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FROM PUBLICATIONS OF THE TECHNICAL UNIVERSITY FOR HEAVY INDUSTRY
Series C, Machinery, Volume 37(1982), Fasc. 1-2, pp. 79-95.

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Optimal design of welded frames by complex programming method

MISKOLC, 1982

OPTIMAL DESIGN OF WELDED FRAMES BY COMPLEX PROGRAMMING METHOD

by
K. JÁRMAI

The complex method is described by means of a detailed flow chart. A procedure is proposed for the discrete variables. The application of the method to the optimal design of rod structures constructed from parts of welded I-sections is demonstrated by a numerical example of a single-bay pitched-roof portal frame with eight variables. Relating to this example, the effect of the number of complex points, that of the values of the reflection coefficient as well as the constant of the convergence criterion is investigated. The comparison of numerical results obtained by the complex as well as backtrack methods shows the advantages and disadvantages of both methods.

1. Introduction

Optimization means finding the best possible solution among several feasible alternatives, for any problem – an ambition common to all. Thus, the aim of a structural engineer is to design optimum structures. In this endeavor, experience, intuition and mathematical insight can play a large part. The measure of a good engineering structure is its cost or its weight. The design of a structure is characterized by a set of parameters, specifying the configuration of the structure and the properties of the structural members. The set of structure parameters may be divided into two subsets: (1) *Parameters* determined a priori because of site conditions, code requirements, aesthetic considerations of other relevant factors and (2) *design variables* to be determined by the designer, subject to special constraints, for instance code requirements. The goal of optimal design is the determina-

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Manuscript received: 05. 12. 1981.

tion of those values of design variables, subject to relevant constraints, that achieve some measure of merit which may be expressed as a function of the design variables.

Analytical optimization methods generally are applicable only to simple structures. For the optimization of complicated structural assemblies programming methods should be employed. Programming methods are also necessary if the design variables are discrete in nature, or if the objective function is not available in analytical form.

Methods of optimization by nonlinear programming can be divided roughly into two categories, i. e., *gradient* and *nongradient* techniques.

One of the *gradient* techniques is the *sequential unconstrained minimization technique* (SUMT) [1]. It can be efficiently used for the solution of nonlinear optimization problems [2]. Unfortunately the calculations of the gradients throw difficulties in the way of applicability.

A *nongradient*, widely accepted systematical search procedure is the so called *backtrack* method [3]. This combinatorial discrete programming method can be successfully applied to optimization problems if the number of unknowns is not too large [4, 5]. It is simple and advantageous because the engineer usually has a limited selection of discrete member sizes available for a particular design.

Another *nongradient* search technique is the powerful *complex* method of Box [6].

The complex method with a particular view towards application to optimization problems with a set of discrete design variables is presented herein. The method has the advantage, however, that any objective function can be employed [7, 8].

The present paper describes the *complex* method applied to the minimum weight design of welded steel frames using design variables that specify member sizes. Constraint of maximum stress, local buckling and minimum size are satisfied by using decisions made in the optimization process to select member sizes. At first, the results obtained by the minimum weight design using continuous variables are compared to the results obtained using discrete variables.

Secondly, the efficiencies of the *backtrack* and *complex* methods are compared.

2. The complex method

A general optimization problem may be defined as follows. Minimize a nonlinear objective function

$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

subject to inequality explicit constraints of the n independent variables

$$x_i^L \leq x_i \leq x_i^U \quad i = 1, \dots, n \quad (2)$$

in which x_i^L and x_i^U are lower and upper bounds, respectively, and subject to nonlinear inequality implicit constraints of the $(m-n)$ dependent variables

