \ , \tag{16}$$

in which V is the structural volume.

For fillet welds using GMAW-M (gas metal arc welding with mixed gas)

$$V = BLt_F + (\varphi - 1)\, LA_S; \quad \kappa = \varphi; \quad T_2 = 0.3258x10^{-3} a_W^2 \, (\varphi - 1)\, 2L \,. \tag{17}$$

Here $a_W = 0.5t_3$ but $a_{Wmin} = 4$ mm.

Numerical example. Given data: $B = 6000\,\text{mm}$, $L = 3000\,\text{mm}$, $N = 1.974 \times 10^7$ [N], $f_y = 235\,\text{MPa}$, $E = 2.1 \times 10^5\,\text{MPa}$, $G = E/2.6$, $\rho = 7.85 \times 10^{-6}\,\text{kg/mm}^3$, $\Theta_{dW} = 3$.

The results of the optimization can be seen in Figure 3. The optimal dimensions are as follows: $t_F = 11$, $t_3 = 8$ mm.

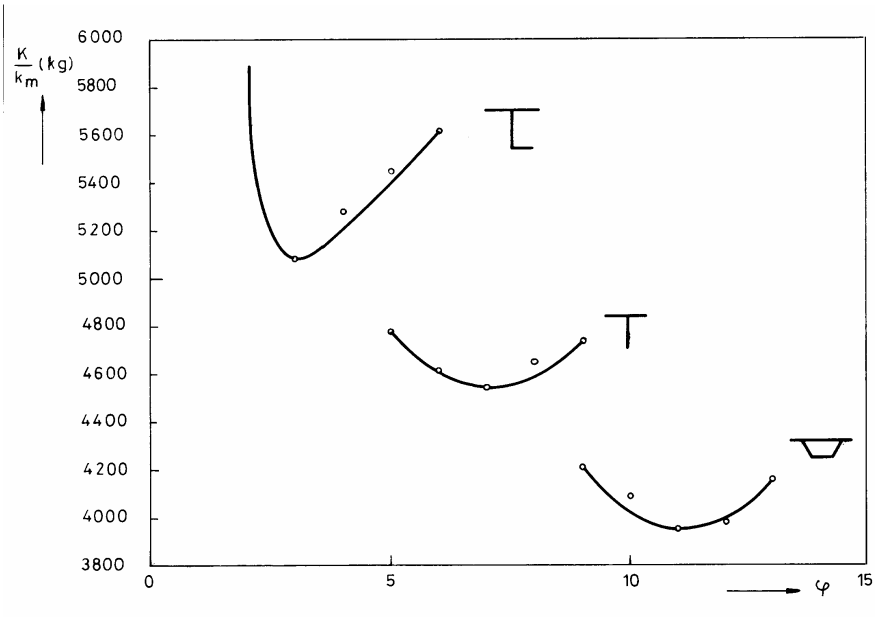

Figure 3. Cost curves in the region of the optimum number of flat, *L*- and trapezoidal stiffeners for $k_f/k_m = 2$ kg/min

For an unstiffened plate we have

$$\gamma_S = 0\,, \quad \alpha_{R0} = 1\,, \quad \alpha_R = 0.5\,, \quad h = t_F\,, \quad D = Et_F^3/10.92$$

and

$$\sigma_{cr} = \frac{6.25\pi^2 E t_F^2}{10.92 B^2}\,; \quad \lambda = \frac{84.45}{t_F}\,. \tag{18}$$

The buckling constraint

$$\frac{N}{A} = \frac{1.974x10^7}{6x10^3 t_F} \leqslant \sigma_U \tag{19}$$

is fulfilled with t_F =51 mm (64.5<66.4 MPa).

The cost of the unstiffened plate is

$$\frac{K}{k_m} = \rho BLt_F = 7206\ \text{kg}.$$

It can be seen that the cost of the optimized stiffened plate with trapezoidal ribs (3940 kg) is by 82% cheaper than the unstiffened one. Thus, in this case the economy of the stiffened structural version is evident.

3. An orthogonally stiffened square plate loaded by bending

A square plate with simply supported edges, loaded by uniformly distributed normal load is considered [1, 7]. The plate is stiffened by flat ribs in two directions (Figure 4). The ribs are continuous in one direction, in the other direction they are interrupted and welded to the others by double fillet welds. The size of a fillet weld is $a_W = 0.4t_S$, but $a_{Wmin} = 4$mm.

Data: $b = 6$ m, $p_0 = 5 \times 10^{-3}$ N/mm^2, the yield stress for steel is 235 MPa, the admissible stress is $\sigma_{adm} = 120$ MPa, the elastic modulus is $E = 2.1 \times 10^5$ MPa, $\rho = 7.85 \times 10^{-6}$ kg/mm^3.

In the optimization procedure we search for the optimum values of the following variables: t_F, h, t_S and φ. The number of stiffeners is $\varphi - 1$.

It is assumed that the base plate is welded with butt welds from 4 strips of dimensions 6m $\times$ 1.5m. Then the stiffeners are welded to the base plate by double fillet welds. Finally the interrupted ribs are welded to the other ribs by double fillet welds.

For the butt welds GMAW-M technology is used, thus, the welding time depends on the thickness of the base plate as follows [1]:

$$T_2' = \begin{cases} 3b \times 0.1861 \times 10^{-3} t_F^2 & \text{for } t_F \leqslant 15\,\text{mm} \\ 3b \times 0.1433 \times 10^{-3} t_F^{1.9035} & \text{for } t_F > 15\,\text{mm} \end{cases} . \tag{20}$$

For longitudinal fillet welds the GMAW-M technology is used, thus, the welding time is

$$T_2'' = 4b(\varphi - 1) \times 0.3258 \times 10^{-3} a_W^2 , \tag{21}$$

for transverse fillet welds the SMAW (shielded metal arc welding) technology is assumed, thus

$$T_2''' = 4h(\varphi - 1)^2 \times 0.7889 \times 10^{-3} a_W^2 . \tag{22}$$

The volume of the structure is

$$V = b^2 t_F + 2(\varphi - 1) b h t_S . \tag{23}$$

The number of assembled elements is $\kappa = 3 + \varphi^2$. The cost function can be formulated as

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left[\Theta_{dW}\left(\kappa \rho V\right)^{0.5} + 1.3\left(T_2' + T_2'' + T_2'''\right)\right] , \tag{24}$$

where $\Theta_{dW} = 3$.

