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Optimum design of a belt-conveyor bridge constructed as a welded ring-stiffened cylindrical shell

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Abstract

In the structural optimization of a ring-stiffened cylindrical shell the unknown variables are the shell thickness as well as the thickness and the number of flat rings. The shell diameter enables to realize a belt-conveyor structure inside of the shell. The uniformly distributed vertical load consists of dead and live load. The design constraints relate to the local shell buckling strength, to the panel ring buckling and to the deflection of the simply supported bridge. The cost function includes the material and fabrication costs. The fabrication cost function is formulated according to the fabrication sequence and includes also the cost of forming of shell elements into the cylindrical shape as well as the cost of cutting of the flat plate ring-stiffeners. Since the shell thickness does not depend on number of ring-stiffeners (n), the n_{opt} is calculated for a selected region of n.

IIW-Thesaurus keywords:

structural optimization, shell buckling, welded stiffened shells, welded structures, fabrication costs

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List of symbols

A_r cross-sectional area of a ring stiffener

A_T thermal impulse due to welding

A_w cross-sectional area of a weld

C coefficient Eq. 17

c_0 specific heat

E elastic modulus

f_y yield stress

h_r stiffener height

I arc current

I_r moment of inertia of a ring stiffener

I_x moment of inertia of the shell cross-section

K cost

K_M material cost

K_F fabrication cost

k_M material cost factor

k_F fabrication cost factor

L span length

L_e shell effective width

L_r distance of rings

M bending moment

n number of ring stiffeners

p factored load intensity

p_0 unfactored load intensity

Q_T specific heat input caused by welding

R shell radius

R_0 radius Figure 1

T_a, T_b times Table 1

t thickness

t_r ring stiffener thickness

U arc voltage

u_{max} maximal radial deformation

V volume

v_w welding speed

w deflection

y_G distance of the gravity centre

Z factor Eq.17

α_0 coefficient of thermal expansion

β reduction factor Eq.15

η_0 coefficient of thermal efficiency

κ number of elements to be assembled

Θ difficulty factor

λ Eq.6

ρ material density

ρ_0 factor Eq.18

σ normal stress

σ_E buckling stress

σ_{cr} critical buckling stress

ψ coefficient Eq.18

ω quotient Eq.21

1. Introduction

Stiffened shells are widely used in offshore structures, bridges, towers, etc. Rings and/or stringers can be used to strengthen the shape of cylindrical shells. Shells can be loaded by axial compression, bending, external or internal pressure or by combined load.

Design rules for the shell buckling strength have been worked out by ECCS [1], API [2] and DNV [3]. The optimum design of stiffened shells has been treated in some of our articles [4, 5, 6]. The optimum design of a stiffened shell belt-conveyor bridge has been treated in [7]. The buckling behaviour of stiffened cylindrical shells has been investigated by several authors, e.g. Harding [8], Dowling and Harding [9], Ellinas et al [10], Frieze et al [11], Shen et al [12], Tian et al [13]

In the calculation of shell buckling strength the initial imperfections should be taken into account. These imperfections are caused by fabrication and by shrinkage of circumferential welds. A calculation method for the effect of welding has been worked out by the first author [14] and it is used in the calculation of the local shell buckling strength.

In the present study the design rules of Det Norske Veritas (DNV) are used for ring-stiffened cylindrical shells. The shape of rings is a simple flat plate, which is welded to the shell by double fillet welds. In the calculation of the fabrication cost the cost of forming the shell elements into the cylindrical shape and the cutting of the flat ring-stiffeners is also taken into account.

The shell is a supporting bridge for a belt-conveyor, simply supported with a given span length of L = 60 m and radius of R = 1800 mm (Figures 1,2). The intensity of the factored uniformly distributed vertical load is p = 16.5 N/mm + self mass. Factored live load is 12 N/mm, dead load (belts, rollers, service-walkway) is 4.5 N/mm. For self mass a safety factor of 1.35 is used, which is prescribed by Eurocode 3 (note that ECCS gives 1.3). The safety factor for variable load is 1.5. The flat plate rings are uniformly distributed along the shell. Note that the belt-conveyor supports are independent of the ring stiffeners, they can be realized by using local plate elements.
The unknown variables are as follows: shell thickness t, stiffener thickness t_r and number of stiffeners n.

