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Laboratory P and S Wave Velocity Measurements to Confirm the Developed Petrophysical Model for Acoustic Hysteresis

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SUMMARY

It is well known that acoustic wave propagation under pressure is very nonlinear and the elastic properties of rocks are hysteretic, which behavior is important for mechanical understanding of reservoirs during depletion. Pressure strongly influences the elastic parameters of rocks, thus wave velocities too. In this study longitudinal and transverse wave velocities measured in laboratory on sandstone samples under pressure are presented. The uniaxial loading of the samples was carried out by an automatic acoustic test system. Measurement data were processed by a joint inversion technique based on the developed petrophysical model which describes the relationship between acoustic P/S wave velocities and pressure for loading and unloading phases. After Birch we assume that the main factor determining the pressure dependence is the closure of pores. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The quality checked joint inversion results showed that the misfits between measured and calculated data are small, proving that the proposed petrophysical model can be applied well in practice.

Introduction

There is a growing claim to predict rock physical parameters more accurate at non-conventional conditions also. Geophysics has a wide palette to determine these parameters, for example acoustic velocity, porosity, permeability, elastic moduli and it is well known that pressure has a strong influence on them. The change of acoustic wave velocity propagating in rocks under pressure is highly nonlinear and the quasistatic elastic properties of rocks are hysteretic (Ji et al. 2007). Characterization of hysteretic behavior is important for mechanical understanding of reservoirs during depletion. Therefore a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which delivers the connection between the propagation velocity of acoustic waves (both P and S) and rock pressure both in case of loading and unloading periods as well as explains the mechanism of acoustic hysteresis.

Acoustic hysteresis may be caused by the irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions. The first theory assumes that the microcracks closed during loading do not reopen during subsequent unloading (Walsh and Brace 1964). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original dimensions at lower pressures (Birch 1960). By idea of the improvement of contact conditions (Hashin and Shtrikman 1963) in a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

Describing acoustic hysteresis of P and S wave velocity

For an analytical description of the nonlinear velocity vs. pressure relationship, exponential functions are most commonly used. In these empirical equations only the regression parameters are given and no physical explanation of the process is provided. At the development of the physical based-model Birch's (1960) qualitative considerations was followed. We assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure -from the unloading state-, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Therefore we introduce the parameter V as the unit pore volume of a rock. If a stress increase $d\sigma$ is created in a rock let us assume that the change of pore volume dV is directly proportional to the applied stress increase $d\sigma$ and also the pore volume V . One can describe the two assumptions with the following differential equation

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \quad (1)$$

where λ_V is new material quality dependent petrophysical parameter (Dobróka and Somogyi Molnár 2012) and V_0 is the pore volume at stress-free state ($\sigma = 0$). The negative sign represents that with increasing stress the pore volume decreases ($\lambda_V > 0$). We assume also a linear relationship between the infinitesimal change of the P wave propagation velocity $d\alpha$ - due to stress increase - and dV

$$d\alpha = -\kappa_P dV, \quad (2)$$

where κ_P is a positive proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining Eqs. (1-2) and solve the differential equation one can obtain

$$d\alpha = \kappa_P \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \rightarrow \alpha = K - \kappa_P V_0 \exp(-\lambda_V \sigma), \quad (3)$$

where K is an integration constant. At stress-free state ($\sigma=0$) the propagation velocity α_0 can be measured which is computed from Eq. (3) as $\alpha_0 = K - \kappa_p V_0$. With this result and introducing the notation $\Delta\alpha_0 = \kappa_p V_0$ Eq. (3) can be rewritten in the following form

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - \exp(-\lambda_v \sigma)). \quad (4)$$

Eq. (4) provides a theoretical connection between the propagation velocity and rock pressure for loading. The acoustic wave velocity increases from α_0 to α_{max} (at high pressure, when approximately all the pores are closed). So, $\Delta\alpha_0$ can be considered the velocity-drop caused by the presence of pores at zero pressure. Petrophysical characteristic λ_v is the logarithmic stress sensitivity of the velocity-drop (Dobróka and Somogyi Molnár 2012). Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded, hence decreasing velocity is observed. This effect is outside of our present investigations.