Constraint on compressive stress in the central faceplate element is expressed by

$$\sigma_{\max} = \sigma_{\max.1} + \sigma_{f.\max} \leqslant \sigma_{adm} . \tag{25}$$

Here $\sigma_{\max.1}$ is caused by the bending of the whole plate, $\sigma_{f.\max}$ is the normal stress due to local bending of the plate elements, σ_{adm} is the admissible stress:

$$\sigma_{\max.1} = \frac{c_M p b^2}{I_x/a} y_G . \tag{26}$$

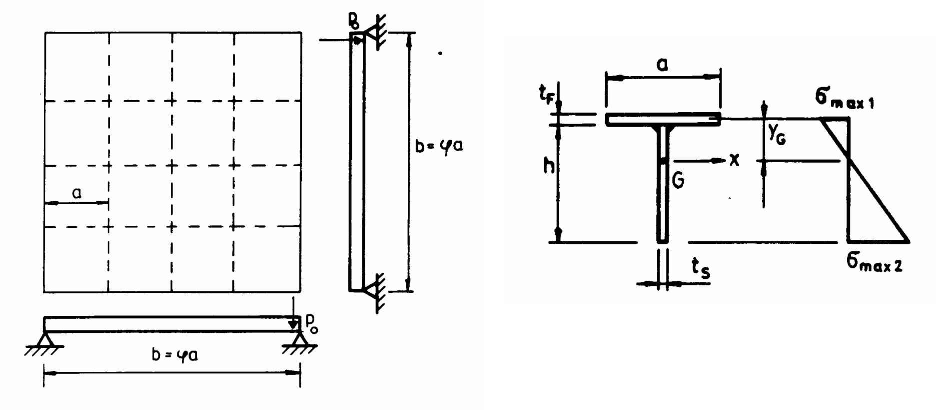

Figure 4. A simply supported transversely uniformly loaded square plate stiffened by flat ribs

The uniformly distributed normal load p contains also the self weight, approximately $p = 1.1p_0$. Since the torsional stiffness of the open section ribs is very small, the stiffened plate can be calculated as an orthotropic one having zero torsional stiffness. Schade [8] calculated for this case $c_M = 0.1102$.

As shown in Figure 4, the distance of the centroidal axis y_G can be calculated as

$$y_G = \frac{h}{2}\frac{1}{1+\alpha}\,, \quad \alpha = \frac{at_F}{ht_S} = \frac{bt_F}{\varphi h t_S} \tag{27}$$

and the moment of inertia is

$$I_x = \frac{h^3 t_S}{12}\frac{1+4\alpha}{1+\alpha}\,. \tag{28}$$

It should be noted that the admissible stress is selected so low that it is not necessary to calculate with an effective width of the face plate, thus $a = b/\varphi$.

The local bending stress can be calculated by means of formulae valid for isotropic square plates with clamped edges (Timoshenko [9])

$$\sigma_{f.\max} = \frac{5.13x10^{-2}p_0 a^2}{t_F^2/6} = \frac{0.3078 p_0 b^2}{\varphi^2 t_F^2}\,. \tag{29}$$

Constraint on the maximum tensile stress in the central ribs can be written as

$$\sigma_{\max.2} = \sigma_{\max.1}(1+2\alpha) \leqslant \sigma_{adm}\,. \tag{30}$$

Constraint on local buckling of the central face plate element compressed from both sides (Farkas & Jármai [10], Volmir [11]). For a plate compressed on one side the buckling factor is $k = 4$. Instead of this value we calculate with $k = 2$. For $k = 4$ Eurocode 3 [12] gives for the limiting slenderness

$$(a/t_F)_{\lim} = 42\varepsilon, \quad \varepsilon = (235/f_y)^{0.5}, \tag{31}$$

f_y is the yield stress, but instead of using the yield stress we can calculate with the maximum stress. Thus, for $k = 2$ the buckling constraint can be written as

$$a/t_F \leqslant 42\varepsilon_1/\sqrt{2} \approx 30\varepsilon_1, \quad \varepsilon_1 = (235/\sigma_{\max.1})^{0.5} \ . \tag{32}$$

Constraint on local buckling of ribs (it is assumed that $\sigma_{\max.2}$ can also be compressive)

$$h/t_S \leqslant 14\varepsilon_2 \ , \qquad \varepsilon_2 = (235/\sigma_{\max.2})^{0.5} \ . \tag{33}$$

Constraint on shear buckling of ribs at the plate edges can be formulated as

$$\tau = \frac{0.42pb^2}{ht_S\varphi} \leqslant \frac{\tau_{ub}}{\gamma_b} = \frac{5.34\pi^2 E}{12(1-\nu^2)\gamma_b}\left(\frac{t_S}{h}\right)^2 \quad \text{for} \quad \frac{\tau_{ub}}{\gamma_b} \leqslant \tau_{adm} \tag{34}$$

and

$$\tau \leqslant \tau_{adm} \quad \text{for} \quad \frac{\tau_{ub}}{\gamma_b} \geqslant \tau_{adm} \ . \tag{35}$$

The factor 0.42 is considered, since the distribution of edge reactions is not uniform along the edges (Timoshenko [9]). $\tau_{adm} = \sigma_{adm}/\sqrt{3}$ is the admissible shear stress, ν is the Poisson ratio.

Constraint on residual distortion due to shrinkage of welds is formulated as follows.

Although the stiffeners are welded along two directions, we do not multiply residual deflection by 2. A multiplying factor of 1.5 is used considering the fact that the longitudinal double fillet welds are intermittent due to interruption of ribs and the residual plastic zones of the continuous welds affect the deflection caused by intermittent welds. Thus the distortion constraint is formulated as

$$f = 1.5Cb^2/8 \leqslant f_{adm} \ , \tag{36}$$

where the admissible deflection is assumed to be $f_{adm} = b/1000 = 6$ mm.

The orthotropic plate bending theory is valid only in the case when the number of stiffeners is more than 3, thus

$$\varphi \geqslant 4 \ . \tag{37}$$

It can be seen that the objective function and the design constraints are highly nonlinear. For the constrained function minimization Rosenbrock's hillclimb mathematical programming method is used (a detailed description can be found in Farkas & Jármai [10]) complemented by an additional discretization of the continuous optima considering rounded dimensions and integer numbers of stiffeners.