$$x_i^L \leq x_i \leq x_i^U \quad i = n + 1, \dots, m \quad (3)$$

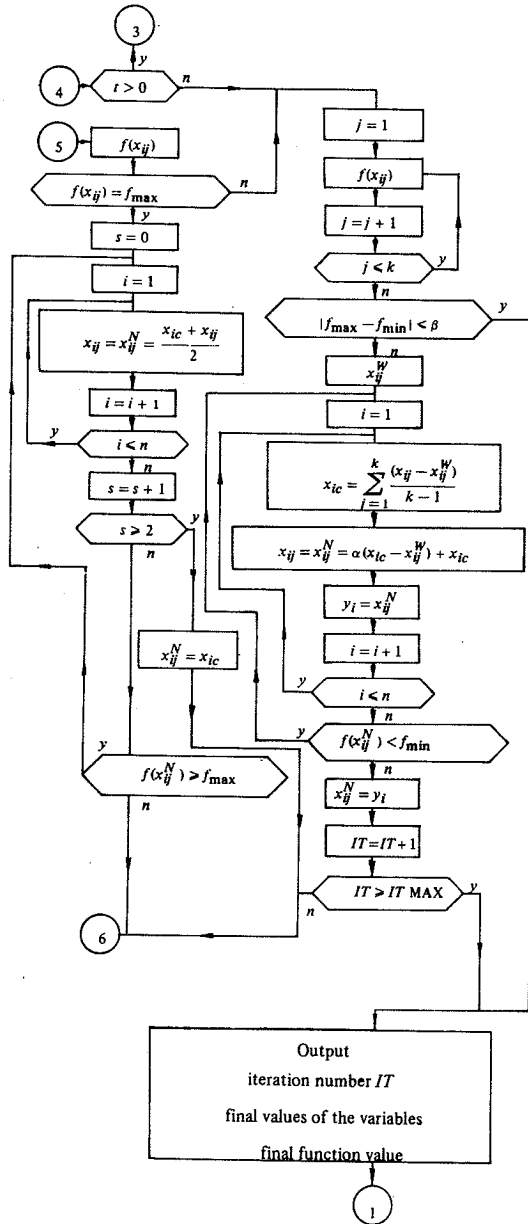


Fig. 1b.

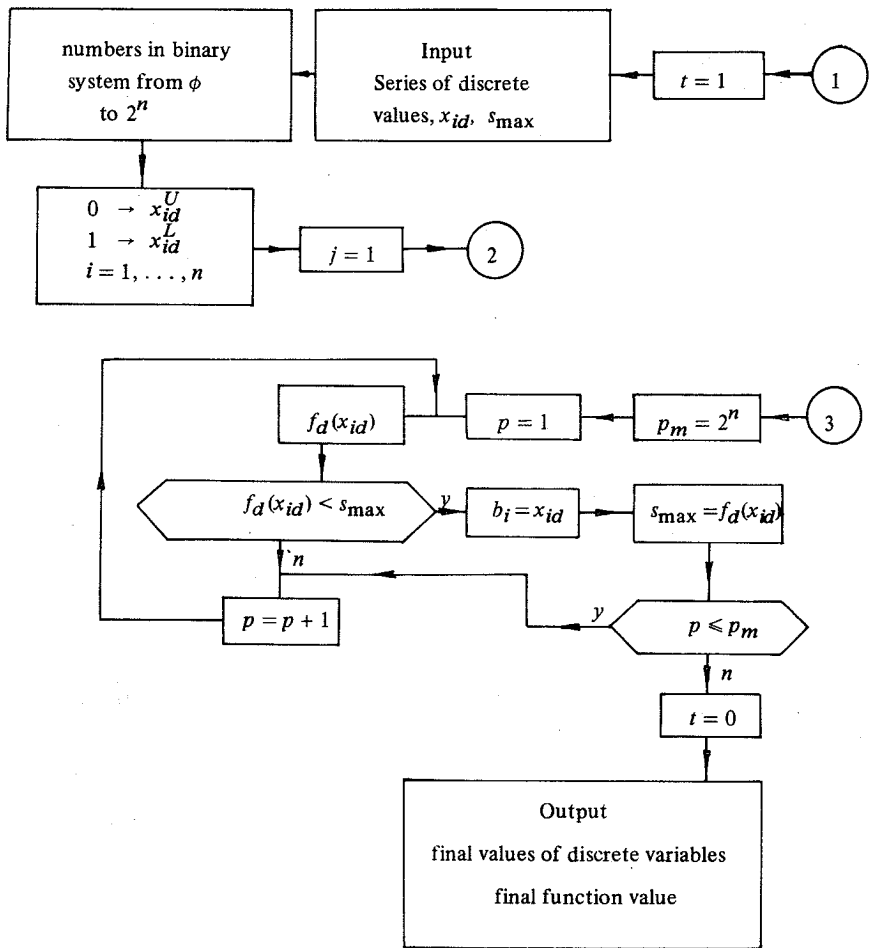


Fig. 1c.