To characterize the unloading phase, $v=V_0-V$ as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed pores start to open again, so decreasing velocity can be measured. Therefore we assume dv (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease $d\sigma$

$$dv = \lambda'_v v d\sigma \rightarrow v = v_m \exp(-\lambda'_v (\sigma_m - \sigma)), \quad (5)$$

where λ'_v is another new material characteristic constant and v_m is the closed pore volume at maximum pressure value σ_m . After Birch (1960) there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during loading do not reopen completely during unloading. This irreversibility is denoted by two different parameters λ_v and λ'_v in our model. Combining Eq. (2) and Eq. (5) by using the formulas $dV=-dv$ and $\kappa_p v_m = \Delta\alpha_m$ one can find

$$\alpha = \alpha_m - \Delta\alpha_m (1 - \exp(-\lambda'_v (\sigma_m - \sigma))). \quad (6)$$

Eq. (6) shows the propagation velocity – pressure function of unloading phase. In the two limiting cases (at pressure value $\sigma=\sigma_m$ and $\sigma=0$) Eq. (6) gives α_m and $\alpha_1 = \alpha_m - \Delta\alpha_m (1 - \exp(-\lambda'_v \sigma_m))$, respectively (notation $\alpha(0) = \alpha_1$ was used), thus the following formula can be formed (similar to Eq. (4))

$$\alpha = \alpha_1 + \Delta\alpha_1 (1 - \exp(-\lambda'_v \sigma)), \quad (7)$$

where $\Delta\alpha_1 = -\Delta\alpha_m \exp(-\lambda'_v \sigma_m)$.

Since the base of the model is the change of pore volume (which is independent of the direction of loading) the described model conditions are valid for S waves also. Following the same procedure similar model equations

$$\beta = \beta_0 + \Delta\beta_0 (1 - \exp(-\lambda_v \sigma)), \quad \beta = \beta_1 + \Delta\beta_1 (1 - \exp(-\lambda'_v \sigma)) \quad (8)$$

can be obtained for loading and unloading phases, respectively.

Case study

To confirm the reliability of the model velocity data sets were processed. The pulse transmission technique was used for P and S wave velocity measurements. We performed measurements on many different sandstone samples which were subjected to uniaxial stresses by an automatic acoustic test

system. The digitally controlled test system includes a pressure cell, an ultrasonic 2-channel testing device and a load frame (Fig. 1). Wave velocities - as a function of pressure - were measured at adjoining pressures during loading and unloading phases. A 256-fold stacking was applied to increase the signal/noise ratio. To avoid the destruction of the samples we loaded them only up to 1/3 of the uniaxial strength. One typical test result (Sample A: fine-grained sandstone) is presented in the paper. Our measurements showed that in the lower pressure range, the increase in velocities with increasing pressure is steep and nonlinear. This is due to the closure of pore volume, which significantly affects the elastic properties of rock and thereby the velocities. In the higher pressure range, the increase in velocities (with increasing pressure) become moderate as the closeable pore volume lessens. A slight difference can be found between the characteristics of the loading and unloading curves which can be explained by the phenomenon of acoustic hysteresis.

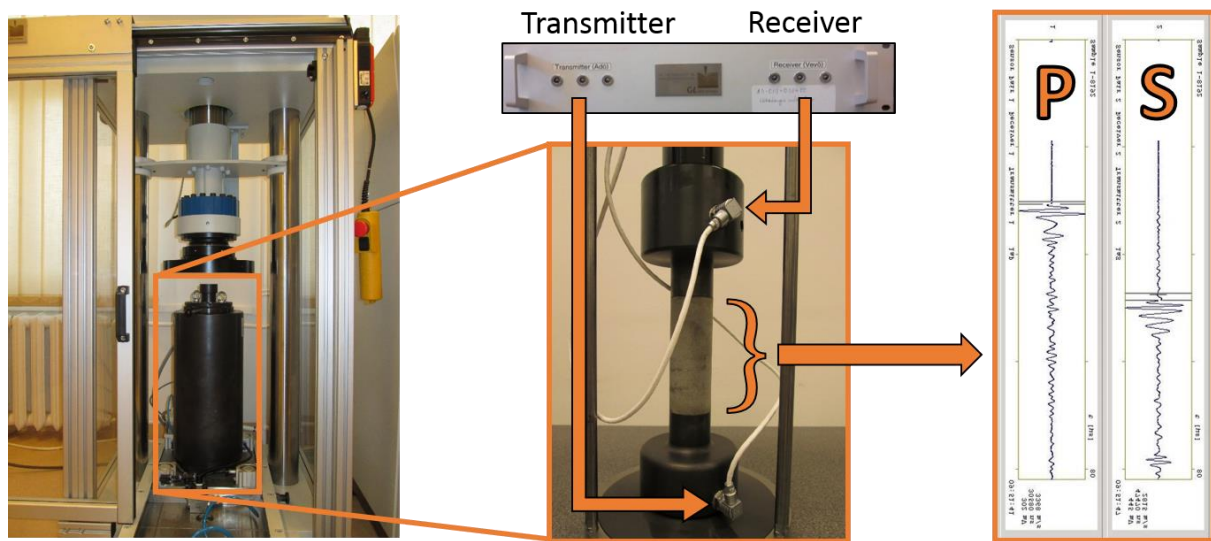


Figure 1 Experimental setup. Left: load frame and pressure cell. Middle: ultrasonic device, sandstone sample between transmitter and receiver built in the pressure stamps. Right: P and S wave arrivals.