We do not consider the case of an unstiffened shell, since to assure a stable cylindrical shape, a certain number of ring-stiffeners should be used. In the present study we consider a range of ring numbers n = 6 – 30. The range of thicknesses t and t_r is taken as 4 – 20 mm, rounded to 1 mm.

2. The design constraints

2.1 Local buckling of the flat ring-stiffeners (Fig. 1.)

According to DNV

$$\frac{h_r}{t_r} \le 0.4\sqrt{\frac{E}{f_y}} \tag{1}$$

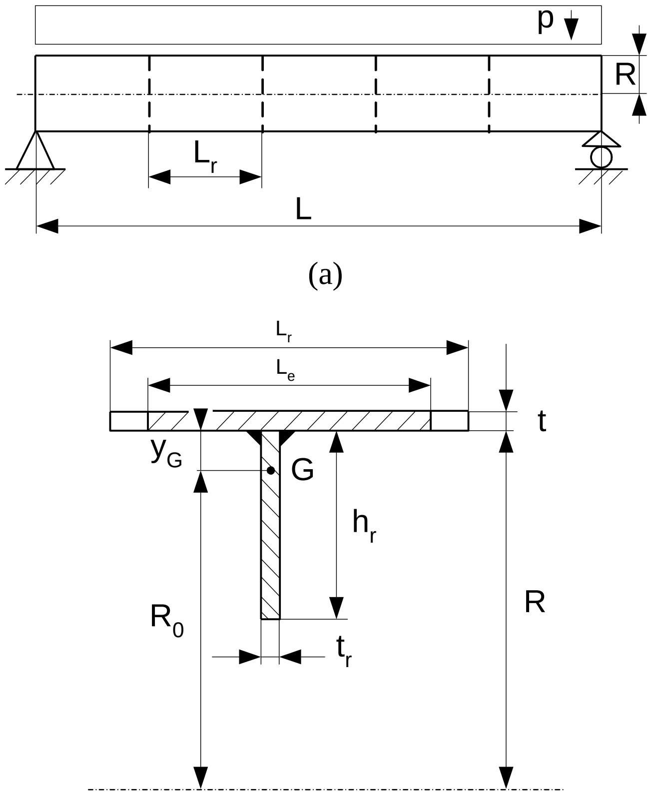
Considering this constraint as active one, for $E = 2.1 \times 10^5$ MPa and yield stress f_y = 355 MPa one obtains

$$h_r = 9t_r. \tag{2}$$

2.2 Constraint on local shell buckling (as unstiffened) (Fig. 3.)

$$p = 16.5 + 1.35\rho(2R\pi t + nA_r); \qquad \rho = 7.85x10^{-6}\ \text{kg/mm}^3; \quad A_r = h_r t_r \tag{3}$$

$$M_{\max} = \frac{pL^2}{8}; \tag{4}$$



(a)

(b)

Figure 1. (a) A simply supported belt conveyor bridge constructed as a ring stiffened cylindrical shell, (b) the cross-section of a ring stiffener including the effective width of the shell

$$\sigma_{\max} = \frac{M_{\max}}{\pi R^2 t} \leq \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \tag{5}$$

$$\lambda^2 = \frac{f_y}{\sigma_E}, \sigma_E = (1.5-50\beta) C \frac{\pi^2 E}{10.92}\left(\frac{t}{L_r}\right)^2 \tag{6}$$

$$L_r = \frac{L}{n+1} \tag{7}$$

The factor of (1.5-50 β) in Eq. (6) expresses the effect of initial radial shell deformation caused by the shrinkage of circumferential welds and can be calculated as follows [14].