The optimization results in the following values: for $k_F/k_m = 2$, $t_F = 11$, $h = 250$, $t_S = 14$ mm, $\varphi = 4$ and the corresponding cost is $K/k_m = 9317$ kg. The maximum deflection in the center due to the normal load is

$$w_{\max} = 0.0083\frac{p_0 b^4}{B_x} = 6.06 < \frac{b}{300} = 20 \text{ mm} \ , \tag{38}$$

where

$$B_x = \frac{EI_x}{a} = 8.8684 \times 10^9 \,\text{Nmm} \ , \quad I_x = \frac{h^3 t_S}{12}\frac{1+4\alpha}{1+\alpha} = 6.3346 \times 10^7 \,\text{mm}^4 \ , \tag{39}$$

$$\alpha = \frac{at_F}{ht_S} = 4.7143 \ .$$

The thickness required for an unstiffened plate to fulfil the deflection constraint of

$$w_{\max} = 0.065 \frac{p_0 b^4}{E t_F^3/10.92} \leqslant w_{adm} = 20\,\text{mm} \tag{40}$$

is $t_{F0} = 103$ mm and the corresponding cost is $K/k_m = \rho b^2 t_{F0} = 29108$ kg.

It can be seen that the stiffened plate which fulfils the stress and deflection constraints is much cheaper (9317 kg) than the unstiffened one (29108 kg). It can be concluded that a plate subject to normal load is very weak against deflection, thus stiffening in this case is also economic.

From the above cases of stiffened plates it can be concluded that their stiffening is economic, since they are very sensitive to buckling as well as to large transverse deformation and stiffening eliminates these sensitivities.

4. **A ring-stiffened cylindrical shell subject to external pressure**

The design constraints are formulated according to API rules [2].

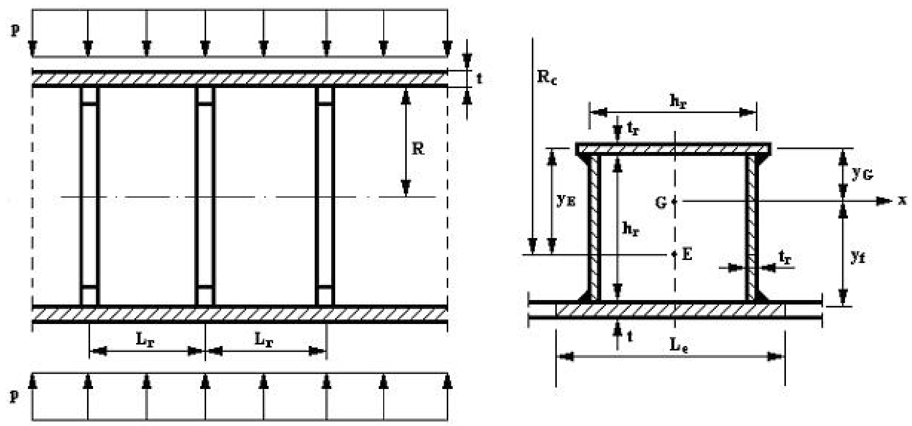

Figure 5. Ring-stiffened cylindrical shell

Geometric characteristics. Data: length of shell $L_b = 15$ m, welded from 5 m long segments, radius of shell $R = 1850$ mm, intensity of the external pressure $p = 0.5$ MPa, yield stress of steel $f_y = 355$ MPa. To avoid tilting of the ring-stiffeners, the welded square box section is used, which is characterized by height h_r and thickness t_r. Considering the local buckling constraint of the stiffener flange active, we use the following correlation between height and thickness

$$t_r = \delta_r h_r; \delta_r = 1/42\varepsilon; \varepsilon = \sqrt{235/f_y}; \delta_r = 1/34\ . \tag{41}$$

The cross-sectional area of a stiffener is

$$A_r = 3 h_r t_r = 3 \delta_r h_r^2\ . \tag{42}$$

The distances of centroid G are as follows (Figure 5)

$$y_G = \frac{h_r}{3}\,, \qquad y_r = \frac{2h_r}{3} + \frac{t}{2}\,, \tag{43}$$

$$y_E = \frac{L_e t\left(h_r + \frac{t}{2}\right) + \delta_r h_r^3}{3\delta_r h_r^2 + L_e t}\,, \tag{44}$$

$$L_e = \begin{cases} 1.1\sqrt{2Rt} & \text{if } \; M_x = \dfrac{L_r}{\sqrt{Rt}} > 1.56\,, \\ L_r & \text{if } \; M_x \leqslant 1.56\,. \end{cases} \tag{45}$$

The distance of the centroid E of the cross-section consisting of the stiffener and the effective part of the shell is characterized by

$$R_C = R - \left(h_r - y_E + \frac{t}{2}\right)\,. \tag{46}$$

The moment of inertia of the stiffener and the effective part of shell is

$$I_{er} = \frac{\delta_r h_r^4}{6} + A_r y_r^2 K_G + \frac{L_e t^2}{12}; \qquad K_G = \frac{L_e t}{A_r + L_e t}\,, \tag{47a}$$

where

$$K_G = \frac{L_e t}{A_r + L_e t}\,. \tag{47b}$$

Design constraints

Constraint on local shell buckling

$$\gamma_b p \frac{R}{t} \leqslant \eta_L \sigma_{UL}\,, \tag{48}$$

where $\gamma_b = 1.5$ is the safety factor and the plasticity reduction factor η_L is calculated in function of $\delta_L = \sigma_{UL}/f_y$ as follows

$$\eta_L = \begin{cases} 1 & \text{if } \; \delta_L \leqslant 0.55\,, \\ \dfrac{0.45}{\delta_L} + 0.18 & \text{if } \; 0.55 < \delta_L \leqslant 1.6\,, \\ \dfrac{1.31}{1 + 1.15\delta_L} & \text{if } \; 1.6 < \delta_L < 6.25\,, \\ \dfrac{1}{\delta_L} & \text{if } \; \delta_L \geqslant 6.25\,. \end{cases} \tag{49}$$

The ultimate local buckling strength is

$$\sigma_{UL} = \alpha_L p_{eL} \frac{R}{t} K_L\,, \tag{50}$$

where $\alpha_L = 0.8$ is the imperfection factor, and for our numerical example $K_L = 1$.

$$p_{eL} = \begin{cases} \dfrac{1.27E}{A^{1.18}+0.5}\left(\dfrac{t}{R}\right)^2 & \text{if} \quad M_x > 1.5 \ \text{ and } \ A = M_x - 1.17 < 2.5 \ , \\ \dfrac{0.92E}{A}\left(\dfrac{t}{R}\right)^2 & \text{if} \quad 2.5 < A < 0.208Rt \ , \\ 0.836C_P^{-1.061}E\left(\dfrac{t}{R}\right)^3 & \text{if} \quad 0.208 < C_P = \dfrac{A}{R/t} < 2.85 \ , \\ 0.275E\left(\dfrac{t}{R}\right)^3 & \text{if} \quad C_P > 2.85 \ . \end{cases} \tag{51}$$