Proving the validity and applicability of the introduced velocity model, we present the interpretation of measurement data. The parameters appearing in the model equations (in case of loading and unloading) can be determined by processing measurement data based on joint inversion method (the Damped Least Squares Method was used). The estimated model parameters are summarized in Table 1. For the characterization of the accuracy of inversion estimates the measure of fitting in data space (D) is also provided. The table contains the mean averages as well, which indicate that the parameters are in moderate correlation.

Table 1 Model parameters, data misfits and mean spreads estimated by joint inversion using the developed model.

P wave								
Sample	Loading			Unloading			RMS (%)	S
	α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ_v (1/MPa)	α_1 (km/s)	$\Delta\alpha_1$ (km/s)	λ'_v (1/MPa)		
A	3,56	1,06	0,0212	3,56	0,94	0,0401	0,39	0,59
S wave								
Sample	Loading			Unloading			RMS (%)	S
	β_0 (km/s)	$\Delta\beta_0$ (km/s)	λ_v (1/MPa)	β_1 (km/s)	$\Delta\beta_1$ (km/s)	λ'_v (1/MPa)		
A	2,29	0,51	0,0292	2,31	0,46	0,0395	0,26	0,5

With these model parameters the pressure-dependent acoustic P and S wave velocities were calculated for the whole pressure range by Eqs. (4) and (7)-(8). The results are shown in Fig. 2, where the solid line shows the calculated velocity-pressure function produced by the velocity model, while symbols represent the measured data. It can be seen that the distances between the measured and calculated data are small, which are also confirmed by the RMS values (Table 1). Inversion results prove that the petrophysical model describing acoustic hysteresis applies well in practice in case of P and S waves, too.

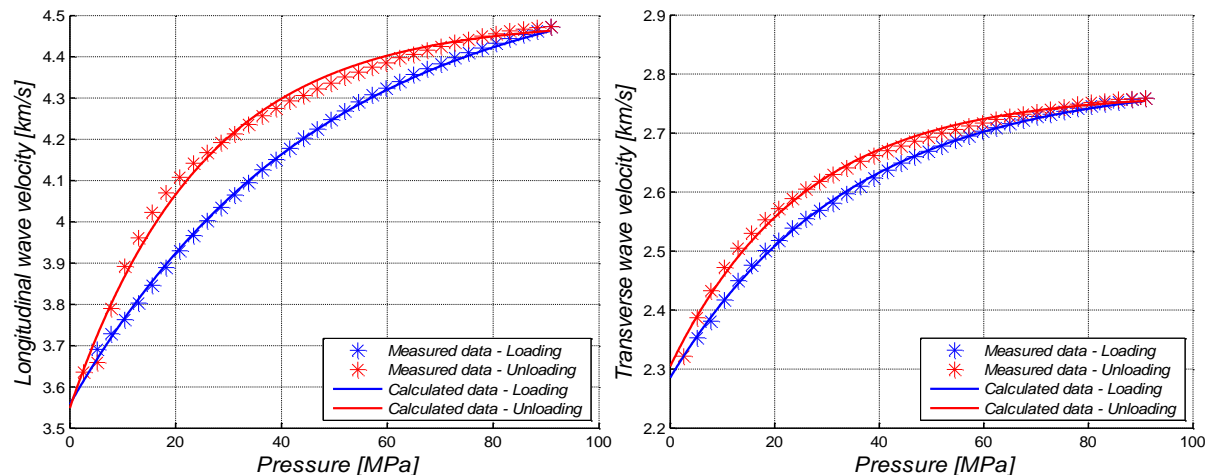


Figure 2 P and S wave velocities as a function of pressure for Sample A

Conclusions

In this paper based on Birch's theory, a petrophysical model describing acoustic hysteresis was presented. It provides the connection between the P and S wave velocity and rock pressure, both in case of loading and unloading phases. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The model (valid only in reversible/elastic range) is based on the idea that pore volume changes with pressure. To test the model laboratory P and S wave velocity measurements on sandstone samples were carried out. Measured data were processed by a quality checked joint inversion technique based on the developed petrophysical model. Inversion results showed that the misfits between measured and calculated data are small, proving that the proposed petrophysical model can be applied well in practice.

Acknowledgements

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