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COST OPTIMIZATION OF WELDED STEEL BEAMS*

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Summary

The optimization problem is formulated as a mathematical programming one. The objective function expresses the approximate structural cost which depends on mass and number of elements in the structure. A possible way of formulation of the optimization problem is shown for welded steel I-section beams. A numerical example illustrates the difference in the results of the cost and volume optimization. Homogeneous and hybrid I-beams made of three types of steel are compared with each other. The constrained Rosenbrock method is used to find the optimum.

1. Introduction

The widely spread approach to the optimization of welded steel beams based on the mass minimization is suitable if the type of steel and the shape of cross-section has been known in advance. When a hybrid beam or a girder with unknown shape of cross-section and distribution of stiffeners is to be optimized the solution providing a cost minimum differs from that which yields a mass minimum. Thus, if the mass minimization is not a special aim the cost of manufactured and erected structure should be accepted as objective function.

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The aim of the present paper is to show a possible formulation of the cost functio and to illustrate the effect of the labour cost on the volume of an optimal welded I -bear

2. The cost function

Usually the main beam parameter (span length) has been defined in advance. Ther fore the cost of transportation and erection can approximately be regarded as proportio to the beam mass. Then, using the economical estimation technique for metal structures [1] the cost of manufactured and erected beam can be written as follows:

$$C = k \left(k^m \sum_i c_i^m G_i + k^L \sum_i c_i^L \sqrt{G_i n_i} + c^t \sum_i G_i \right). \tag{1}$$

where k, k^m, k^L are the coefficients reflecting additional expenditures for erection (k) and manufacturing (k^m, k^L); c_i^m, c_i^L are the cost coefficients for the ith type of steel an for the labour for structure made of it, respectively; c^t is the cost factor of transportatio and erection; G_i, n_i are the mass and number of elements made from the ith type of stee respectively (if an element of structure is assembled from several details, n_i is the numbe of details made of the ith type of steel).

In the expression (1) the dependence of the shophours (T) on the structural mass (and number of details (n) has been approximated in the following manner [1]:

$$T = \xi \sqrt{G n}$$

where ξ is a coefficient.

3. Design constraints and objective function for a welded I-beam

If the length of beam span is given, the parameters to be optimized are the dimen- sions and shape of cross-section, number and distribution of additional elements, types of steels. The parameter values providing a minimum of structural cost should be searched i the feasible region limited by the constraints on strength, overall and local buckling, de- formation and sizes of structure. The formulation of these constraints and the objective function (1) as functions of the parameters to be optimized yields a mathematical pro- gramming problem, which can be solved by a computer method.

In order to illustrate this, consider a steel beam (homogeneous or hybrid) of const doubly symmetric I-cross-section under arbitrary load (*Fig. 1*). So the web and flanges are the beam elements. Using the main expressions from [2] the constraints can be writt in the following form:

(a) Stress constraint

$$\sqrt{\sigma^2 + 3\tau^2} \leqslant R_{uf}, \quad \text{where} \tag{2}$$

$$\sigma = \frac{M(F, I_y, r)}{W_y}\ ; \quad \tau = \frac{Q(F, I_y, r)}{h\,t_w}\ ;$$

$$W_y = \frac{b\,t_f\,(h+t_f)^2 + \dfrac{h^3\,t_w}{6}\,\dfrac{R_{yw}}{R_{uf}}\left(3 - \dfrac{R_{yw}^2}{R_{uf}^2}\right)}{h + 2\,t_f}$$

(b) Deflection constraint

$$I_y = \frac{h^3\,t_w}{12} + \frac{b\,t_f}{2}\,(h+t_f)^2 \geqslant \frac{D_j\,(F, I_y, r)}{\gamma_j\,LE}\ , \quad \text{where} \tag{3}$$

$$D_j = \int_0^L \overline{M}_j\,M\,(F, I_y, r)\,dx\ ;$$

(c) Web buckling constraint

$$\frac{t_w}{h} \geqslant \beta \sqrt[4]{\frac{\sigma_w^2 + 20\,\tau^2}{\sigma_w^2 + 3\,\tau^2}}\ \sqrt{\frac{R_{uw}}{R_{ul}}}\,. \tag{4}$$