Constraint on general shell buckling

$$\gamma_b p \frac{R}{t} \leqslant \eta_G \sigma_{UG} \ , \tag{52}$$

where the plasticity reduction factor η_G is calculated in function of $\delta_G = \sigma_{UG}/f_y$ with the same formulae as in the case of η_L – see equation (44):

$$\sigma_{UG} = \frac{\alpha_G}{1.2} p_{eG} \frac{R}{t} K_G \ . \tag{53}$$

Here $\alpha_G = 0.8$ is the imperfection factor, K_G is defined by equation $(42)_2$, and a factor of 1.2 is recommended to avoid the mode interaction (coupled instability):

$$p_{eG} = \frac{E\frac{t}{R}\lambda_G^4}{(n^2-1)\left(n^2+\lambda_G^2\right)^2} + \frac{EI_{er}\left(n^2-1\right)}{L_r R_C^2 R} \ , \tag{54}$$

where

$$\lambda_G = \frac{\pi R}{L_b} = \frac{1850\pi}{15000} = 0.3875,$$

n is the value which gives the minimum value of p_{eG}, $n_{min} = 2$, $n_{max} = 10$. For our case $n = 2$ is used.

The cost function includes the material, fabrication and painting costs:

$$K = K_M + K_F + K_P \ . \tag{55}$$

The material cost is

$$K_M = k_M \rho V \ , \tag{56}$$

k_M [\$/kg] is the material cost factor and V is the volume of the structure:

$$V = 2\pi R t L_b + n_r \left[4\pi\delta_r h_r^2\left(R - \frac{h_r}{2}\right) + 2\pi\delta_r h_r^2\left(R - h_r\right)\right] \ , \tag{57}$$

where n_r is the number of ring-stiffeners.

For a fabrication phase it is

$$K_F = k_F\left(\Theta_{dW}\sqrt{\kappa\rho V} + 1.3C_W a_W^n L_W\right) \ , \tag{58}$$

where k_F (\$/min) is the fabrication cost factor, Θ_{dW}=3 is the difficulty factor expressing the complexity of a structure regarding assembly, the first member calculates time for assembly and tacking, κ is the number of structural parts to be assembled, the second member calculates the time of welding and additional works (changing the electrode, deslagging, chipping). The additional works are considered by a factor of 1.3. L_W is the weld length, a_W is the weld size, C_W and n are given for different welding technologies and weld type (butt or fillet).

The fabrication cost function is formulated according to the fabrication sequence as follows.

(1) Welding of a shell segment from 3 parts without stiffeners with GMAW-C butt welds, number of structural parts to be assembled is 3

$$K_{F1} = 3\sqrt{3\rho V_S} + 1.3x0.2245x10^{-3}t^2x3L_S \ , \tag{59}$$

where $L_S = 3000$ mm, $V_S = 2R\pi t L_S$.

(2) Welding of a ring-stiffener from 3 plate parts with 2 fillet welds of GMAW-C (gas metal arc welding with CO_2), weld size $a_W = 0.7t_r$

$$K_{F2} = 3\sqrt{3\rho V_r} + 1.3x0.3394x10^{-3}a_W^2 x4\pi\left(R - h_r\right) \ , \tag{60}$$

where $V_r = 4\pi\delta_r h_r^2\left(R - \frac{h_r}{2}\right) + 2\pi\delta_r h_r^2\left(R - h_r\right)$.

(3) Welding of $n_r/5$ stiffeners to a shell segment with 2 fillet welds of size $a_W = 0.7t_r$, GMAW-C

$$K_{F3} = 3\sqrt{\left(\frac{n_r}{5} + 1\right)\rho V_3} + 1.3 \times 0.3394 \times 10^{-3}a_w^2 x4\pi R n_r/5 \ , \tag{61}$$

where $V_3 = V_S + V_r n_r/5$.

(4) Welding of 5 stiffened shell segments together with butt welds GMAW-C

$$K_{F4} = 3\sqrt{5\rho 5V_3} + 1.3 \times 0.2245 \times 10^{-3}t^2 \times 8R\pi \tag{62}$$

The total material cost is

$$K_M = k_M\rho 5V_3 \ . \tag{63}$$

The total fabrication cost is

$$K_F = k_F\left(5K_{F1} + n_r K_{F2} + 5K_{F3} + K_{F4}\right) \ . \tag{64}$$

The painting cost is

$$K_P = k_P\left[2R\pi L_b + 2R\pi\left(L_b - n_r h_r\right) + 2\pi\left(R - h_r\right)h_r + 4\pi\left(R - \frac{h_r}{2}\right)\right] \ . \tag{65}$$

In the numerical example the following cost factors are used: $k_M = 1.0$ \$/kg, $k_F = 1.0$ \$/min and $k_P = 28.8 \times 10^{-6}$ \$/mm^2.

The results of the optimum design are as follows: $n_r = 15$, $h_r = 160$, $t = 10$ mm and the total minimum cost is $K = 38964$ \$.

Comparing the ring-stiffened design with an *unstiffened* one, it is calculated that, for the unstiffened version a shell thickness of $t = 34$ mm is required. In the case considered here, where $M_x = 59.81$, $A = 58.64$, $C_P = 1.1094$, $p_{el} = 0.9761$ MPa and $1.5 \times 0.5 = 0.75 < 0.8 \times 0.9761 = 0.78$ MPa is satisfactory. The costs of the unstiffened version are as follows: $K_{F1} = 2140$ \$, $K_{F4} = 9916$ \$, $K_F = 20616$ \$, $K_M = 46535$ \$, $K_P = 10043$ \$, the total cost is $K = 77194$ \$. This is 100(77194-38964)/38964 = 98% higher than that of the stiffened version.

It can be concluded that, in the case of cylindrical shells subject to external pressure, ring-stiffening is economic, since the shell is very sensitive to buckling due to external pressure.

5. Ring-stiffened cylindrical shells subject to axial compression or bending

According to DNV design rules (Det Norske Veritas [4]) the constraint on local shell buckling is expressed by

$$\sigma_{\max} \leqslant \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \tag{66}$$

$$\lambda^2 = \frac{f_y}{\sigma_E}, \qquad \sigma_E = (1.5-50\beta)\, C \frac{\pi^2 E}{10.92}\left(\frac{t}{L_r}\right)^2 \tag{67}$$

$$L_r = \frac{L}{n+1} \,. \tag{68}$$

The factor of (1.5-50β) in equation (51) expresses the effect of initial radial shell deformation caused by the shrinkage of circumferential welds and can be calculated as follows (Farkas [7], Farkas & Jármai [1]).