The maximum radial deformation of the shell caused by the shrinkage of a circumferential weld is

$$u_{\max} = 0.64 A_T \sqrt{R/t} \tag{8}$$

where $A_T t$ is the area of specific strains near the weld. According to our results [15]

$$A_T t = \frac{0.3355 Q_T \alpha_0}{c_0 \rho} \tag{9}$$

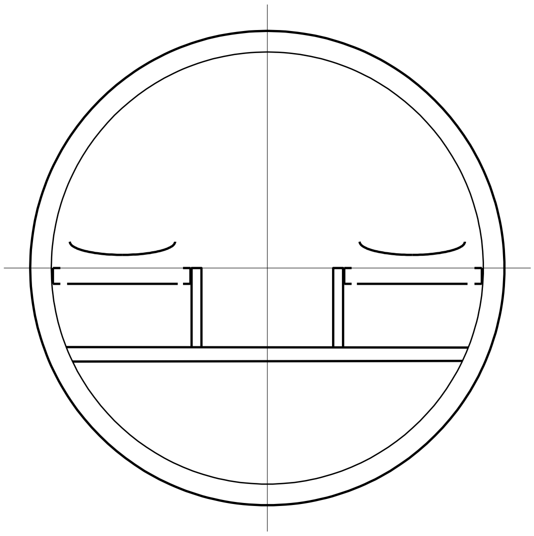


Figure 2. Cross-section of a belt conveyor bridge with two belt conveyors and a service walkway in the middle.

For steels it is

$$A_T t = 0.844x10^{-3} Q_T \quad (A_T t \text{ in mm}^2,\ Q_T \text{ in J/mm}) \tag{10}$$

$$Q_T = \eta_0 \frac{UI}{v_W} = C_A A_W \tag{11}$$

For manually arc welded butt welds it is

$$Q_T = 60.7 A_W \quad (A_W \text{ in mm}^2) \tag{12}$$

When $t \le 10$ mm, $A_W = 10t$ (13)

When $t > 10$ mm, $A_W \cong 3.05 t^{1.45}$ (14)

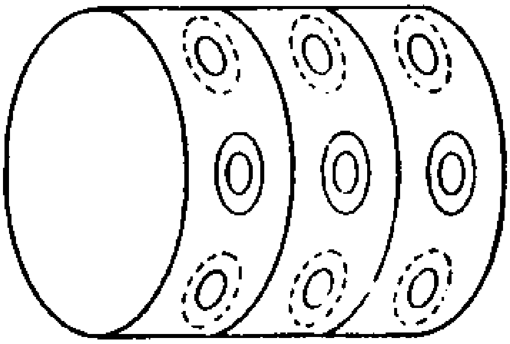


Figure 3. Top-view of the shell with local buckling

Introducing a reduction factor of β for which

$$0.01 \le \beta = \frac{u_{max}}{4\sqrt{Rt}} \le 0.02 \quad (15)$$

and the imperfection factor for shell buckling strength should be multiplied by $(1.5 - 50\beta)$.

For $\beta \le 0.01 \quad \beta = 0.01$, for $\beta \ge 0.02 \quad \beta = 0.02$. (16)

Furthermore

$$C = \psi \sqrt{1 + \left(\frac{\rho_0 \xi}{\psi}\right)^2}, Z = 0.9539 \frac{L_r^2}{Rt} \quad (17)$$

$$\psi = 1, \xi = 0.702Z, \rho_0 = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5} \quad (18)$$

It can be seen that σ_E does not depend on L_r, since in Eq. (6) L_r^2 is in nominator and in C (Eq.17) it is in denominator. The fact that the buckling strength does not depend on the shell length is first derived by Timoshenko and Gere [16]. Note that API design rules [2] give another formulae. On the contrary, in the case of external pressure the distance between ring-stiffeners plays an important role [4,6].

2.3. Constraint on panel ring buckling (Fig. 4.)

Requirements for a ring stiffener are as follows:

$$A_r = h_r t_r \ge \left(\frac{2}{Z^2} + 0.06\right) L_r t \quad (19)$$

$$I_r = \frac{h_r^3 t_r}{12} \cdot \frac{1 + 4\omega}{1 + \omega} \ge \frac{\sigma_{max} t R_0^4}{500 E L_r} \quad (20)$$

$$R_0 = R - y_G; y_G = \frac{h_r}{2(1+\omega)}; \omega = \frac{L_e t}{h_r t_r} \quad (21)$$

