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**Design of economic, stiffened box girders, experiments for the
local buckling interaction**

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DESIGN OF ECONOMIC, STIFFENED BOX GIRDERS, EXPERIMENTS FOR THE LOCAL BUCKLING INTERACTION

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Summary

Experiments were carried out in order to determine the interaction between local buckling of webs and flanges in welded, stiffened box girders. The results and the developed computer-aided interactive decision support system was used to work out an economic design of welded box girders with longitudinally stiffened webs and compression flange. Two objective functions and five inequality constraints have been used.

1. Introduction

The first papers on structural optimization by using mathematical programming raised a great interest in development of the related numerical techniques. In two decades of intensive research a wide range of algorithms has been developed and tested.

The aim of structural optimization is to achieve lighter or lower-cost structures that satisfy requirements on safety and fabrication. In general, the load or load conditions are given, and a set of design limitations such as stress, displacement or stability are imposed.

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The values associated with load and strength terms are predominantly treated in a non-statistical, deterministic fashion, and it is usually implied that specification or design codes provide adequate safety factor values to cover any likelihood of failure. Most optimization studies to date report on how a structure should be proportioned for the given load and strength terms. However, it is clear that the implied safety factor may have a greater effect on the cost or weight than any of the subsequent results of mathematical programming techniques.

This is the reason, why we have carried out some experiments on the interaction of local stability of webs and flanges at stiffened, welded box girders, to determine more exact values of the limit slendernesses, to determine the real safety relating to the limit state.

2. Experiments on the stiffened, welded box girders

Longitudinally stiffened webs and flanges allow larger plate slendernesses and reduce the mass of structure. Very slender webs and flanges buckle simultaneously and it is difficult to consider this interaction in the design. Thus, the aim of our experiments was to determine limit slendernesses when the interaction of web and flange buckling may be neglected.

Frieze determined in [1] the values of limit slendernesses for welded box girders, without longitudinal stiffeners. For steel Fe 360 for pure bending, the limit slendernesses of webs and flanges are 145 and 30, respectively.

A BS 5950 standard categorized the sections to four groups:

- *plastic* cross sections are those in which all elements subject to compression comply with the yield stress value. A plastic hinge can be developed with sufficient rotation capacity to allow redistribution of moments within the structure,
- *compact* cross sections are those in which all elements subject to compression comply the yield stress value. The full plastic moment capacity can be developed but local buckling may prevent development of a plastic hinge with sufficient rotation capacity to permit plastic design,
- *semi-compact* sections are those in which the stress at the extreme fibres can reach the design strength but local buckling may prevent the development of the full plastic moment,
- *slender* sections are those which contain slender elements subject to compression due to moment or axial load. Local buckling may prevent the stress in a slender section from reaching the design strength.

Regarding the section as a semi-compact one we made a suboptimization for the model beams, using various slendernesses for webs and flanges (see *Table 1*). The suboptimization was carried out by the Hillclimb procedure of Rosenbrock [2, 3].

The *objective function* is the mass of the structure to be minimized. The minimum cross section represents the minimum mass.

Table 1.

Signs of sections using various limit slenderness ratios for webs and flanges of box girders

Limit slenderness ratios		for webs			
		130	140	160	200
for flanges	30	130-30	140-30	-	-
	40	130-40	140-40	-	-
	60	130-60	140-60	160-60	200-60

$$A = 2(ht_w + bt_f) \rightarrow \min \quad (1)$$

where h , t_w , b , t_f are the unknown variables according to Fig. 1.

Steel grade Fe 360 of yield stress $R_y = 230$ MPa is used.

The moment of inertia and the section modulus are as follows:

$$I_x = \frac{h^3 t_w}{6} + \frac{bt_f h^2}{2} \quad (2)$$

$$W_x = \frac{h^2 t_w}{3} + bt_f h. \quad (3)$$

The constraints according to the BS 5400 [7] are the following:

– static stress due to uniaxial bending

$$\sigma_{\max} = \frac{M_x}{W_x} \leq R_{adm} \quad (4)$$

where $M_x = \frac{FL}{4}$ at midspan

$R_{adm} = 200$ MPa is the admissible stress;

– buckling of the upper flange due to compression:

$$\frac{M_x}{R_{adm} K_{1C} W_x} \leq 1 \quad (5)$$

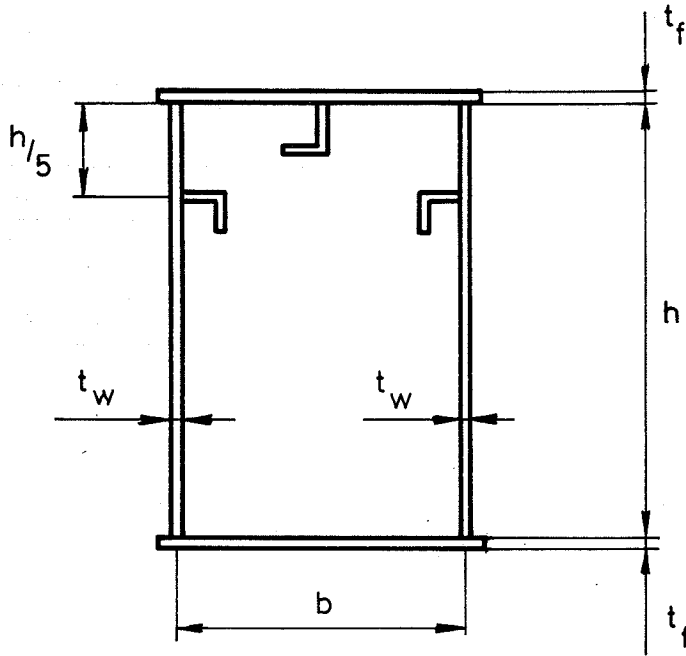


Fig. 1. Cross section of the stiffened, welded box girders.