(d) Flange buckling constraint

$$\frac{t_f}{b} \geqslant \delta \sqrt{\frac{\sigma_f}{R_{u1}}}\ \sqrt{\frac{R_{uf}}{R_{u1}}}\,. \tag{5}$$

(e) Size constraints

$$t_0 \leqslant t_w = \alpha\,t_f\,, \quad \text{where} \quad \alpha = 0{,}5 \div 1{,}5. \tag{6}$$

Here σ, σ_w, σ_f are the normal stress in dangerous cross-section of the beam and its values for web and flanges yielding minima of the right parts of (4) and (5), respectively; τ is the shear stress; $M(F, I_y, r)$, $Q(F, I_y, r)$ are the bending moment and shear force in any cross-section (both of them depend on the external force vector F and on the ratio of I_y to r); r is the rigidity of elastic-pliable support; I_y, W_y are the moment of inertia and the modulus of cross-section, respectively; R_{uf}, R_{yw}, R_{ul} are the ultimate limiting stresses for flanges, web and mild steel, respectively; R_{yw} is the yield stress for web in the case of a hybrid beam ($R_{ym} = R_{uf}$ in the case of a homogeneous beam); $\overline{M}_j$ is the bending moment in any cross-section due to the virtual load for the jth deflection constraint; γ_j is the allowable ratio of a beam deflection in the jth direction to the beam span; E is the modulus of elasticity; β, δ are constants; t_0 is the limiting value for web thickness.

In the inequality (2) values of $M(F, I_y, r)$ and $Q(F, I_y, r)$ should be taken for the dangerous cross-section to provide maximum of the left part of (2). The expression (3) is related only to the elastic deflection. The lateral buckling of the beam is assumed to be eliminated by suitable bracings.

The objective function (1) for an I-beam (*Fig. 1*) can be written in the form:

$$C = k\,[k^m \rho L\,(c_N^m h t_w + 2\,c_f^m b t_f)\,] + \\ + k^L \sqrt{\rho L}\,(c_w^L\, h t_w n_w + c_f^L \sqrt{2\, b t_f n_f}) + \\ + c^t \rho L\,(h t_w + 2\, b t_f) \tag{7}$$

where ρ is the material density.

4. Numerical example

The mathematical formulation (2)–(7) of the optimization problem enables to show numerically the difference between the beams having minimal cost and mass, respectively.

Consider a simply supported ($r = r_1 = r_2 = 0$) beam of span $L = 12$ m under a uniformly distributed load $p = 10^4$ N/m. Thus $M_{max} = p L^2/8 = 10^4 \cdot 12^2/8 = 1{,}8 \cdot 10^5$ [Nm]. The midspan deflection is limited: $D_j/(\gamma_j EL) = 2{,}7 \cdot 10^6/(5 \cdot 10^{-3} \cdot 12 \cdot 2{,}1 \cdot 10$ $= 2{,}143 \cdot 10^{-4}$ [m^4]. For a beam manufacture one can use three types of steel with following characteristics: $R_u = 210$; 290; 380 MPa; $R_y = 250$; 340; 450 MPa; $c^m = 108{,}5$; 137,1; 159,7 rouble/t; $c^L = 7{,}40$; 8,14; 8,73 rouble/t^{0,5}. It is assumed that the web and each flange are assembled from three details (sheets). Thus $n_w = 3$; $n_f = 2 \cdot 3 = 6$. It is also accepted that $k = 1{,}148$; $k^m = 5{,}303$; $k^L = 4{,}14$; $\beta = 1/145$; $\delta = 1/30$; $c^t = 6{,}1$ roub $R_{ul} = 210$ MPa; $\rho = 7{,}8$ t/m^3; $t_0 = 4 \cdot 10^{-3}$ m.