For computational reasons it is advisable to use the same subscripts for explicit and implicit variables. If linear constraint are present or equality constraints are involved, other methods should prove to be more efficient.

The algorithm proceeds as follows: (see flow chart in Fig. 1. a, b, c).

2.1. Generation of starting complex

In the first iteration cycle ($IT = 0$) an original „complex” is generated. The complex contains $k \geq n + 1$ feasible points or vertices in an n - dimensional design space. It is assumed that an initial feasible point exists. The remaining $(k-1)$ points are generated randomly. For clarity a two-subscript notation is used, the first subscript indicating the

coordinate of the point and the second one indicating the point number. Thus, the j th point is generated from random numbers r_{ij} and from the upper bound and lower bound on the i th independent variable as follows:

$$x_{ij} = x_i^L + r_{ij}(x_i^U - x_i^L) \quad i = 1, \dots, n \quad (4)$$

and

$$j = 2, \dots, k.$$

The random numbers have a uniform distribution over the interval 0–1.

The point generated in the foregoing fashion will satisfy the explicit constraints but not necessarily the implicit constraints. If an implicit constraint is violated, the trial point is moved halfway towards the centroid of the previous points as follows:

$$x_{ic} = \frac{1}{k-1} \sum_{j=1}^{k-1} x_{ij} \quad (5)$$

and take a new point

$$x_{ij}^{\text{New}} = \frac{x_{ic} + x_{ij}^{\text{Old}}}{2} \quad (6)$$

The process is repeated a few times. If the design space is convex, halving will eventually lead to a feasible point (Fig. 2.).

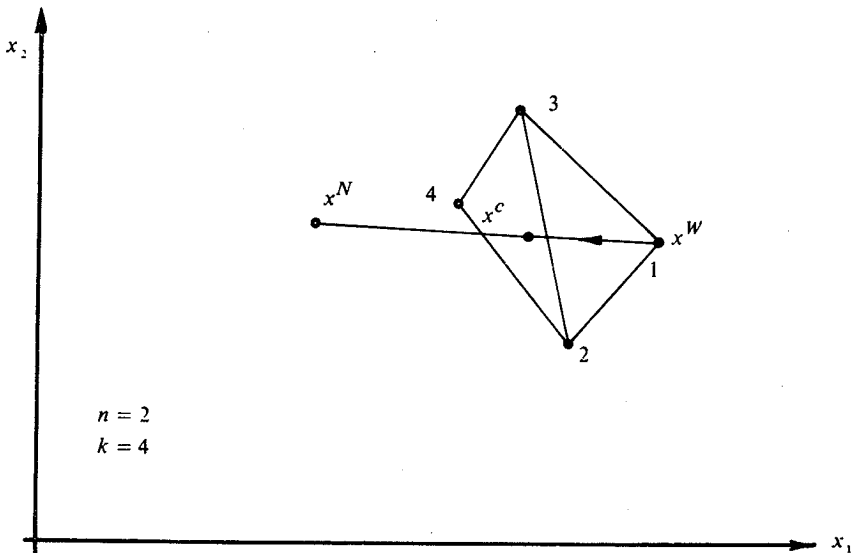


Fig. 2. Complex in two dimensions

2.2. Search procedure

Since the points are generated in a random fashion, they are located throughout the feasible design space. The function values are evaluated at each of these points. The points with the worst, and the best function values are determined. The convergence criterion checked:

$$|f_{\max} - f_{\min}| < \beta .$$

If it is not fulfilled, the worst point x_{ij}^{Worst} , for which $f(x_{ij}^{\text{W}}) = f_{\max}$, is rejected and replaced with a new one

$$x_{ij}^{\text{New}} = \alpha(x_{ic} - x_{ij}^{\text{W}}) + x_{ic} \quad (7)$$

where

$$x_{ic} = \frac{1}{k-1} \sum_{j=1}^k (x_{ij} - x_{ij}^{\text{W}}) \quad i = 1, \dots, n \quad (8)$$

is the centroid of the remaining points.

This new point is first examined to see whether it satisfies the explicit constraints. If not it is moved a small distance δ inside the violated limit. If so, a further check is made to see whether any of the implicit constraints are violated. If the function value is less than the worst value and all the implicit constraints are satisfied, then this point is accepted as an improvement.

