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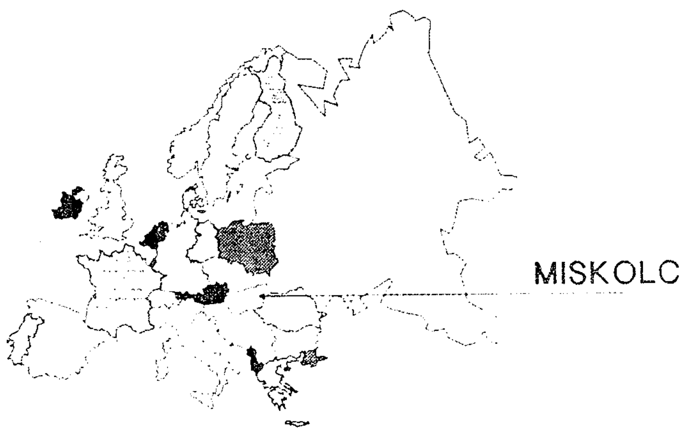
I. kötet



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MINIMUM COST DESIGN OF PLATES STIFFENED ON ONE SIDE SUBJECT TO UNIFORM NORMAL LOAD

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SUMMARY

The cost function consists of material and fabrication costs. The grid of ribs is constructed from cold-formed channels. The variables of a square plate are as follows: number of distances between ribs, stiffener height, thickness of ribs, thickness of cover plate. In the optimization procedure the values of variables are sought, which minimize the cost function and fulfil the design constraints on stresses, stiffener web shear buckling, deflection and thickness limitations. An illustrative numerical example is computed for a simply supported square plate carrying a uniformly distributed normal load.

INTRODUCTION

Stiffened plates are significant structural parts of welded steel structures. There are two main types of stiffened plates: plates stiffened on one side and cellular plates (Farkas 1984). Our aim is to compare these two types. Here we treat only plates stiffened on one side, cellular plates are investigated in another study. For purposes of comparison we consider here the special case of square symmetry.

THE COST FUNCTION

It is assumed that the fabrication has the following steps. First the grid of ribs is welded from cold-formed channels or from welded I-beams. The grid nodes should be completely welded to be able to carry the bending moments and shear forces. Then the cover plate elements are welded to the grid with fillet welds (Fig.1.). Note that other fabrication methods can also be used, for instance, first the whole cover plate may be welded from plate strips, then the cover plate is welded to the grid from inside with fillet welds.

The ribs are continuous in one direction, in the other direction they are intermittent. There are two types of nodes, one for peripheral and another for internal nodes.