The maximum radial deformation of the shell caused by the shrinkage of a circumferential weld is

$$u_{\max} = 0.64 A_T \sqrt{R/t}\,, \tag{69}$$

where $A_T t$ is the area of specific strains near the weld. According to our results (Farkas & Jármai [13])

$$A_T t = \frac{0.3355 Q_T \alpha_0}{c_0 \rho} \,. \tag{70}$$

For steels it is

$$A_T t = 0.844 \times 10^{-3} Q_T \qquad (A_T t\,,\ \text{in mm}^2\ \ Q_T\ \ \text{in J/mm})\,, \tag{71}$$

$$Q_T = \eta_0 \frac{UI}{v_W} = C_A A_W \,. \tag{72}$$

For manually arc welded butt welds

$$Q_T = 60.7 A_W \qquad (A_W\ \ \text{in mm}^2)\,. \tag{73}$$

Here

$$A_W = \begin{cases} 10t & \text{if} \quad t \leqslant 10 \ \text{ mm}\,, \\ 3.05 t^{1.45} & \text{if} \quad t > 10 \ \text{ mm}\,. \end{cases} \tag{74}$$

Introducing a reduction factor of β for which

$$0.01 \leqslant \beta = \frac{u_{\max}}{4\sqrt{Rt}} \leqslant 0.02 \tag{75}$$

the imperfection factor for shell buckling strength should be multiplied by $(1.5-50\beta)$.

$$\text{For} \quad \beta \leqslant 0.01\,, \quad \beta = 0.01\,; \qquad \text{for} \quad \beta \geqslant 0.02 \quad \beta = 0.02. \tag{76}$$

For $t > 9$ mm $1.5 - 50\beta = 1$.

Furthermore

$$C = \psi\sqrt{1 + \left(\frac{\rho_0 \xi}{\psi}\right)^2}\,, \qquad Z = 0.9539\frac{L_r^2}{Rt}\,. \tag{77}$$

In the case of bending it is

$$\psi = 1\,, \qquad \xi = 0.702Z\,, \qquad \rho_0 = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5}\,. \tag{78}$$

In the case of axial compression

$$\psi = 1\,, \qquad \xi = 0.702Z\,, \qquad \rho_0 = 0.5\left(1 + \frac{R}{150t}\right)^{-0.5}\,. \tag{79}$$

It can be shown that the critical buckling stress does not depend on the distance of ring stiffeners (L_r).

If

$$\rho_0\xi > 10\,, \qquad C \approx \rho_0\xi = 0.702 \times 0.9539\rho_0\frac{L_r^2}{Rt} \tag{80}$$

and

$$\sigma_E = 0.702 \times 0.9539\rho_0\frac{\pi^2 Et}{10.92R} \tag{81}$$

does not depend on L_r.

From equation (80) we obtain

$$L_r > 3.8644\sqrt{\frac{Rt}{\rho_0}}\,. \tag{82}$$

E.g. for $R = 1800$, $t = 18$ mm in the case of axial compression $\rho_0 = 0.3873$; $L_r > 1117$ mm, in the case of bending $\rho_0 = 0.4330$; $L_r > 1057$ mm. Therefore it can be concluded that the shell thickness can be decreased only if $L_r < 1000$ mm. This means a very dense stiffening, the welding cost of which is very high and the stiffened shell is uneconomic. The fact that the buckling strength does not depend on the shell length is first derived by Timoshenko and Gere [14].

Ring-stiffening cannot be economic for axially compressed or bent cylindrical shells, since the shell is not very sensitive to buckling for such loads, its stiffness against buckling is large. Ring-stiffening can be used to ensure the accurate cylindrical shape of the shell. In this case the designer can select a realistic domain of the number of ring-stiffeners and can search for the optimum stiffener number in this region minimizing the cost function (Farkas et al. [15]), but this minimal cost will be higher than that of an unstiffened shell.

6. A stringer-stiffened cylindrical shell loaded by bending

The shell is a supporting bridge for a belt-conveyor, simply supported with a given span length of $L = 60$ m and radius of $R = 1850$ mm (Figures 6, 7 and 8). The intensity of the factored uniformly distributed vertical load is $p = 26.0$ N/mm + self mass. Factored live load is 20.0 N/mm, dead load (belts, rollers, service-walkway) is 6.0 N/mm. For self mass a safety factor of 1.35 is used, which is prescribed by

Eurocode 3 (1992). The safety factor for variable load is 1.5. Outside stringers are made of half-rolled I-section (Universal Beam sections). The yield stress of the steel is $f_y = 355$ MPa.

Inside stiffeners are uneconomic, since their moment of inertia in the calculation of central deflection is much lower than that of outside ones. Instead of rolled I-section stiffeners it is better to use half-rolled I-sections, since welding the flange to the curved shell surface is difficult.

Variables: shell thickness t, number and cross-sectional area of longitudinal stiffeners (stringers) n_s, A_s.

Constraints

(1) *Shell buckling (unstiffened curved panel buckling)*

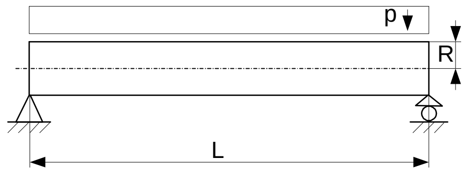

Figure 6. A simply supported cylindrical shell subject to uniformly distributed normal load

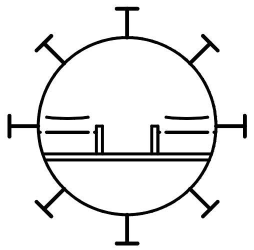

Figure 7. Cross-section of a belt conveyor bridge with two belt conveyors and a service walkway in the middle

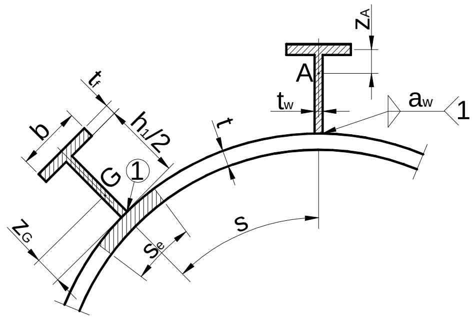

Figure 8. Part of the stiffened shell

$$\sigma_a = \frac{M}{R^2 \pi t_e} \leqslant \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \tag{83}$$

$$\lambda^2 = \frac{f_y}{\sigma_E}; \qquad t_e = t + \frac{A_s}{2s}; \qquad s = \frac{2R\pi}{n_s} \tag{84}$$

$$M = \frac{pL^2}{8}; \qquad p = 26.0 + 1.35\rho\left(2R\pi t_e\right); \quad \rho = 7.85 \times 10^{-5}\,\mathrm{N/mm^3} \tag{85}$$

$$\sigma_E = C\left(1.5 - 50\beta\right)\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{s}\right)^2 \tag{86}$$

$$C = 4\sqrt{1+\left(\frac{\rho_e \xi}{4}\right)^2}; \qquad Z = \frac{s^2}{Rt}\sqrt{1-\nu^2} \tag{87}$$

$$\rho_e = 0.5\left(1+\frac{R}{150t}\right)^{-0.5}; \qquad \xi = 0.702Z\,. \tag{88}$$

Note that the calculation of β is detailed in Section 5.