The effect of shear can be neglected for the simply supported beam considered in the numerical example. In order to simplify the calculation we can also use the following approximate expressions:

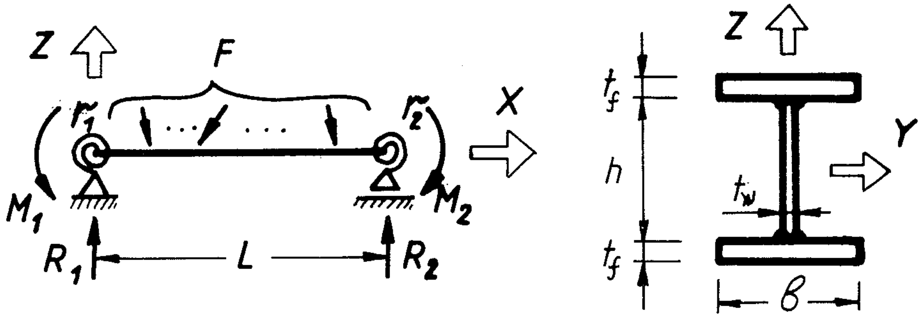

Fig. 1. Scheme of a beam of welded I-section

$$I_y \approx \frac{h^2}{12}\,(ht_w + 6\,bt_f)\,; \qquad W_y \approx h\left(\frac{ht_w}{6} + bt_f\right).$$

Finally, taking into account the accepted values, the mathematical model of the problem to be solved can be written in the form (dimensions in N, m and rouble):

minimize

$$C = 569{,}8\,(c_w^m\;ht_w + 2\,c_f^m\,bt_f) + + 79{,}64\,(c_w^L\sqrt{ht_w} + 2\,c_f^L\sqrt{bt_f}\,) + + 655{,}5\,(ht_w + 2\,bt_f) \tag{8}$$

subjected to

$$W_y \approx h\left[bt_f + \frac{ht_w}{12}\cdot\frac{R_{yw}}{R_{uf}}\left(3 - \frac{R_{yw}^2}{R_{uf}^2}\right)\right] \geqslant \frac{1{,}8\cdot 10^5}{R_{uf}}\;; \tag{9}$$

$$I_y \approx \frac{h^2}{12}\,(ht_w + 6\,bt_f) - 2{,}143\cdot 10^{-4}\,; \tag{10}$$

$$\frac{t_w}{h} \geqslant \frac{1}{145} \sqrt{\frac{\sigma_w}{R_{u1}}} \sqrt{\frac{R_{uw}}{R_{u1}}} = \frac{\sqrt{\sigma_w R_{uw}}}{3{,}045 \cdot 10^{10}} \; ; \tag{11}$$

$$\frac{t_f}{b} \geqslant \frac{1}{30} \sqrt{\frac{\sigma_f}{R_{u1}}} \sqrt{\frac{R_{uf}}{R_{u1}}} = \frac{\sqrt{\sigma_f R_{uf}}}{6{,}3 \cdot 10^{9}} \; . \tag{12}$$

$$(0{,}5 \div 1{,}5)\, t_f = t_w \geqslant 4 \cdot 10^{-3} \; . \tag{13}$$

The following rules can be recommended for calculations of σ_w u σ_f:

(a) for homogeneous beams:
if the stress constraint (9) is active then

$$\sigma_w \approx \sigma_f = R_u \; ;$$

if the deflection contraint (10) is only active then

$$\sigma_w \approx \sigma_f = 1{,}8 \cdot 10^5 / W_y \; ;$$

(b) for hybrid beams:
if the stress constraint (9) is active then

$\sigma_w = R_{yw}$ and $\sigma_f = R_{uf}$; if the deflection constraint (10) in only active then $\sigma_f = 1{,}8 \cdot 10^6 / W_y$

and if

$\sigma_f \geqslant R_{yw}$ then $\sigma_w = R_{yw}$,

if $\sigma_f < R_{yw}$ then $\sigma_w \approx \sigma_f$.