where the slenderness of flange

$$\lambda_f = \frac{b}{2t_f} \sqrt{\frac{R_y}{355}} \quad (6)$$

if	$\lambda_f \leq 24$	then	$K_{1c} = 1$	}	(7)
	$24 < \lambda_f \leq 47$		$K_{1c} = \left(\frac{24}{\lambda_f}\right)^{0,75}$		
	$47 < \lambda_f \leq 130$		$K_{1c} = \left(\frac{26}{\lambda_f}\right)^{0,85}$		
	$130 < \lambda_f \leq 300$		$K_{1c} = 0,274 - \frac{\lambda_f}{7000}$		

– buckling the upper part of the web:

$$\sqrt{\left(\frac{0,8 \frac{M_x}{W_x}}{R_{adm} K_{2c}}\right)^2 + \left(\frac{\frac{F}{t_w a_e}}{R_{adm} K_{3c}}\right)^2} \leq 1 \quad (8)$$

$$a_e = 50 + 3 (h_r - 5 - t_f) \quad (\text{mm}) \quad (9)$$

where a_e is the length of load bearing

h_r is the height of rail (at crane girders)

The slenderness of the upper part of web is

$$\lambda_w = \frac{h}{5t_w} \sqrt{\frac{R_y}{355}} \quad (10)$$

if $\lambda_w \leq 24$ then $K_{2c} = 1$

$$24 < \lambda_w \leq 47 \quad K_{2c} = \left(\frac{24}{\lambda_w}\right)^{0,75}$$

$$47 < \lambda_w \leq 130 \quad K_{2c} = \left(\frac{26}{\lambda_w}\right)^{0,85}$$

$$130 < \lambda_w \leq 300 \quad K_{2c} = 0,274 - \frac{\lambda_w}{7000} \quad (11)$$

For the computation of wheel compression the slenderness is:

$$\lambda_e = \frac{h_e}{t_w} \sqrt{\frac{R_y}{355}} \quad (12)$$

if $\lambda_e \leq 24$ then $K_{3c} = 1$

$$24 < \lambda_e \leq 43 \quad K_{3c} = \left(\frac{24}{\lambda_e}\right)^{0,5}$$

$$43 < \lambda_e \leq 59 \quad K_{3c} = \left(\frac{28}{\lambda_e}\right)^{0,68}$$

$$\begin{aligned}
 59 < \lambda_e \leq 90 & \quad K_{3c} = \left(\frac{30}{\lambda_e}\right)^{0,75} \\
 90 < \lambda_e \leq 130 & \quad K_{3c} = \left(\frac{36}{\lambda_e}\right)^{0,9} \\
 130 < \lambda_e \leq 200 & \quad K_{3c} = 0,38 - \frac{\lambda_e}{2000} \\
 200 < \lambda_e \leq 300 & \quad K_{3c} = 0,33 - \frac{\lambda_e}{4000} \quad (13)
 \end{aligned}$$

where $h_e = 1,9 \sqrt{\frac{a_e h}{5K_w}}$ (14)

$$K_w = \left(3,4 + \frac{2,2h}{5a_d}\right) \left(0,4 + \frac{a_e}{2a_d}\right) \quad (15)$$

a_d is the distance between diaphragms.

Check of the stiffeners both at webs and flanges.

The dimensions of the stiffener can be seen in *Fig. 2*.

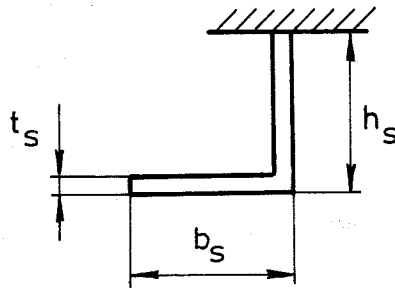


Fig. 2. Dimensions of the stiffener.