We continue with the next iteration cycle ($IT = 1$). However, if either the function value of the new point is equivalent or larger than the worst value or any of the implicit constraints are violated, the point is replaced by selecting another point halfway towards the centroid from its present position.

$$x_{ij}^{\text{New}} = \frac{x_{ic} + x_{ij}^{\text{W}}}{2} \quad i = 1, \dots, n \quad (9)$$

in which x_{ij}^{New} is the i th coordinate of this new point. This point is checked for feasibility.

If all constraints are satisfied, but the point remains the worst one, i. e. $f(x_{ij}^{\text{New}}) = f_{\max}$, a new point must be calculated:

$$x_{ij}^{\text{New}} = \frac{x_{ic} + x_{ij}^{\text{W}}}{2} \quad i = 1, \dots, n .$$

If $f(x_{ij}^{\text{New}}) = f_{\max}$, we continue with step 2.2. until some predetermined criterion (convergence, iteration number) stops the process.

The value of α is chosen to be greater than unity in order to continually enlarge the complex and to compensate for moves halfway toward the centroid. The values of α and k , suggested by Box, are 1.3. and $2n$, respectively, although the speed of convergence does not appear to be prejudiced adversely by using alternative values. If $k \leq n + 1$ vertices are used in the complex, it tends to collapse.

2.3. Modifications of the original algorithm

Some modifications, including those suggested by some authors [9, 10, 11] were implemented. The following briefly describes the main modifications of the complex algorithm and some salient features of the computer program.

Expansion Step

If on reflection of the worst point, (7), none of the explicit or the implicit constraints is violated and the function value of the point is better than the best point value, this direction of search is considered to be promising. In this case an expansion step is taken by simply reflecting the point again in (7), and the results compared.

If unsuccessful, the point is restored to the previous position. This procedure allows a rapid movement of the complex towards the optimum.

Limitation of Halving Steps

The halving steps bring the new point closer to the centroid in a geometric progression. The centroid was always a feasible point in this study. Instead of continuously halving the distance between the wrong reflected point and the centroid, it is halved only twice, and then the centroid is used.

Variation in size of complex (k), in convergence and reflexion coefficients (β , α)

Since the result may be improved by using an other value of k , it is useful to repeat the computation with other k -values. The program varies the value of k automatically if it is needed. The decreasing of the convergence coefficient (β) improves the result, but increases the computer time remarkably.

The iteration number (IT) is reducible with the variation of value of reflexing coefficient (α) between certain bounds.

The program varies the values of α and β automatically if it is needed.

Discrete member sizes

To make the search more practicable it is advisable to use discrete member sizes. The original program was extended with a secondary search to find discrete optimum sizes in such a way, that not only are the explicit and implicit constraints satisfied but the merit function takes its minimum as well. It is assumed that the optimal discrete sizes are near to the optimal continuous ones.

Starting from the optimal continuous values, the secondary search chooses the nearest discrete sizes for each continuous size from the series of discrete values. The number of chosen discrete sizes for one continuous size can be two, three or more. The possible variations can be got by using binary, ternary or larger systems. In our numerical example we use the *binary system*, two discrete sizes, upper and lower, belonging to one continuous

us value. In a binary system number the figure zero means the upper discrete size, the figure one means the lower one. The first 2^n number in binary system gives the all possible variations. Each variation is tested, whether the explicit and implicit constraints are satisfied, and the optimal values minimizing the merit function are determined.

3. Minimum weight design of welded plane frames

The elastic or plastic minimum weight of steel frames has been investigated by a number of authors. Plastic design usually leads to a linear programming problem [12, 13]. For the elastic design nonlinear programming procedures have been applied [14, 15].