Before solving the problem (8)–(13) some numerical investigations of the objectiv function and feasible region have been carried out for the homogeneous beam with $R_{uw} = R_{uf} = R_{u1} = 210$ MPa and $c^m_w = c^m_f = 108{,}5$ rouble/t but $c^L_w = c^L_f = 74{,}0$ rouble i.e. c^L was increased by ten times to raise an influence of a labour cost on a beam one. The load was also multiplied by ten ($p = 10^5$ N/m) in order to make active only the stre constraint (9). In this case the local buckling constraints (11) and (12) take the form:

$$145\, t_w \geqslant h \; ; \qquad 30\, t_f \geqslant b \; .$$

Moreover they are practically active and it can be assumed that

$$t_w = h/145 \quad ; \quad t_f = b/30.$$

Using this approximation and the above accepted values the following expressions for the objective function (8) and for the stress and deflection constraints (9) and (10) can be written:

$$C = 430{,}9 \ h\,(h + 1{,}136) + 4179{,}7\, b\,(b + 0{,}515)\,; \tag{14}$$

$$h\,(h^2 + 29\, b^2) = 7{,}456\,; \tag{15}$$

$$h^2\,(h^2 + 29\, b^2) = 3{,}733. \tag{16}$$

In addition the volume function takes the form

$$V = 12\,(h\, t_w + 2\, b\, t_f) = 0{,}8\,(0{,}103\, h^2 + b^2). \tag{17}$$

Fig. 2 shows the contours of the cost and volume (dotted lines) functions (14) and (17) and the limiting curves for stress and deflection constraints (15) and (16). It can be seen that the cost optimum (point 1) differs from the volume one (point 2) if variable b is limited for example by $0{,}2\,M$ to avoid the lateral instability.

In the course of solution of the problem (8)–(13) the following beams have been investigated: three types of homogeneous beams made of steel 210, 290 and 380, respectively, and three types of hybrid beams made of steels 290/210, 380/210 and 380/290 (steels of greater strength are used for flanges). The optimization of each type of beams was carried out by computer program realizing Rosenbrock's method for „Commodore 64" microcomputer.

5. The Hill algorithm

The procedure is based on the „automatic" method proposed by Rosenbrock [3]. The method of rotating coordinates can be considered as a further development of the Hooke and Jeeves method. At the Hill algorithm, the coordinate system is rotated in each stage of minimization in such a manner that the first axis is oriented towards the locally estimated direction of the valley and all the other axes are made mutually orthogonal and normal to the first one. No derivatives are required. The algorithm proceeds as follows:

(*i*) Before starting the minimization process, define a set of „initial" step lengths, S_i, $i = 1, 2, \ldots, N$, to be taken along the search directions M_i, $j = 1, 2, \ldots, N$, a starting point that satisfies the constraints and does not lie in the boundary zones, and evaluate the objective function. The boundary zones are defined as follows:

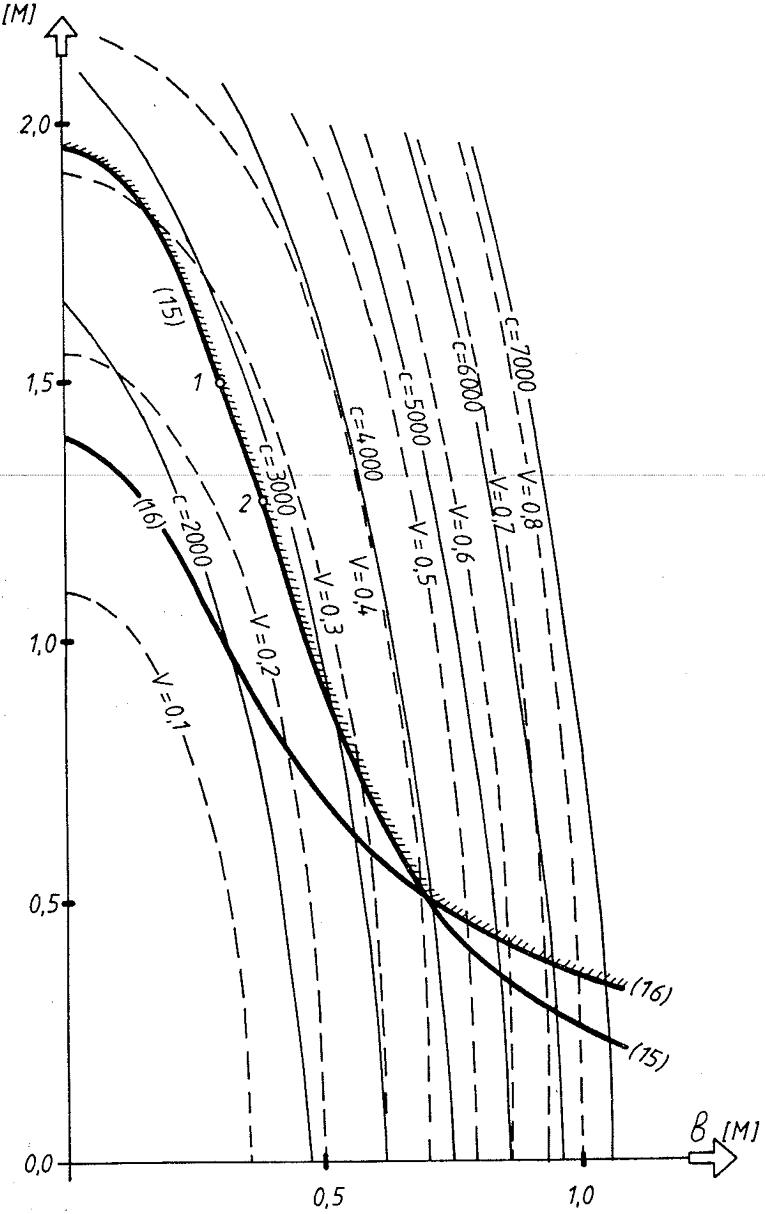

Fig. 2. Results of numerical investigation of the objective functions in the feasible region

Lower zone:

$$x_i^L \leqslant x_i \leqslant x_i^L + (x_i^U - x_i^L) \cdot 10^{-4} \tag{18}$$

Upper zone:

$$x_i^U \geqslant x_i \geqslant x_i^U - (x_i^U - x_i^L) \cdot 10^{-4} \tag{19}$$

$$i = 1, 2, \ldots, M$$

(*ii*) After each function evaluation, the following steps are carried out:

Define by f^0 the current best objective function value for a point where the constraints are satisfied, and f^x where in addition to this the boundary zones are not violated. f^0 and f^x are initially set equal to the objective function value at the starting point.

(*iii*) The first variable x_1 is stepped a distance S_1 parallel to the axis, and the function evaluated. If the current point objective function value, f, is worse (greater or less) than f^0 or if the constraints are violated, the trial point is a failure and S_1 decreased by a factor β, $0 < \beta \leqslant 1{,}0$, and the direction of movement reversed. If the move is termed a success, S_1 increased by a factor α, $\alpha \geqslant 1{,}0$. The new point is retained, and a success is recorded. The values of α and β are usually taken as 3,0 and 0,5 respectively.

(*iv*) Continue the search sequentially stepping the variables, x_i, a distance S_i parallel to the axis. The same acceleration or deceleration and reversal procedure is followed for all variables, until at least one step has been successful and one step has failed in each of the N directions.