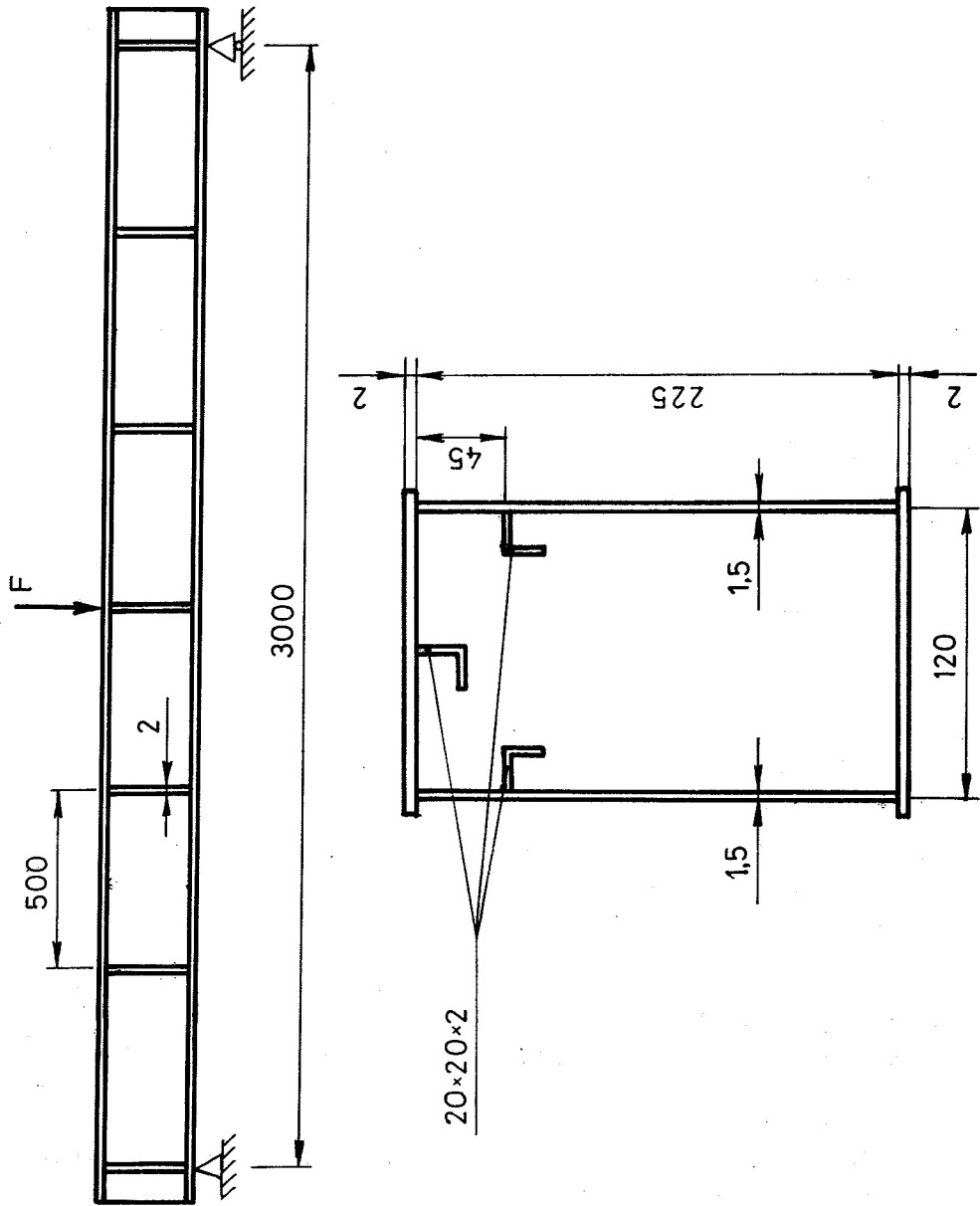


Fig. 3. Cross section of 130-30 type girder.

If $b_s \leq h_s$ then must be

$$\frac{b_s}{t_s} \sqrt{\frac{R_y}{355}} < 11 \quad \text{and} \quad \frac{h_s}{t_s} \sqrt{\frac{R_y}{355}} \leq 7$$

or if

$$\frac{l_s}{b_s} \sqrt{\frac{R_y}{355}} > 7 \quad \text{then must be}$$

$$\frac{h_s}{t_s} \sqrt{\frac{R_y}{355}} = 6,2 + \frac{31,6}{\frac{l_s}{b_s} \sqrt{\frac{R_y}{355}} - 10}$$

where l_s is the length of stiffener.

If $\frac{b}{t} \sqrt{\frac{R_y}{355}} \leq 30$ there is no prescription

on to $\frac{h_s}{t_s} \sqrt{\frac{R_y}{355}}$.

So it is, if $\frac{h}{t_w} \sqrt{\frac{R_y}{355}} \gg 30$.

Using the Hillclimb method, where the objective function is the minimum area of cross-section, we have got the following results for various slendernesses (see *Table 2*). The span of girders $L = 3$ m, the distance between diaphragms is $a_d = 0,5$ m. The stiffeners are angle profiles $20 \times 20 \times 2$ mm, so the stiffeners are rigid enough.

Two model girders have been investigated. The cross sections can be seen in *Figs. 3, 4*. The yield strength of the steel was 300 MPa.

Table 3 and *4* shows that residual displacements, local buckling occur both at webs and at upper flanges near to midspan, as can be seen in *Fig. 5*., nearly simultaneously, when the stress reached the yield strength.

For plates with residual welding stresses an empirical formula proposed by Faulkner [4] can be used for the computation of effective width:

Table 2.