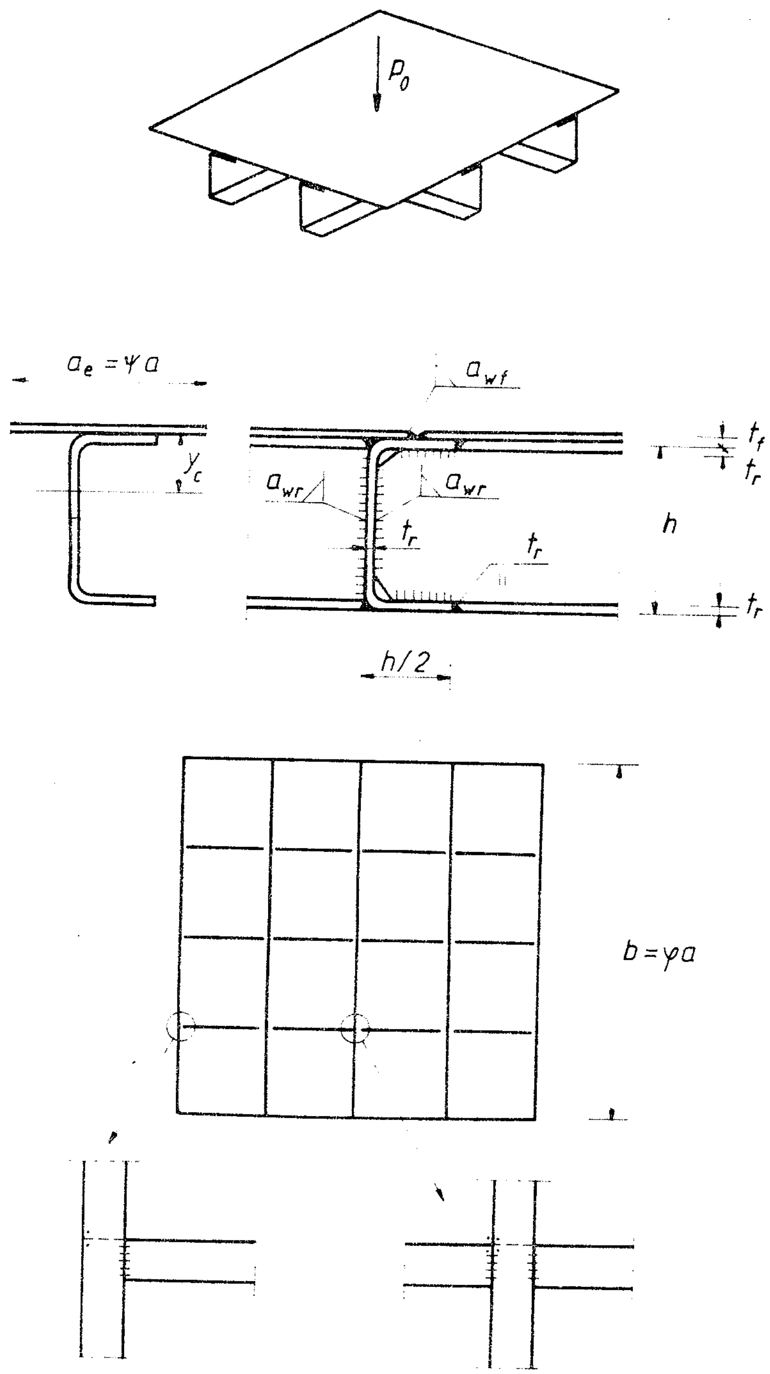

Fig.1. General view and node details of a plate orthogonally stiffened on one side by cold-formed channel-section ribs

The cross-sectional area of a rib is approximately $2ht_r$ where h is the height, t_r is the constant thickness. The integer number of distances between ribs in one direction is $\varphi = b/a$, where b is the side length of the plate. Thus, the number of ribs in one direction is $\varphi+1$. Assuming that all ribs have the same cross-sectional area, the whole volume of the square stiffened plate is

$$V = b^2 t_f + 4bht_r(\varphi + 1) \tag{1}$$

where t_f is the thickness of the cover plate. The cost consists of the material and fabrication costs

$$K = K_m + K_f = k_m \rho V + k_f \sum_i T_i \tag{2}$$

k_m (\$/kg) and k_f (\$/min) are the material and fabrication cost factors, respectively, ρ is the material density, T_i are the fabrication times in min. The method proposed by Pahl and Beelich (1982) is used here for the calculation of fabrication times. Note that this method has also been used in other studies (Farkas 1988, 1990a, b).

a/ Preparation, assembly and tacking

$$T_1 = C_1 \delta \sqrt{\rho V} \sqrt{\varkappa} \quad ; \quad C_1 = 1.0 \text{ min/kg}^{0.5} \tag{3}$$

where δ is the difficulty factor. According to the values described by Farkas (1988) we take here $\delta = 3$. Furthermore $\varkappa$ is the number of structural elements

$$\varkappa = \varphi^2 + \varphi + 3 + \varphi(\varphi - 1) = 2\varphi^2 + 3 \tag{4}$$

b/ Welding

$$T_2 = \sum_i C_{2i} a_{wi}^{1.5} L_{wi} \tag{5}$$

$C_2' = 0.8\text{x}10^{-3}$ min/(mm$^{1.5}$xmm) for manual arc welding

$C_2'' = 0.5\text{x}10^{-3}$ min/(mm$^{1.5}$xmm) for CO_2-welding

a_w and L_w are the size and length of welds in mm. For CO_2-welded fillet welds, joining the cover plate elements to the grid, one obtains

$$T_2' = 0.5\text{x}10^{-3}\text{x}4b\varphi a_{wf}^{1.5} \tag{6}$$

For manual-arc-welded joints of grid nodes we get

$$T_2'' = 0.8\text{x}10^{-3}\left\{(\varphi-1)^2\left[4ha_{wr}^{1.5} + 2ht_r^{1.5}\right]\right\} \quad \text{(internal nodes)} \tag{7}$$

$$T_2''' = 0.8\text{x}10^{-3}\left\{4\varphi(2ha_{wr}^{1.5} + ht_r^{1.5})\right\} \quad \text{(peripheral nodes)} \tag{8}$$

Thus

$$T_2 = T_2'+T_2''+T_2''' = 2\text{x}10^{-3}\, b\varphi a_{wf}^{1.5} + 1.6\text{x}10^{-3}\, h(\varphi^2+1)(2a_{wr}^{1.5}+t_r^{1.5}) \tag{9}$$

c/ Eledrode changing, weld slagging and chipping

$$T_3 = C_3 \sqrt{\delta} \sum_i a_{wi}^{1.5} L_{wi} \; ; \quad C_3 = 1.2 \times 10^{-3} \ \mathrm{min}/(\mathrm{mm}^{1.5} \times \mathrm{mm})$$

With $\delta = 3$ we obtain

$$T_3 = 2.07846 \times 10^{-3} \left\{ 4b\varphi a_{wf}^{1.5} + 2h(\varphi^2 + 1)\left(2a_{wr}^{1.5} + t_r^{1.5}\right)\right\}$$