3.1. Design data

Geometrical conditions

We illustrate the application of the complex and backtrack method by the numerical example of a plane frame, made of welded I – section bars as shown in Fig. 3. We con-

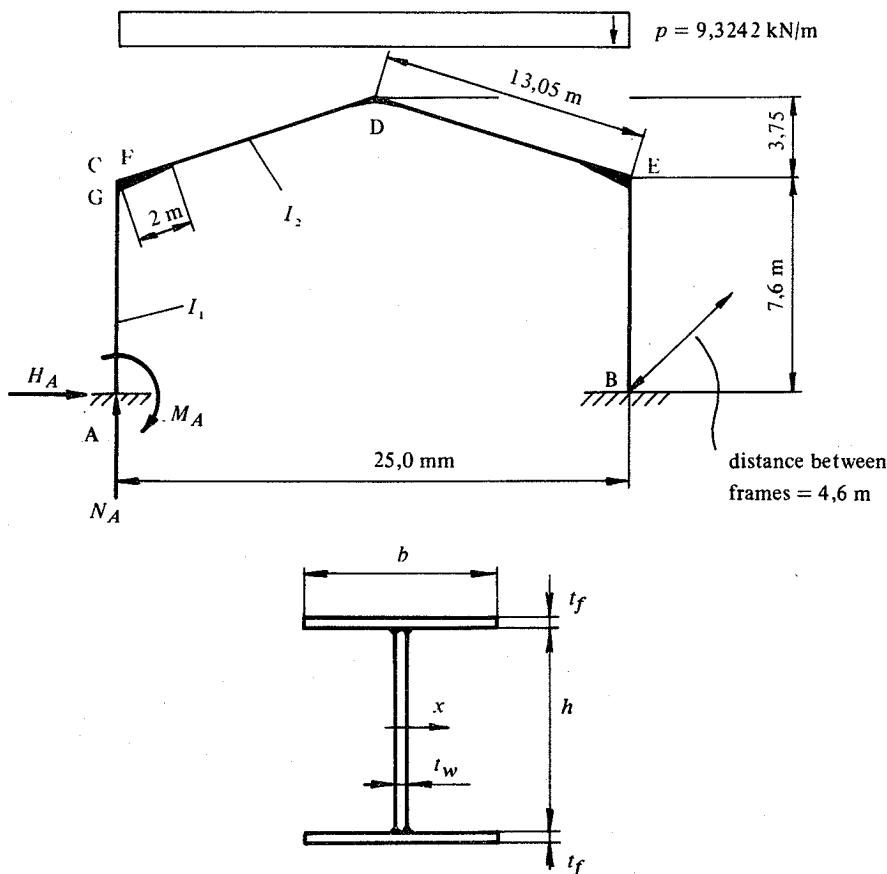


Fig. 3. Geometrical and loading conditions of the plane frame

sider the sizes of columns and rafters to be different. So the number of unknown variables is *eight*.

Loading conditions

The load applied to the frame is also shown in *Fig. 3*. The intensity of the uniformly distributed normal load is

$$p = (1,1 \cdot 0,57 + 1,4 \cdot 1,0) \cdot 4,6 = 9,3242 \text{ [kN/m}^2\text{]} \quad (10)$$

Material: steel Fe 360 used.

3.2. Design constraints

a) *Stress constraints*

Maximum elastic stresses in the columns and rafters due to bending and compression must be lower than the admissible stress.

We introduce the ratio of moments of inertia

$$\omega = \frac{I_2}{I_1}$$

For the columns, the maximum stress occurs at the point *A*. By using the equations of *Glushkov* [16] we get

$$M_1 = |M_A| \text{ [kNm]} = \frac{3266,8895 \omega + 989,5919}{1,0175 \omega^2 + 13,8624 \omega + 0,73031} \quad (12)$$

$$N_1 = |N_A| \text{ [kN]} = \frac{2ps^2}{l} = 116,6 \text{ [kN]} \quad (13)$$

For the rafters the maximum stress occurs at the point *F*, at the end of haunching

$$M_2 = |M_F| = 8,174696 H_A - |M_A| - 206,2564 \quad (14)$$

$$N_2 = |N_F| = 28,373 + 0,957826 H_A \quad (15)$$

where

$$M_C = \frac{4963,8363 \omega + 88,6588}{1,0175 \omega^2 + 13,86245 \omega + 0,73031} \quad (16)$$

$$H_A = (|M_A| + M_C) / 7,6 \quad (17)$$

The maximum stresses in the columns and rafters are as follows

$$\sigma_{Mi} + \sigma_{Ni} = M_i/W_i + N_i/A_i \leq R_u \quad (18)$$

considering the I-section sizes as shown in *Fig. 3*. the cross section areas are

$$A_i = h_i t_{wi} + 2b_i t_{fi} \quad (19)$$

the section moduli are

$$W_i = h_i (b_i t_{fi} + h_i t_{wi}/6) \quad (20)$$

the moments of inertia are

$$I_i \cong W_i h_i / 2 \quad (21)$$

$$i = 1, 2 .$$

The subscripts 1 and 2 are valid for columns and rafters, respectively.