Results of suboptimization, using various slendernesses

Force (N)	Dimensions (mm)				Sign of section	Area of cross section (mm ²)	σ_{max} (MPa)	Buckling constraints for	
	h	t_w	b	t_f				flange	web
4800	225	1,5	120	2,0	130-30	1155	54,0	0,998	0,804
3800	225	1,5	120	1,5	130-40	1035	53,6	0,995	0,796
5200	225	1,5	180	1,5	130-60	1215	53,1	0,994	0,793
5700	260	1,5	120	2,0	140-30	1260	53,9	0,996	0,807
4550	260	1,5	120	1,5	140-40	1140	53,5	0,994	0,799
6200	260	1,5	180	1,5	140-60	1320	53,4	0,999	0,801
7300	300	1,5	180	1,5	160-60	1440	52,9	0,990	0,800
12600	375	1,5	180	1,5	200-60	1665	53,2	0,993	0,801

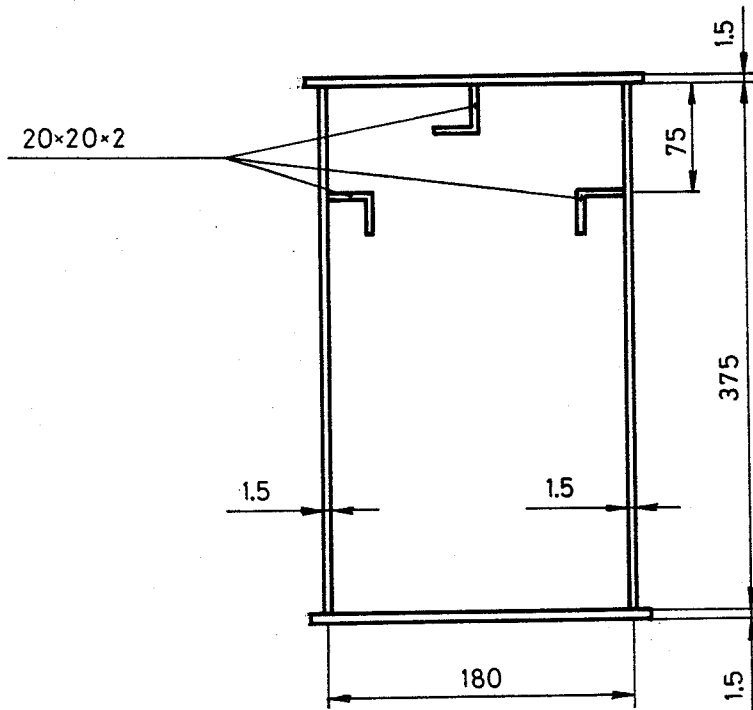


Fig. 4. Cross section of 200-60 type girder.

Table 3.

Static stress and deflection measurement of girder type 130-30

Force (kN)	σ_{\max} (MPa)	Deflection at midspan (mm)	Remarks
5	41,9	2,4	
10	85,5	4,1	
15	128,6	6,0	
20	172,6	8,0	
25	214,2	10,0	
30	241,5	12,0	
35	281,7	14,3	
39	314,9	17,3	residual displacement

Table 4.

Static stress and deflection measurement of girder type 200-60

Force (kN)	σ_{\max} (MPa)	Deflection at midspan (mm)	Remarks
10	42,0	3,7	
20	87,1	6,5	
30	132,5	9,6	
40	196,2	13,2	
50	245,3	17,5	
60	294,4	25,0	
64	315,0	32,1	buckling at web and flange

$$\psi = \frac{2}{\lambda_p} - \frac{1}{\lambda_p^2} - \frac{\sigma_c(\vartheta)}{R_y} = \frac{s_e}{s} \quad (16)$$