In the numerical computations we take first $a_{wf} = 0.7t_f$ and $a_{wr} = 0.7t_r$. First we treat h, t_f and t_r as continuous variables and compute the optima for each integer values of φ. Then, by using a complementary computer algorithm, we determine the optimal discrete values of h, t_f and t_r, a_{wf} and a_{wr} on the basis of a series of available discrete values. In this procedure we use rounded values a_w as follows (in mm)

for t	2	2.5	3	3.5	4	4.5	5	5.5	6
a_w	1.5	2	2.5	2.5	3	3.5	3.5	4	4.5

In order to give an internationally usable solution, the following ranges of k_m and k_f may be considered. For steel Fe360 k_m = 0.5 - 1.2 $/kg, for fabrication including overheads k_f = 15-30 $/manhour = 0.25-0.50 $/min. Thus, the ratio k_f/k_m may vary in the range 0-1, so we consider the values k_f/k_m = 0; 0.25; 0.50; 0.75 and 1.0.

THE DESIGN CONSTRAINTS

a/ Constraint on compressive stress in the central upper cover plate element

$$\sigma_{max.1} + \sigma_{f.max} \le R_{adm} \tag{12}$$

$\sigma_{max.1}$ is caused by the bending of the whole plate, $\sigma_{f.max}$ is the normal stress due to the local bending of the cover plate elements, R_{adm} is the admissible stress. $\sigma_{max.1}$ can be calculated by means of the orthotropic plate theory

$$\sigma_{max.1} = \frac{c_M \, p \, b^2}{I_x/a} \, y_C \tag{13}$$

Since the torsional stiffness of the open section ribs is very small the stiffened plate can be calculated as an orthotropic one having torsional stiffness $H = 0$. Schade (1941) has calculated for this case a value of $c_M = 0.1102$. Furthermore, $p = 1.1p_o$, p_o is the intensity of the uniform normal load, the effect of self weight is considered by the factor of 1.1.

According to Fig.1., the distance of the centroidal axis y_C can be calculated as

$$y_C = \frac{2ht_r}{2ht_r + a_e t_f} \cdot \frac{h}{2} = \frac{h}{2} \cdot \frac{1}{1+\alpha} \quad ; \quad \alpha = \frac{a_e t_f}{2ht_r} \qquad (14)$$

$a_e = \psi a$ is the effective width of the compressed cover plate section (Farkas 1984). The most simple formula for ψ has been proposed by Usami and Fukumoto (1982), taking into account the effect of residual welding stresses and initial imperfections

$$\psi = \frac{0.75}{\lambda_p} \quad ; \quad \lambda_p = \frac{a}{t_f}\sqrt{\frac{\sigma_{max.1}}{E}}; \quad \psi \le 1 \qquad (15)$$

E is the Young modulus. The moment of inertia can be expressed as

$$I_x = a_e t_f y_C^2 + 2ht_r\left(\frac{h}{2} - y_C\right)^2 + \frac{h^3 t_r}{3} = \frac{h^3 t_r}{6} \cdot \frac{2+5\alpha}{1+\alpha} \qquad (16)$$

Substituting (13) into (15) we get a quadratic equation for ψ, the solution of which is

$$\psi = \frac{5x0.75^2 \varphi^3 t_f^3 h E}{12 c_M p b^4}\left(1 + \sqrt{1 + \frac{96 c_M p b^3 t_r}{25x0.75^2 \varphi^3 t_f^4 E}}\right) \qquad (17)$$

The local bending stress $\sigma_{f.max}$ can be calculated by means of formulae valid for isotropic plates with clamped edges (Timoshenko 1959)

$$\sigma_{f.max} = 6c_f p_o a^2/t_f^2 \quad ; \quad c_f = 5.13x10^{-2} \qquad (18)$$

thus

$$\sigma_{f.max} = 0.3078 p_o b^2/(\varphi t_f)^2 \qquad (19)$$

b/ Constraint on maximal tensile stress in the central ribs

$$\sigma_{max.2} = \sigma_{max.1} \cdot \frac{h - y_C}{y_C} = \sigma_{max.1}(1 + 2\alpha) \le R_{adm} \qquad (20)$$

c/ Constraint on shear buckling of rib webs at the edges

$$\tau = \frac{c_q p b a}{h t_r} = \frac{0.42 p b^2}{h t_r} \le \frac{\tau_{ub}}{\gamma_b} = \frac{5.34\pi^2 E}{12(1-\nu^2)\gamma_b}\left(\frac{t_r}{h}\right)^2 \quad \text{for} \quad \frac{\tau_{ub}}{\gamma_b} \le \tau_{adm} \qquad (21)$$

$$\tau = \frac{0.42 p b^2}{h t_r} \le \tau_{adm} \quad \text{for} \quad \frac{\tau_{ub}}{\gamma_b} > \tau_{adm} \qquad (22)$$