(2) *Stringer panel buckling*

$$\sigma_a = \frac{M}{R^2 \pi t_e} \leqslant \sigma_{crp} = \frac{f_y}{\sqrt{1+\lambda_p^4}} \tag{89}$$

$$\lambda_p^2 = \frac{f_y}{\sigma_{Ep}}; \qquad \sigma_{Ep} = C_p \frac{\pi^2 E}{10.92}\left(\frac{t}{L}\right)^2 \tag{90}$$

$$C_p = \psi_p \sqrt{1+\left(\frac{0.5\xi_p}{\psi_p}\right)^2}; \qquad Z_p = 0.9539\frac{L^2}{Rt} \tag{91}$$

$$\xi_p = 0.702Z_p; \qquad \gamma_s = 10.92\frac{I_{sef}}{st^3} \tag{92}$$

$$\psi_p = \frac{1+\gamma_s}{1+\frac{A_s}{2s_e t}}\,. \tag{93}$$

Since DNV rules give the effective shell part s_e by too complicated an iteration, we use the simpler ECCS (1988) rules:

$$s_E = 1.9t\sqrt{\frac{E}{f_y}} \tag{94}$$

$$\text{If } s_E < s\,, \quad s_e = s_E; \qquad \text{if } s_E > s\,, \quad s_e = s\,.$$

I_{sef} is the moment of inertia of a cross section containing the stiffener and a shell part of width s_e

$$I_{sef} = s_e t\left(\frac{t}{2}+z_G\right)^2 + \frac{s_e t^3}{12} + \frac{t_w}{12}\left(\frac{h_1}{2}\right)^3 + \\ + \frac{h_1 t_w}{2}\left(\frac{h_1}{4} - z_G\right)^2 + \frac{b t_f^3}{12} + b t_f\left(\frac{h_1+t_f}{2} - z_G\right)^2 \tag{95}$$

$$z_G = \frac{h_1^2 t_w/8 + h_1 b t_f/2}{h_1 t_w/2 + b t_f + s_e t} \,. \tag{96}$$

(3) *Deflection constraint.* The moment of inertia is calculated here approximately by the formula of $\pi R^3 t_e$, while the exact expression is given by equation (132).

$$w_{\max} = \frac{5 p_0 L^4}{384 E \pi R^3 t_e} \leqslant w_{allow} = \frac{L}{\phi}; \qquad \phi = 500 - 1000 \tag{97}$$

$$p_0 = 20/1.5 + 6.0/1.35 + \rho 2 R \pi t_e = 17.78 + \rho 2 R \pi t_e \,. \tag{98}$$

The selected UB rolled I-sections are given in Table 1.

Table 1. Characteristics of the selected UB rolled I-sections

UB Profile	h mm	b mm	t_w mm	t_f mm	A mm^2	$I_y \times 10^{-4}$ mm^4
$152 \times 89 \times 16$	152.4	88.7	4.5	7.7	2032	834
$168 \times 102 \times 19$	177.8	101.2	4.8	7.9	2426	1356
$203 \times 133 \times 25$	203.2	133.2	5.7	7.8	3187	2340
$254 \times 102 \times 25$	257.2	101.9	6.0	8.4	3204	3415
$305 \times 102 \times 28$	308.7	101.8	6.0	8.8	3588	5366
$356 \times 127 \times 39$	353.4	126.0	6.6	10.7	4977	10172
$406 \times 140 \times 46$	403.2	142.2	6.8	11.2	5864	15685
$457 \times 152 \times 60$	454.6	152.9	8.1	13.3	7623	25500
$533 \times 210 \times 92$	533.1	209.3	10.1	15.6	11740	55230
$610 \times 229 \times 113$	607.6	228.2	11.1	17.3	14390	87320
$686 \times 254 \times 140$	683.5	253.7	12.4	19.0	17840	136300
$762 \times 267 \times 173$	762.2	266.7	14.3	21.6	22040	205300
$838 \times 292 \times 194$	840.7	292.4	14.7	21.7	24680	279200
$914 \times 305 \times 224$	910.4	304.1	15.9	23.9	28560	376400

The characteristic data of the UB rolled I-sections can approximately be expressed by the main parameter of section height h (approximately equalling the first number of the profile name) as follows:

$$A_S = 1093.24394022488 + 0.0336839947 h^2 \,, \tag{99}$$

$$t_f = \sqrt{34.552565817 + 0.0006518757864 h^2} \,, \tag{100}$$

$$b = \sqrt{4676.099669 + 0.11159269 h^2} \,, \tag{101}$$

$$t_w = \sqrt{16.154183 + 4.228419 x 10^{-5} h^2 \ln h} \,. \tag{102}$$

The cost function

Fabrication sequence:

(1) Fabrication of 20 shell elements of length 3 m without stiffeners. For one shell element 2 axial butt welds are needed (GMAW-C) (K_{F1}). The cost of forming a shell element into the cylindrical shape is also included (K_{F0}).

(2) Welding of an unstiffened shell unit from 4 shell segments of 3m length with 3 circumferential butt welds (K_{F2}).

(3) Welding the n_s stiffeners to the unit with $2n_s$ fillet welds of size a_w and length 12 m (K_{F3}), $a_w = 0.3t_w$, $a_{wmin} = 3$ mm.
(4) Welding the 5 units together with 4 butt welds and butt welds connecting the half UB stiffeners (K_{F4}).