where the slenderness

$$\lambda_p = \frac{s}{t} \sqrt{\frac{\sigma_{\max}}{E}} \quad (17)$$

the residual stress

$$\frac{\sigma_e}{R_y} = \frac{2\eta}{\vartheta - 2\eta} \quad (18)$$

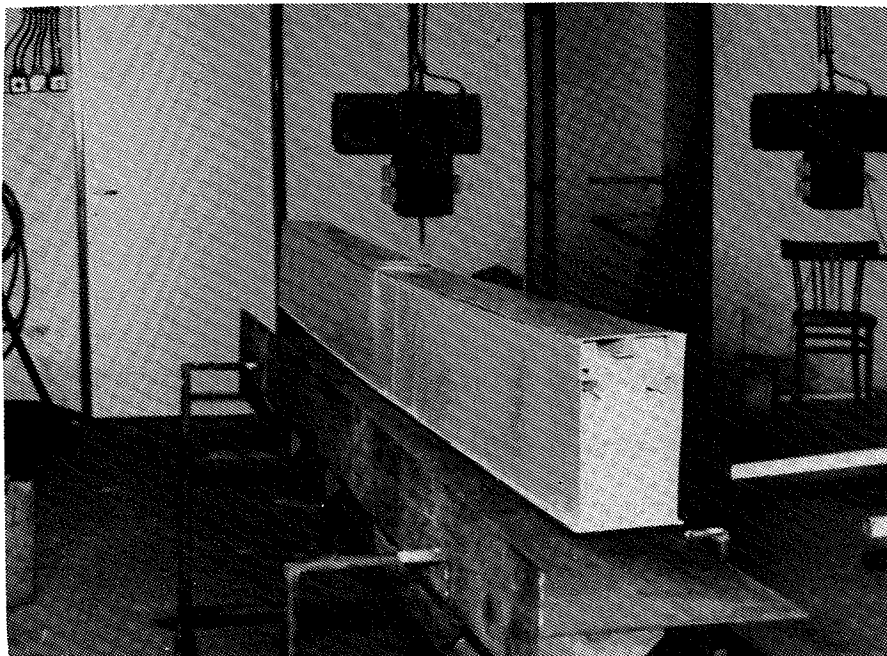


Fig. 5. Local buckling of 130–30 type girder.

$\eta = 3$ can be used for lightly welded parts.

$$\vartheta = \frac{s}{t}. \quad (19)$$

For the section 130–30 the computation of effective width is as follows:

$$\lambda_p = \frac{60}{2,0} \sqrt{\frac{314,9}{2,1 \cdot 10^5}} = 1,162$$

$$\vartheta = \frac{s}{t} = \frac{60}{2} = 30$$

$$\psi = 0,73$$

$$s_e = s \psi = 43,8 \text{ (mm)}.$$

So the effective width of flange is $b_e = 87,6$ (mm). The moment of inertia for this section, regarding the effect of stiffeners is

$$I_x = 7,36 \cdot 10^6 \text{ (mm}^4\text{)}.$$

The section modulus is

$$W_x = 6,48 \cdot 10^4 \text{ (mm}^3\text{)}.$$

The maximum normal stress due to bending at midspan is

$$\sigma_{\max} = 329 \text{ (MPa)}.$$

There is a good agreement between the measured and calculated values of stress (see *Table 3*).

For the section 200–60 the computation of effective width is as follows:

$$\lambda_p = \frac{90}{1,5} \sqrt{\frac{315}{2,1 \cdot 10^5}} = 2,323$$

$$\vartheta = \frac{s}{t} = \frac{90}{1,5} = 60$$

$$\psi = 0,564; \quad s_e = s\psi = 50,78 \text{ (mm)}.$$

So the effective width of flange is $b_e = 101,6 \text{ (mm)}$.

The moment of inertia for this section, regarding the effect of stiffeners is: $I_x = 2,875 \cdot 10^7 \text{ (mm}^4\text{)}$. The section modulus is $W_x = 1,533 \cdot 10^5 \text{ (mm}^3\text{)}$. The maximum normal stress due to bending at midspan is $\sigma_{\max} = 313,1 \text{ (MPa)}$.

The agreement with the measured value of σ is very good (see *Table 4*).

According to Klöppel, Scheer [5], the buckling coefficients due to pure bending and shear are $k_B = 110,8$; $k_\tau = 10$, respectively.

For specimen 200–60, if the concentrated load is 30 kN, the computation, according to the Hungarian standard [8], is as follows:

$$F = 30 \text{ kN}; \quad \sigma_B = 132,5; \quad \tau_Q = 26,6 \text{ MPa}; \quad \sigma_r = 140,2 \text{ MPa}$$

so the reduced buckling coefficient k_r is as follows:

$$k_r = \frac{\sigma_r}{\sqrt{\left(\frac{\sigma_B}{k_B}\right)^2 + \left(\frac{\tau_Q}{k_\tau}\right)^2}} = 48,07$$

the slenderness ratio λ_0 is

$$\lambda_0 = \frac{3,3}{\sqrt{k_r}} \left(\frac{h}{t_w}\right) = 118,98$$

the equivalent slenderness is $\lambda_E = \pi \sqrt{\frac{E}{R_y}} = 82,3$

$$\bar{\lambda}_0 = \frac{\lambda_0}{\lambda_E} = 1,45$$

so the factor φ_b is

$$\varphi_b = \frac{1}{\bar{\lambda}_0^2} = 0,48.$$

There is no buckling, if

$$\sigma_r \leq 1,1 \varphi_b R_u$$

$$140,2 \leq 157,8 \text{ MPa.}$$

According to the Hungarian standard, buckling of web does not occur. If the force is higher than $F = 30 \text{ kN}$, buckling may occur or the safety factor is too small according to the code.

The results of the measurements show, that higher stresses up to 300 MPa are admissible. Buckling of webs and flange occur near to the yield stress. The interaction does not lower the buckling capacity of individual plates, therefore they can be designed separately.

The measurement and calculation of stiffened box girders show, that the local buckling occurs simultaneously both at webs and flange. Using stiffeners not only at webs, but at the upper flange we can reach higher limit slendernesses, which give a good way to mass reduction. For flange the limit slenderness ratio can be 60, using one stiffener. For webs it depends on the category of section, according to BS 5950.