The limiting stress for steel Fe 360 can be taken as $R_u = 200$ MPa.

b) Local web buckling constraints

In the case of bending and compression the buckling constraint may be expressed as

$$t_{wi}/h_i \geq c_1 .$$

According to Frieze [17], for a web made of steel Fe 360 subject to bending it is $c_1 = 1/145$.

For a web subject to bending (σ_M), compression (σ_N) and shear (τ) the following approximate interactive formula may be used [18]:

$$\frac{1}{c_1} = 145 \sqrt[4]{\frac{(1 + \sigma_N/\sigma_M)^2 + 3(\tau/\sigma_M)^2}{1 + 173(\sigma_N/\sigma_M)^2 + 20(\tau/\sigma_M)^2}} .$$

Thus, in our case, neglecting the effect of shear, we can write

$$\frac{h_i}{t_{wi}} \leq 145 \sqrt[4]{\frac{(1 + \sigma_{Ni}/\sigma_{Mi})^2}{1 + 173(\sigma_{Ni}/\sigma_{Mi})^2}} ; \quad i = 1, 2 . \quad (22)$$

c) Local flange buckling constraints

In the case of a compressed flange the buckling constraint can be written as

$$t_{fi}/b_i \geq c_2 .$$

For a flange made of steel Fe 360 it is $c_2 = 1/30$. Thus, we use the following constraints:

$$b_i/t_{fi} \leq 30 ; \quad i = 1, 2 . \quad (23)$$

3.3. Objective Function

When the weight of the structure is selected as the objective to be minimized in (1), this quantity is computed as a sum of member weights.

$$F(x_i) = \sum_{i=1}^p \rho A_i l_i \quad (24)$$

where A_i and l_i are the area and length of the i th member, respectively and ρ is the material density.

Since ρ is the same for each member, the volume of the frame can be taken as objective function:

$$V(x_i) = 2 \sum_{i=1}^2 l_i A_i \quad (25)$$

$$V[\text{mm}^3] = V(x_i) = 2(7,6A_1 + 13,05A_2) \cdot 10^5. \quad (26)$$

Thus, (26) is a linear function of the unknown sizes.

3.4. Results of computations by complex method

The computer program was written in Fortran IV and run on computer CDC 3300 and ODRA 1304, respectively. The eight variables frame problem was computed with the following bounds of parameters:

	bounds	best values
k	9 – 20	~ 16
α	1,1 – 2	1,3 – 1,7
β	1200(0,92%) – 200(0,15%)	600(0,5%)

Table 1. Bounds and final values of continuous and discrete variables of the numerical example treated by complex method [mm]

Run number	1		2		3		bounds on explicit variables		starting point
k	14		16		18		upper	lower	
α	1,3		1,3		1,3				
β	600		1200		600				
$x_1 = h_1$ [mm]	524,2	540	608,4	600	611	620	840	520	840
$x_2 = t_{w1}$	4,46	4,5	6,8	5	5,1	5,5	12	4	12
$x_3 = b_1$	275,6	280	192,5	200	189,5	200	420	100	420
$x_4 = t_{f1}$	9,9	10	12,1	13	12,8	13	20	4	20
$x_5 = h_2$	617,8	620	538,2	560	577,0	580	740	420	740
$x_6 = t_{w2}$	5,7	6	5,2	5,5	5,3	5,5	12	4	12
$x_7 = b_2$	106,7	110	165,1	170	162,7	170	420	100	420
$x_8 = t_{f2}$	8,7	9	7,8	8	6,8	7	20	4	20
$f(x_i) = V \cdot 10^{-8}$ [mm ³]	2,590	2,708	2,743	2,760	2,599	2,762			
number of iteration	154	154	101	101	173	173			
CPU time on CDC 3300 [s]	109,4	111,6	105	106,8	114	114,6			