The material cost is

$$K_M = k_{M1} 5\rho_1 V_2 \ , \tag{103}$$

$$V_2 = 4V_1 + n_S \frac{A_s L}{2 \times 5} \ , \qquad V_1 = 3000 \times 2R\pi t \ . \tag{104}$$

According to data obtained from a Hungarian manufacturing company (Jászberényi Aprítógépgyár, Crushing Machine Factory, Jászberény), K_{F0} can be expressed in function of shell thickness and diameter as follows (valid for $t = 4 - 40$ mm and $2R = 1500 - 3500$ mm, for width of 3000 mm)

$$K_{F0} = k_F \Theta e^{\mu} \ , \qquad \mu = 6.8582513 - 4.527217t^{-0.5} + 0.009541996\,(2R)^{0.5} \ , \tag{105}$$

$$K_{F1} = k_F \left(\Theta \sqrt{\kappa \rho_1 V_1} + 1.3 \times 0.152 \times 10^{-3} t^{1.9358} \times 6000 \right) \ , \tag{106}$$

$$\Theta = 2; \ \kappa = 2; \ \rho_1 = 7.85 \times 10^{-6} \mathrm{kg/mm}^3 \ ,$$

$$K_{F2} = k_F \left(\Theta \sqrt{4x4\rho_1 V_1} + 1.3x0.152x10^{-3} t^{1.9358} 6R\pi \right) \ , \tag{107}$$

$$k_F = 1.0\,\$/\mathrm{min}, \qquad k_{M1} = 1.0\$/\mathrm{kg} \ ,$$

where Θ is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled.

$$K_{F3} = k_F \left(\Theta \sqrt{(n_s + 1)\,\rho_1 V_2} + 1.3 \times 0.3394 \times 10^{-3} a_W^2 2L n_s/5 \right) \ , \tag{108}$$

$$K_{F4} = k_F \left(\Theta \sqrt{5 \times 5 \rho_1 V_2} \right) + \\ + k_F 1.3x0.152x10^{-3} \left(8R\pi t^{1.9358} + n_S \frac{h_1}{2} t_w^{1.9358} + n_S b t_f^{1.9358} \right) \ , \tag{109}$$

$$h_1 = h - 2t_f \ . \tag{110}$$

The cost of painting is

$$K_P = k_P \left(4R\pi L + n_s \frac{A_L L}{2} \right); \qquad k_P = 14.4 \times 10^{-6} \$/\mathrm{mm}^2 \ , \tag{111}$$

$$A_L = 2h_1 + 4b \ . \tag{112}$$

The total cost is

$$K = K_M + 20K_{F1} + 20K_{F0} + 5K_{F2} + 5K_{F3} + K_{F4} + K_P \ . \tag{113}$$

In order to compare the stiffened shell with the unstiffened one, the constraints of an *unstiffened shell* are given as follows.

(1) *Shell buckling*

$$\sigma_b = \frac{M}{R^2 \pi t} \leqslant \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} , \tag{114}$$

$$\lambda^2 = \frac{f_y}{\sigma_E} , \tag{115}$$

$$\sigma_E = C\,(1.5 - 50\beta)\,\frac{\pi^2 E}{10.92}\left(\frac{t}{L}\right)^2 , \tag{116}$$

$$C = \sqrt{1 + (\rho_e \xi)^2}; \qquad Z = \frac{L^2}{Rt} 0.9539 , \tag{117}$$

$$\rho_e = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5}; \qquad \xi = 0.702Z . \tag{118}$$

(2) *Vertical deflection*

$$w_{\max} = \frac{5 p_0 L^4}{384 E \pi R^3 t} \leqslant w_{allow} = \frac{L}{\phi} . \tag{119}$$

The cost function

Fabrication sequence:

(1) Fabrication of 20 shell elements of length 3 m without stiffeners. For one shell element 2 axial butt welds are needed (GMAW-C) (K_{F1}). The cost of forming a shell element into the cylindrical shape is also included (K_{F0}).
(2) Welding the 20 units together with 19 butt welds (K_{F2}).

The material cost is

$$K_M = k_{M1} 20 \rho_1 V_1 , \tag{120}$$

$$V_1 = 3000x2R\pi t , \tag{121}$$

$$K_{F0} = k_F \Theta e^{\mu}; \qquad \mu = 6.8582513 - 4.527217 t^{-0.5} + 0.009541996\,(2R)^{0.5} , \tag{122}$$

$$K_{F1} = k_F\left(\Theta\sqrt{\kappa \rho_1 V_1} + 1.3 \times 0.152 \times 10^{-3} t^{1.9358} \times 6000\right) , \tag{123}$$

$$\Theta = 2; \quad \kappa = 2; \quad \rho_1 = 7.85x10^{-6}\mathrm{kg/mm}^3 ,$$

$$K_{F2} = k_F\left(\Theta\sqrt{20x20\rho_1 V_1} + 1.3 \times 0.152 \times 10^{-3} t^{1.9358} 38R\pi\right) , \tag{124}$$

$$k_F = 1.0\,\$/\mathrm{min}, \quad k_{M1} = 1.0\$/\mathrm{kg} .$$

The cost of painting is

$$K_P = k_P\,(4R\pi L)\,; \quad k_P = 14.4x10^{-6}\$/\mathrm{mm}^2 . \tag{125}$$

The total cost is

$$K = K_M + 20K_{F1} + 20K_{F0} + K_{F2} + K_P . \tag{126}$$

A numerical example using manual calculation

Assuming that the deflection constraint is active, i.e. selecting a low allowable deflection with a value of $\phi = 1000$, it is possible to determine the required thickness of an *unstiffened* shell. From the deflection constraint – see equation (119) – one obtains an equation for the required thickness

$$t \geqslant \frac{5L^3\phi}{384E\pi R^3}\left(17.78 + \rho 2R\pi t\right) \ . \tag{127}$$

For $L = 60$ m, $R = 1850$ mm, $E = 2.1 \times 10^5$ MPa, $\phi = 1000$, $\rho = 7.85 \times 10^{-5}\text{N/mm}^3$

$$t \geqslant 0.6733\left(17.78 + 0.9125t\right) \ . \tag{128}$$

Solving this equation we get $t = 32$ mm.

Checking this unstiffened shell for buckling we obtain the following values – see equations (114-118): $p = 65.42$ N/mm, $M = 29.439 \times 10^9$ Nmm, $\sigma_{\max} = 85.56$ MPa, $Z = 58.01 \times 10^3$, $\xi = 40.72 \times 10^3$, $\rho_e = 0.4578$, $C = 18642$, $\sigma_E = 1006$ MPa, $\sigma_{cr} = 334.8 > 85.56$ MPa, OK.

The cost calculation of the unstiffened shell results in the following values – see equations (120-126): $V_1 = 1.1159 \times 10^9$ mm^3, $K_M = 175195\,\$$, $K_{F0} = 1528\,\$$, $K_{F1} = 1159\,\$$, $K_{F2} = 39517\,\$$, $K_P = 20086\,\$$, the total cost is $K = 288538\,\$$.

Calculation of a stringer-stiffened shell with the same deflection constraint. Equation (128) results in a formula for t_e

$$t_e \geqslant 0.6733\left(17.78 + 0.9125t_e\right) \tag{129}$$

which gives $t_e = 32$ mm. t_e contains all the three unknowns (t, n_s, A_s)

$$t_e = t + \frac{n_s A_s}{4R\pi} \tag{130}$$

since we use for a stiffener only the half cross-section of a rolled I-section ($A_s/2$).