3. Economic design of welded, stiffened box girders

The investigated structure is shown in *Fig. 1*. Using the results of the previous chapter another optimization approach of these type of structures is as follows:

Independent variables

Height of web $h = x(1)$, span length $L = x(2)$. The other dimensions of the beam, using limit slendernesses, are as follows:

$$t_w = 0,2 \frac{h}{30} = \frac{h}{150} = \frac{x(1)}{150} \quad (20)$$

$$b = \frac{h}{2} = \frac{x(1)}{2} \quad (21)$$

$$t_f = \frac{b}{2} \frac{1}{30} = \frac{h}{120} = \frac{x(1)}{120}$$

Objective functions

– volume of the girder

$$V = (2ht_w + 2bt_f)L = \left(2 \cdot x(1) \cdot \frac{x(1)}{150} + 2 \cdot \frac{x(1)}{2} \cdot \frac{x(1)}{120} \right) \cdot x(2)$$

$$V = F(1) \rightarrow \min \quad (22)$$

– deflection of the girder

$$w_{\max} = \frac{FL^3}{48EI_x}$$

$$I_x = 2 \frac{h^3 t_w}{12} + \frac{h^2 b t_f}{2} = \frac{x(1)^3 \frac{x(1)}{150}}{6} + \frac{x(1)^2 \frac{x(1)}{2} \frac{x(1)}{120}}{2}$$

$$w_{\max} = F(2) \rightarrow \min \quad (23)$$

Constraints

– stress constraint

$$\frac{FL}{4W_x} \leq R_{adm}$$

$$\text{where } W_x = 2 \frac{h^2 t_w}{6} + h b t_f = \frac{x(1)^2 \cdot \frac{x(1)}{150}}{3} + x(1) \cdot \frac{x(1)}{2} \cdot \frac{x(1)}{120} \quad (24)$$

The constraint can be written as

$$R_{adm} - \frac{FL}{4W_x} \geq 0$$

– size constraints.

For the independent variable $x_i^{\min} \leq x_i \leq x_i^{\max}; i = 1, 2$.

Four geometric constraints relating to the span length and the height of web:

$$L_{\max} - x(2) \geq 0; \quad x(2) - L_{\min} \geq 0 \quad (25)$$

$$h_{\max} - x(1) \geq 0; \quad x(1) - h_{\min} \geq 0. \quad (26)$$

3.1 Description of the objective function and constraints in the coordinate system of variables

We use the results of the previous chapter. If the concentrated load is 4800N at span length 3m, the maximum stress at midspan is 54 MPa. So we choose an upper and lower limit both for the span and height of girder. They are 2 + 4m, 180 + 400mm, respectively.

Objective functions are as follows:

$$F(1) = 2 \cdot x(2) \cdot \left(\frac{x(1)^2}{150} + \frac{x(1)^2}{240} \right) \quad (27)$$

$$F(2) = \frac{4800}{48 \cdot 2,06 \cdot 10^5} \cdot \frac{x(2)^3}{\frac{x(1)^4}{900} + \frac{x(1)^4}{480}} \quad (28)$$

Inequality constraints are as follows:

$$G(1) = 54 - \frac{4800 \cdot x(2)}{\frac{x(1)^3}{450} + \frac{x(1)^3}{240}} = 54 - \frac{1200}{\left(\frac{1}{450} + \frac{1}{240} \right) x(1)^3} x(2) \geq 0 \quad (29)$$

$$G(2) = 4000 - x(2) \geq 0 \quad (30)$$

$$G(3) = x(2) - 2000 \geq 0 \quad (31)$$

$$G(4) = 400 - x(1) \geq 0 \quad (32)$$

$$G(5) = x(1) - 180 \geq 0. \quad (33)$$

Fig. 6 shows the $F(i)$ and $G(i)$ functions.

At the weighting methods numbers mean the weighting coefficients. For example $M 8,2$ means, that the weighting min-max method is used, and the weighting coefficients are $w_1 = 0.8$, $w_2 = 0.2$ respectively.

3.2 Description of constraints in the coordinate system of objective functions

From equations (27, 28) we get

$$x(2)^5 = \frac{F(1)^2 F(2)}{0,021666^2 \cdot 0,1519} ; x(2) = \frac{\sqrt[5]{F(1)^2 F(2)}}{0,1481248} \quad (34)$$

$$x(1)^{10} = \frac{F(1)^3}{F(2)} \frac{0,1519}{0,021666^3} ; x(1) = 2,6146778 \sqrt[10]{\frac{F(1)^3}{F(2)}} \quad (35)$$