Table 2. Final values of discrete explicit variables of the numerical example obtained by complex and backtrack method, respectively, [mm]

method	backtrack		complex		upper	lower	starting point	step size
	run number	1	2	3	4	bounds on explicit variables		
k			14	16				
α			1,3	1,3				
β			600	800				
total number of combinations	$1955,1 \cdot 10^6$	$548 \cdot 10^6$						
$x_1 = h_1$ [mm]	580	580	540	600	840	520	840	20
$x_2 = t_{w1}$	5	5	4,5	5	12	4	12	0,5
$x_3 = b_1$	240	220	280	200	420	100	420	20
$x_4 = t_{f1}$	11	12	10	13	20	4	20	1
$x_5 = h_2$	500	520	620	560	740	420	740	20
$x_6 = t_{w2}$	5	5	6	5,5	12	4	12	0,5
$x_7 = b_2$	100	120	110	170	420	100	420	10
$x_8 = t_{f2}$	16	12	9	9	20	4	20	1
$f(x_i) =$ $= V \cdot 10^{-8}$ [mm ³]	2,732	2,674	2,708	2,7602				
number of iteration	—	—	154	149				
number of tested combinations	109900	330800	—	—				
CPU time on CDC 3300 [s]	398	1026	111,6	110,4				

Concerning the effectiveness of the computation it was found, that the best values of k are around $2n$ (16), the best values of α are from 1,3 to 1,7 and the best values of β are around 600 (0,5%).

Table 1 contains the bounds of the explicit variables x_1 to x_8 and with continuously varying and discrete variables.

A changing in values of k , α and β may result in a large changing in the final values of variables, but only a small changing in the value of the merit function.

3.5. Comparison of computational results obtained by the backtrack programming method

For the characterization of the complex method it is useful to compare it with other methods. We have solved several optimum design problems with the backtrack programming method. This method was found to be effective and easily manageable [4, 5]. Using discrete variables the complex programming method may be compared to the backtrack one. The eight variables frame problem was solved by the backtrack method too.

Table 2 shows the results.

The complex method is faster than the backtrack one. The CPU time of complex is fifth-tenth part of the backtrack's one, depending on the parameters k , α , β .

The advantages of backtrack method become visible at the final values of the merit functions. The final values obtained by the complex programming method are 3–4% larger than those obtained by the backtrack one.

4. Conclusions

The *complex* method has been treated whereby low cost or weight designs may be generated for realistic computer expense by an approach which does not require the computation of derivatives. The advantage of the procedure is that any reasonable cost or weight model, which reflects the user's practice, may be utilized. The method is especially adaptable to dealing with a *discrete member spectrum*. The method has been demonstrated to be of practical worth on a common structural engineering problem. The results obtained from application of the algorithm was compared with those obtained by the backtrack programming method.

5. Acknowledgement

The author gratefully acknowledges to *professor Dr. J. Farkas* for his help and critical remarks by which this work was supported.

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OPTIMALBEMESSUNG VON GESCHWEIßTEN RAHMEN MITTELS DER KOMPLEX-PROGRAMMIERUNGSMETHODE

von
K. JÁRMAI

Zusammenfassung

Die Komplex-Methode wurde mit einem ausführlichen Flußdiagramm dargestellt und für diskrete Veränderliche erweitert. Das numerische Beispiel eines einschiffigen Portalrahmens illustriert die Anwendung der Methode auf die Optimalbemessung von Stabtragwerken aus Teilen geschweißten I-Profils. Der Einfluß der Anzahl von Komplex-Punkten, des Wertes des Spiegelungskoeffizienten und der Konstante des Konvergenzkriteriums wurde untersucht. Der Vergleich der mit der Komplex- bzw. Backtrack-Methode erhaltenen Ergebnisse zeigt die Vor- und Nachteile einiger Methoden.

ОПТИМАЛЬНОЕ ПРОЕКТИРОВАНИЕ СВАРНЫХ РАМ С ПОМОЩЬЮ КОМПЛЕКС-МЕТОДА ПРОГРАММИРОВАНИЯ

К. ЯРМАИ

Резюме

Комплекс-метод изложен детальной блок-схемой вычислительного процесса и развит для дискретных переменных. Численный пример простой рамы иллюстрирует применение метода к оптимальному проектированию стержневых конструкций с элементами сварного двутаврового профиля. Трактуются эффект числа точек комплекса, величины коэффициента отражения и константы критерия конвергенции. Сравнение результатов полученных комплекс-методом и комбинаторическим методом „бектрек“ показывает выгоды и невыгоды двух методов.