(*v*) Compute the new set of directions $M_{i,j}^{(k)}$ rotating the axes by the following equations:

$$M_{i,j}^{(k+1)} = \frac{D_{i,j}^{(k)}}{\left[\sum_{l=1}^{n} (D_{i,j}^{(k)})^2\right]^{1/2}} \tag{20}$$

where $D_{i,1}^{(k)} = A_{i,1}^{(k)}$

$$D_{i,j}^{(k)} = A_{i,j}^{(k)} - \sum_{l=1}^{j-1} \left[\left(\sum_{n=1}^{j} M_{n,j}^{(k+1)} \cdot A_{n,j}^{(k)} \right) \cdot M_{i,j}^{(k+1)} \right]$$

$$j = 2, 3 \ldots, N \tag{21}$$

$$A_{i,j}^{(k)} = \sum_{l=j}^{N} d_i^{(k)} \cdot M_{i,l}^{(k)} \tag{22}$$

$$i = 1, \ldots, N \quad , \qquad j = 1, \ldots, N$$

d_i -sum of distances moved in the i direction since last rotation of axes.

(vi) Search is made in each of the x directions using the new coordinate axes

$$\text{new } x_i^{(k)} = \text{old } x_i^{(k)} + S_j^{(k)} M_{i,j}^{(k)} \tag{23}$$

(vii) If the current point lies within a boundary zone, the objective function is mo dified as follows:

$$f(\text{new}) = f(\text{old}) - (f(\text{old}) - f^x)(3\lambda - 4\lambda^2 + 2\lambda^3) \tag{24}$$

where

$$\lambda = \frac{\text{distance into boundary zone}}{\text{width of boundary zone}}$$

$$= \frac{x_i^L + (x_i^U - x_i^L) \cdot 10^{-4} - x_i}{(x_i^u - x_i^L) \cdot 10^{-4}} \qquad \text{(lower zone)}$$

$$= \frac{x_i - (x_i^U - (x_i^U - x_i^L) \cdot 10^{-4})}{(x_i^U - x_i^L) \cdot 10^{-4}} \qquad \text{(upper zone)} \tag{25}$$

At the inner edge of the zone, $\lambda = 0$, i.e., the function is unaltered ($f(\text{new}) = f(\text{old})$).
At the constraints, $\lambda = 1$, and thus $f(\text{new}) = f^x$.
For a function which improves as the constraint is approached, the modified function h an optimum in the boundary zone.

($viii$) f^x is set equal to f^0 if an improvement in the objective function has been ol tained without violating the boundary zones or constraints.

(ix) The search procedure to find the continuous values of the variables is termin ed when the convergence criterion is satisfied.

(x) The procedure was modified by a secondary search to find the discrete values c the variables. In details it was written in [4]. A flow sheet illustrating the above procedu is given in *Fig. 3.*