With these equations the constraints are

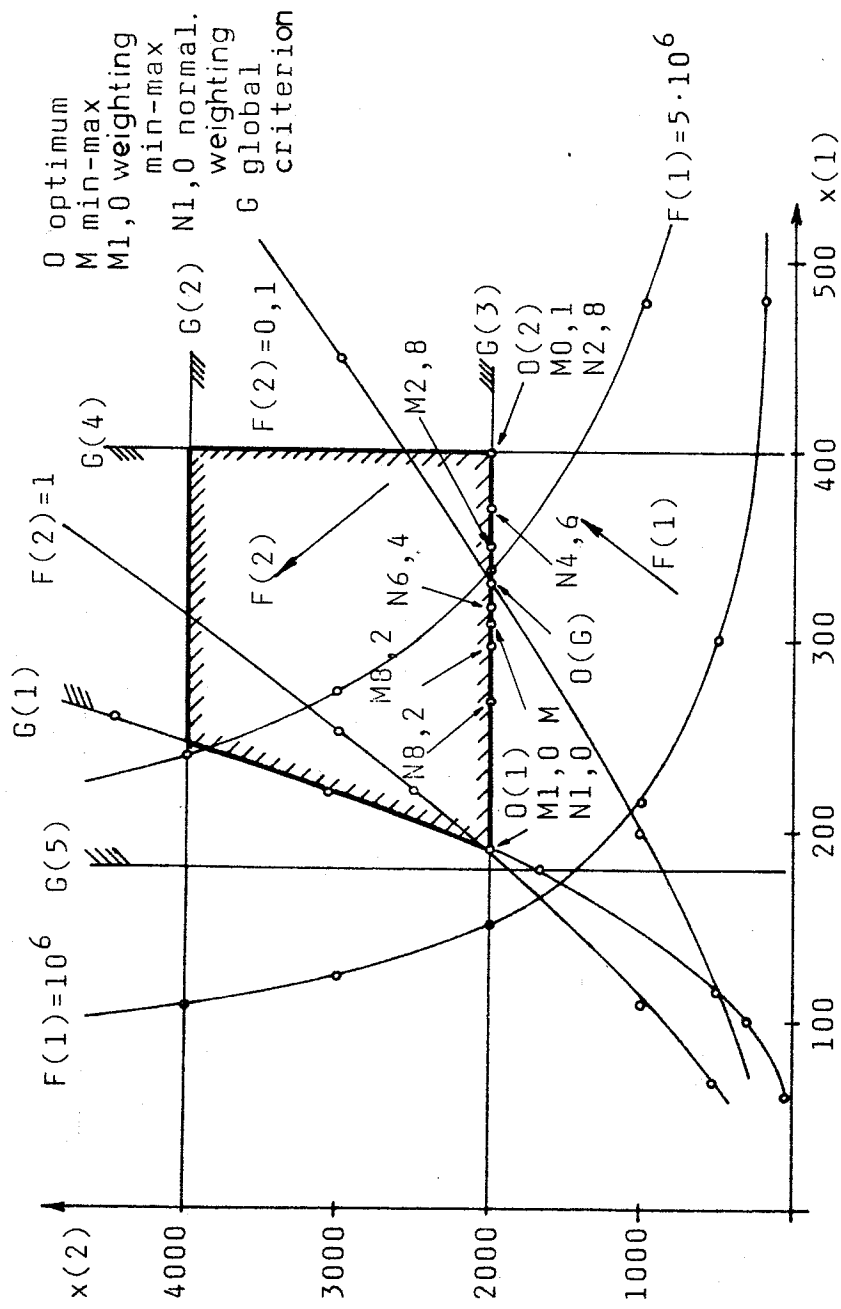


Fig. 6. Description of objective functions and constraints in the coordinate system of variables.