For the minimum cost solution we need a mathematical algorithm, but, to show the cost savings achieved by decrease of the unstiffened shell thickness of 32 mm, we use a manual calculation and take $t = 10$ mm. If we select a rolled I-section of UB $914 \times 305 \times 289$ ($A_s = 36830$ mm^2), then the required number of stiffeners from equation (130) is $n_s = 10$.

The selected UB section has the following dimensions: $h = 926.6$, $h_1 = 862.6$, $b = 307.7$, $t_w = 19.5$, $t_f = 32$ mm, the half surface for the calculation of painting cost is $A_L/2 = 1506$ mm^2/mm.

Note that this UB profile is not included in Table 1.

Checking the stringer-stiffened shell for buckling using equations (83-88).

$p = 38.318$ N/mm, $M = 172.43 \times 10^8$ Nmm, $\sigma_{\max} = 62$ MPa, $s = 1162$ mm, $Z = 69.62$, $\xi = 48.87$, $\rho_e = 0.3346$, $C = 16.83$, $\sigma_E = 237$ MPa, $\sigma_{cr} = 197 > 62$ MPa, OK.

Check of stringer panel buckling using equations (89-96) .

$s_e = 642$ mm, $z_0 = 264.9$ mm, $I_{sef} = 7.476 \times 10^8$ mm^4, $\gamma_s = 7025.6$, $\psi = 1409.3$, $\rho_e = 0.5$, $Z = 185.6 \times 10^3$, $\xi = 130310$, $C = 65166$, $\sigma_E = 343.6$ MPa, $\sigma_{cr} = 247 > 62$ MPa, OK.

Check of deflection. The exact calculation of the moment of inertia for the deflection uses the following formulae (Figure 8):

The distance of the center of gravity for the half UB section is

$$z_A = \frac{h_1 t_w/2\,(h_1/4 + t_f/2)}{h_1 t_w/2 + b t_f} \ . \tag{131}$$

The moment of inertia of the half UB section is expressed by

$$I_x = b t_f z_A^2 + \frac{t_w}{12}\left(\frac{h_1}{2}\right)^3 + \frac{h_1 t_w}{2}\left(\frac{h_1}{4} - z_A\right)^2 \ . \tag{132}$$

The moment of inertia of the whole stiffened shell cross-section is

$$I_{x0} = \pi R^3 t + I_x \sum_{i=1}^{n_s} \sin^2\left(\frac{2\pi i}{n_s}\right) +$$

$$+\left(\frac{h_1 t_w}{2} + b t_f\right)\left(R + \frac{h_1 + t_f}{2} - z_A\right)^2 \sum_{i=1}^{n_s} \sin^2\left(\frac{2\pi i}{n_s}\right) \tag{133}$$

In our case $z_A = 106.7$ mm, $I_x = 3.83 \times 10^8\,\mathrm{mm}^4$, $I_{x0} = I_x + (1.989 + 4.400)10^{11} = 6.393x10^{11}\,\mathrm{mm}^4$, $p_0 = 41.36$ N/mm.

The approximate formula for the moment of inertia gives a smaller value of $I_{x0} \approx \pi R^3 t_e = 5.14x10^{11}\,\mathrm{mm}^4$ because, with equation (130) $t_e = 25.84$ mm.

The exact deflection is $w_{\max} = \frac{5 p_0 L^4}{384 E I_{x0}} = 52 < 60$ mm, OK.

It can be concluded that the approximate formula of I_{x0} gives a value on the safe side. Note that, in the case of inside half UB section stiffeners, this approximate formula overestimates the exact value, thus, outside stiffeners are more effective than inside ones.

Cost calculation of the stiffened shell using equations (103-113).

$V_1 = 3.4872 \times 10^8$ mm^3, $V_2 = 3.6047 \times 10^9$ mm^3, $K_M = 141485$ \$, $K_{F0} = 812.55$ \$, $K_{F1} = 250.3$ \$, $K_{F2} = 1012$ \$, $K_{F3} = 1497$ \$, $K_{F4} = 3241$ \$, $K_P = 33098$ \$, the total cost is $K = 211628$ \$, i.e. stringer stiffening results in 36% cost savings. It should be mentioned that the cutting costs of UB sections can be neglected.

Comparison of the costs for unstiffened and stiffened shells

This comparison is shown in Table 2.

Table 2. Summary of costs (negative difference means cost savings) (Costs in $)

Cost	Unstiffened shell	Stiffened shell	Difference %
Material K_M	175195	141485	-24
Forming K_{F0}	30560	16251	-88
Welding $20K_{F1} + K_{F2}$	62697		
Welding $20K_{F1} + 5K_{F2} + 5K_{F3} + K_{F4}$		20794	-201
Painting K_P	20086	33098	64
Total	288538	211628	-36

It can be seen that the cost savings caused by stringer stiffening are significant in forming and welding costs, but painting for an unstiffened shell is 64% cheaper than that for a stiffened one. It can be concluded that the cost factors of fabrication and painting play an important role in the achievable cost savings.

7. Conclusions

The economy of stiffening is characterized by a cost comparison of stiffened and unstiffened structural versions. For this purpose the own cost calculation method is used, which is developed mainly for welded structures. The cost function includes the costs of material, forming of shell elements into cylindrical shape, assembly, welding and painting and is formulated according to the fabrication sequence.

The economy of stiffening depends on type of structure (plate, cylindrical shell), type of stiffening (rings, stringers), stiffener profile (flat, L-, trapezoidal, rolled I-section etc.), loading (axial compression, bending, external pressure), constraints (buckling, deflection). Therefore the cost comparison is performed for the following cases: (a) a longitudinally stiffened plate loaded by axial compression, (b) an orthogonally stiffened square plate with transverse loading, (c) a ring-stiffened shell subject to external pressure, (d) a ring-stiffened shell loaded by axial compression or bending, (e) a stringer-stiffened shell loaded by bending with a deflection constraint.

Summarizing the above cost comparisons it can be concluded that the economy of these structures shows the following differences:

(1) stiffened plates are always economic, since they are very sensitive to buckling and transverse deflection;

(2) ring-stiffened cylindrical shells are economic only in the case of external pressure, but for axial compression and bending they are uneconomic and can be used only to guarantee the appropriate cylindrical shape;

(3) cylindrical shells stiffened outside by stringers are economic for bending with an active deflection constraint, but for axial compression or bending without a deflection constraint they are uneconomic. In order to decrease the welding cost, the stiffeners should have a cross-sectional area as large as possible and should be welded to the shell with welds as small as possible, thus halved rolled I-section stringers are advantageous for this purpose.

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