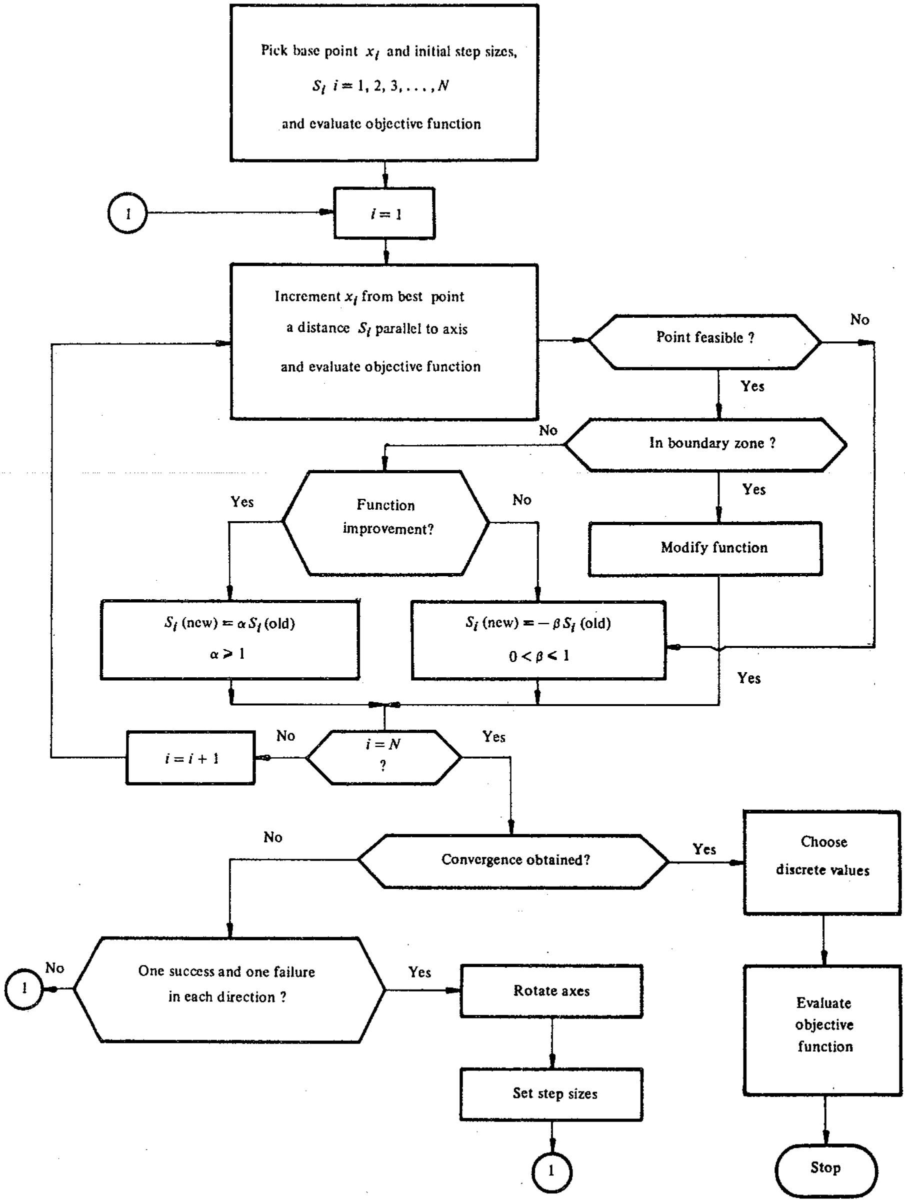

Fig. 3. Flow chart of the Hill algorithm

6. Computational results

Each type of beam has been optimized using the cost and volume objective functic from the same starting point which was taken not far from boundaries of feasible region The results obtained can be only regarded as approximately optimal due to the discretiza tion of variables and limitation of solution time.

The results are presented in *Table 1.* The column 1 contains the costs of optimal beams and their volumes obtained by the cost optimization. It can be seen that the 290/21 hybrid beam has the lowest cost (it has the following dimensions in [m]: $h = 0{,}694$; $t_w =$ $= 0{,}006$; $b = 0{,}052$; $t_f = 0{,}005$). At the same time the 380/290 hybrid beam has the sm lest volume (0,0554 m^3).

The results of the volume optimization are presented in the column 2. The 380/29 hybrid beam has the smallest volume (0,0552 m^3) and the following dimensions [m]: h $= 0{,}662$; $t_w = 0{,}006$; $b = 0{,}079$; $t_f = 0{,}004$. It should be noted that the costs corresp ing the beams of minimum volume are a bit different from those for the beams of mini mum cost. The small difference is due to the cross-section shape and number of details c fined in advance.

The column 3 contains the results of cost optimization with the greater labour cos factors ($c^L = 74{,}0$; $81{,}4$; $87{,}3$ rouble/$t^{0{,}5}$). In this case the 290/210 hybrid beam has tl lowest cost (876,5 roubles) but the more expensive hybrid beam from the steels 380/21(has the smallest volume (0,0558 m^3).