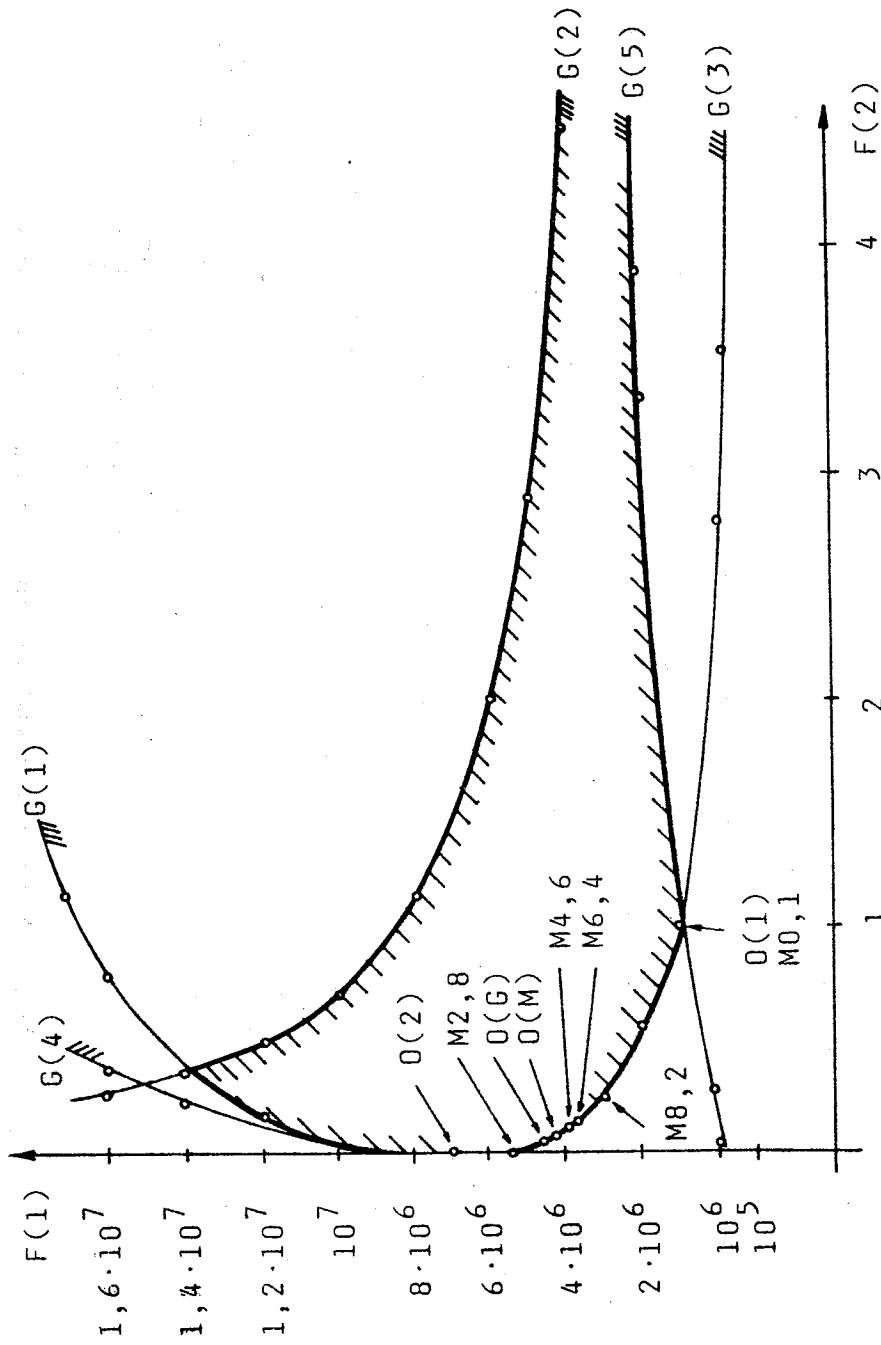


Fig. 7. Description of constraints in the coordinate system of objective functions.

$$G(2) = 4000 - x(2) = 4000 - \frac{\sqrt[5]{F(1)^2 \cdot F(2)}}{0,1481248} \geq 0 \quad (36)$$

$$F(2) \leq \frac{7,3019497 \cdot 10^{13}}{F(1)^2} \quad (37)$$

$$G(3) = x(2) - 2000 = \frac{\sqrt[5]{F(1)^2 \cdot F(2)}}{0,1481248} - 2000 \geq 0 \quad (38)$$

$$F(2) \geq \frac{2,2818593 \cdot 10^{12}}{F(1)^2} \quad (39)$$

$$G(4) = 400 - x(1) = 400 - 2,6146778 \sqrt[10]{\frac{F(1)^3}{F(2)}} \geq 0 \quad (40)$$

$$F(2) \leq \frac{F(1)^3}{7,0213095 \cdot 10^{21}} \quad (41)$$

$$G(5) = x(1) - 180 = 2,6146778 \sqrt[10]{\frac{F(1)^3}{F(2)}} - 180 \geq 0 \quad (42)$$

$$F(2) \geq \frac{F(1)^3}{2,3908 \cdot 10^{18}} \quad (43)$$

$$G(1) = x(1)^3 \cdot 0,0002875 - x(2) \geq 0 \quad (44)$$

$$F(2)^5 \leq 1,5131729 \cdot 10^{-36} F(1)^5 \quad (45)$$

Fig. 7 shows the $G(i)$ functions.

Using the decision support system, we can get the various optima in both coordinate systems. The so called Pareto optima take place between the single-criterion optima $(0(1), 0(2))$.

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**WIRTSCHAFTLICHE BEMESSUNG VON LÄNGSVERSTEIFTEN
KASTENTRÄGERN, VERSUCHE FÜR DIE INTERAKTION
DER LOKALEN BEULUNGEN**

K. JÁRMAI

Zusammenfassung

Bei den geschweißten Trägern mit verripptem Kastenquerschnitt werden für die Bestimmung der Beulungsinteraktion des Steges und des Gurtblechs Versuche durchgeführt. Die Stegbleche und das Gedrückte Gurtblech wurde mit Längsrippen versteift. Bei dem interaktiven entscheidungsunterstützenden Programmsystem wurden die Ergebnisse für die Erarbeitung der wirtschaftlichen Bemessung der geschweißten Träger mit Kastenquerschnitt benutzt. Die Berechnung erfolgte bei zwei Zielfunktionen und fünf Bedingungen.

**ПРОЕКТИРОВАНИЕ ЭКОНОМИЧНЫХ КОРОБЧАТЫХ БАЛОК С ПРОДОЛЬНЫМИ
РЕБРАМИ ЖЕСТКОСТИ, ЭКСПЕРИМЕНТЫ ВЗАИМОДЕЙСТВИЯ
ВЫПУЧИВАНИЯ СТЕНОК И ПОЯСА**

К. ЯРМАИ

Резюме

Эксперименты были проведены со сварными, ребристыми, коробчатым балками, чтобы определить интеракцию между выпучиванием стенок и сжатого пояса. Результаты были использованы у интерактивно программной системе для экономического проектирования сварных ребристых коробчатых балок. Стенки и сжатый пояс были усилены продольными ребрами. В проблеме были использованы две переменных, две целевых функции и пять условий.