τ_{ub} is the ultimate shear buckling stress, γ_b = 1.35 is the safety factor against buckling, ν = 0.3 is the Poisson's ratio and τ_{adm} the admissible shear stress.

d/ Deflection constraint

$$w_{max} = \frac{c_w\, p_o\, b^4}{E\, I_x/a} \le w_{adm} = c^* b \qquad (23)$$

According to Schade (1941), for a torsional stiffness H = 0, it is c_w = 0.0082.

e/ Size constraints are the thickness limitations

$$t_f \ge t_o \quad \text{and} \quad t_r \ge t_o \qquad (24)$$

NUMERICAL EXAMPLE

Data: t_o = 2 mm; p_o = 5 kN/m^2 = 5x10^{-3}MPa; p = 1.1p_o = 5.5 kN/m^2; E = 2.06x10^5 MPa; steel Fe360; R_{adm} = 120 MPa; b = 8 m; ρ = 7850 kg/m^3; ρb^3 = 401.92 kg; $\tau_{adm} = R_{adm}/\sqrt{3}$ = 69 MPa; c^* = 1/2000.

The optimization has been performed by using the "hillclimb" algorithm (Rosenbrock 1960). The computer program has been developed by Jármai (1989).

The results are summarized in Table 1, for rounded optimal dimensions only.

Table 1. Optimal rounded dimensions, V_{min} and K_{min} values

$\frac{k_f}{k_m}$	φ_{opt}	t_f (mm)	h (mm)	t_r (mm)	$\frac{10^4}{b^3} V_{min}$	$\frac{K_{min}}{401.92 k_m}$
1	7	5.5	460	2.5	12.62	31.65
0.75	7	5.5	460	2.5	12.62	26.90
0.50	8	5.0	430	2.5	12.30	22.40
0.25	13	3.5	370	2.0	10.85	17.07
0	14	3.5	355	2.0	11.03	11.03

CONCLUSIONS

1. The φ_{opt} depends on k_f/k_m. For k_f/k_m = 1 it is φ_{opt} = 7 and for k_f/k_m = 0 we get φ_{opt} = 14.

2. Calculating with unrounded optimal values, for $k_f/k_m = 1$, the difference between the costs of V_{min} and K_{min} solutions is 100(31.81 - - 30.875)/30.875 = 3%, thus, savings of 3% in costs may be achieved by using a cost function instead of a volume function.

3. The ratio of the fabrication cost/total cost, for $k_f/k_m = 1$, is 100(30.875 - 12.350)/12.350 = 60%, thus, the fabrication cost plays an important role in the case of stiffened plates.

4. The sensitivity of the objective function to the change of the main variable φ may be characterized by the following data. If φ changes from 7 to 9 (28%), for $k_f/k_m = 1$, the change of the objective function is 100(31.780 - 30.875)/30.875 = 3%; if φ changes from 14 to 11 (27%), for $k_f/k_m = 0$, the change of the objective function is 100(11.257 - 10.822)/10.822 = 4%. Thus, the sensitivity is small.

REFERENCES

Farkas,J. (1984) Optimum design of metal structures. Akadémiai Kiadó, Budapest, Ellis Horwood, Chichester.

Farkas,J. (1988) Consideration of fabrication costs in the optimum design of welded tubular trusses. IIW Doc. XV-677-88. Vienna.

Farkas,J. (1990a) Minimum cost design of tubular trusses considering buckling and fatigue constraints. In "Tubular Structures.Elsevier Applied Science, London-New York, 1990." p.451-459.

Farkas,J. (1990b) Fabrication aspects in the optimum design of welded structures. IIW Doc. XV-725-90. Montreal.

Jármai,K. (1989) Single- and multicriteria optimization as a tool of decision support system. Computers in Industry 11, p.249-266.

Pahl,G., Beelich,K.H. (1982) Kostenwachstumsgesetze nach Ähnlichkeitsbeziehungen für Schweissverbindungen. VDI-Berichte Nr.457. Düsseldorf. p.129-141.

Rosenbrock,H.H. (1960) An automatic method for finding the greatest or least value of a function. Computer J. 3, No.3.p.175-184.

Schade,H.A. (1941) Design curves for cross-stiffened plating under uniform bending load. Trans.Soc.Naval Archit. Marine Eng. 49, p.154-182.

Timoshenko,S.,Woinowsky-Krieger,S. (1959) Theory of plates and shells. 2nd ed. New York, McGraw Hill.

Usami,Ts.,Fukumoto,Y. (1982) Local and overall buckling of welded box columns. J.Struct.Div. Proc.ASCE 108, No.ST3, p.525-542.