Cost optimizations have been carried out by using a greater load ($p = 10^5$ N/m) fo both above mentioned ranges of the labour cost factors. The results are presented in the columns 4 and 5. It can be seen that the hybrid beams from the steels 380/210 have the lowest costs with the lowest volumes.

It is worth mentioning that, generally, hybrid beams are more economical than th homogeneous ones.

Other investigation has been carried out to estimate the influence of the labour co factors on the cost and volume of the 290/210 hybrid beam. The cost optimizations hav been performed in the following range of the labour cost factor: c^L, 10 c^L, 20 c^L, 30 c 40 c^L, 100 c^L. The obtained results are presented in *Table 2.* It can be seen that the eff of the labour cost factor on the volume is not great.

7. Conclusions

The presented approach to the structural cost optimization can be used at the stag of designing if the exact scheme of structure is unknown. It can help a designer to choo a structural scheme, number of elements, type of material and to obtain approximate va lues of variables. The exact values can then be found through the cost optimization by using a more exact cost function.

Table 1. Results of the beam optimization

Type of beam		Cost (Volume values in rouble) m³ for different values of p and c^L				
		p, c^L 1	p, c^L 2	$p, 10 \cdot c^L$ 3	$10 \cdot p, c^L$ 4	$10 \cdot p, 10 \cdot c^L$ 5
homogeneous	210/210	367,1/0,0583	370,1/0,0580	961,4/0,0586	1514,9/0,262	2809,9/0,262
	290/290	428,1/0,0557	430,2/0,0556	995,8/0,0559	1669,7/0,231	2993,9/0,231
	380/380	496,6/0,0559	499,7/0,0558	1110,4/0,0560	1866,2/0,223	3176,3/0,218
hybrid	290/210	359,8/0,0562	360,8/0,0560	876,5/0,0571	1449,8/0,223	2643,3/0,216
	380/210	368,7/0,0561	370,6/0,0553	888,8/0,0558	1285,0/0,181	2543,8/0,192
	380/290	430,2/0,0554	436,4/0,0552	1059,2/0,0618	1596,7/0,204	2827,1/0,208

Table 2. Effect of the labour cost factor on the cost and volume

Labour cost factors	c^L	$10\,c^L$	$20\,c^L$	$30\,c^L$	$40\,c^L$	$100\,c^L$
cost [rouble]	359,8	876,5	1430,2	1973,4	2504,3	5689,4
volume [m³]	0,0562	0,0571	0,0579	0,0585	0,0589	0,0595

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KOSTENOPTIMIERUNG VON GESCHWEIßTEN STAHLTRÄGERN

S. BYKOWSKIJ–K. JÁRMAI

Zusammenfassung

Das Optimierungsproblem wurde als eine mathematische Programmierungsaufgabe formuliert. In der Zielfunktion wurden die Konstruktionskosten näherungsweise ausgedrückt, die von dem Masse und Anzahl der Elementen abhängen. Die Kostenfunktion wurde für geschweißte I-Träger definiert. E numerisches Beispiel zeigte die Differenz zwischen den Ergebnissen der Kosten- bzw. Massenminimierung. Es wurden homogene und hybride (kombinierte) I-Träger aus drei verschiedenen Stählen miteina der verglichen. Die Optima wurden mittels einer Rosenbrock-Methode gefunden.

ОПТИМИЗАЦИЯ СВАРНЫХ СТАЛЬНЫХ БАЛОК ПО СТОИМОСТИ

С. БЫКОВСКИЙ – К. ЯРМАИ

Резюме

Оптимизационная задача формулируется как задача математического программиров ния. В качестве целевой функции применяется стоимость конструкции, которая приближенн определяется массой и количеством элементов в конструкции. Показывается возможны путь формулирования задачи оптимизации сварных двутавровых стальных балок и приводи ся численный пример, показывающий различие в результатах оптимизации по стоимости весу.