$$L_e = \min\left(L_r, L_{e0} = 1.5\sqrt{Rt}\right) \quad (22)$$

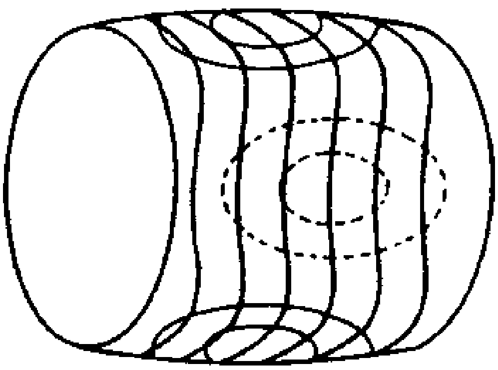


Figure 4. Top-view of panel ring buckling

2.4 Deflection constraint

$$w_{\max} = \frac{5 p_0 L^4}{384 E I_x} \le \frac{L}{500} \tag{23}$$

$$I_x = \pi R^3 t \tag{24}$$

The unfactored load is

$$p_0 = 12/1.5 + 4.5/1.35 + \rho(2R\pi t + nA_r) = 11.33 + \rho(2R\pi t + nA_r). \tag{25}$$

3 The cost function

The cost function is formulated according to the fabrication sequence. A possible fabrication sequence is as follows:

(1) Fabricate 20 shell elements of length 3 m without rings (using 2 end ring stiffeners to assure the cylindrical shape). For one shell element 2 axial butt welds are needed (GMAW-C). The welding of end ring stiffeners is not calculated, since it does not influence the variables. The cost of the forming of the shell element to a cylindrical shape is also included (K_{F0}). According to the time data obtained from a Hungarian production company (Jászberényi Aprítógépgyár, Crushing Machine Factory, Jászberény) for plate elements of 3 m width (Table 1.), the times (T_a + T_b) can be approximated by the following function of the plate thickness (Eq. 26).

Table 1. Time for forming the shell elements of 3m width into circular shape (T_a) ,as well as for reducing the initial imperfections due to forming (T_b).

t (mm)	T_a (min)	T_b (min)	$T_a + T_b$ (min)
6	270	184	454
8	336	204	540
10	395	228	623
15	495	304	799
20	588	374	962
25	680	442	1122
30	744	538	1282
40	834	692	1526

$$K_{F0} = k_F \Theta (212.18 + 42.824t - 0.2483t^2) \quad (26)$$

The cost of welding of a shell element is

$$K_{F1} = k_F \left[\Theta \sqrt{\kappa \rho V_1} + 1.3x0.2245x10^{-3} t^2 \left(2x3000 \right) \right] \quad (27)$$

where Θ is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled

$$\kappa = 2; V_1 = 2R\pi tx3000; \Theta = 2 \quad (28)$$

The first term of Equation 27 expresses the time of assembly and the second calculates the time of welding and additional works [18].

(2) Welding the whole unstiffened shell from 20 elements with 19 circumferential butt welds

$$K_{F2} = k_F \left(\Theta \sqrt{20 \rho V_1} + 1.3x0.2245x10^{-3} t^2 x19x2R\pi \right) \quad (29)$$

(3) Cutting of *n* flat plate rings with acetylene gas [17]

$$K_{F3} = k_F \Theta_c C_c t_r^{0.25} L_c \quad (30)$$

where Θ_c , C_c and L_c are the difficulty factor for cutting, cutting parameter and length respectively, Θ_c =3, C_c= 1.1388, $L_c \approx 2R\pi n + 2\left(R - h_r\right)\pi n$.

(4) Welding *n* rings into the shell with double-sided GMAW-C fillet welds. Number of fillet welds is 2*n*

$$K_{F4} = k_F \left(\Theta \sqrt{\left(n+1\right) \rho V_2} + 1.3x0.3394x10^{-3} a_W^2 x4R\pi n \right) \quad (31)$$

$a_W = 0.5t_r$, but a_{Wmin} = 3 mm. $V_2 = 20V_1 + 2\left(R - \frac{h_r}{2} \right) \pi h_r t_r n$ (32)

a_W is taken so that the double fillet weld joint be equivalent to the stiffener thickness.

The total material cost is $K_M = k_M \rho V_2$ (33)

The total cost is $K = K_M + 20(K_{F0} + K_{F1}) + K_{F2} + K_{F3} + K_{F4}$ (34)

k_M = 1 \$/kg; k_F = 1 \$/min

4. Results of the optimum design

The optimization has been worked out using the Hillclimb technique [18]. Results can be found in Table 2. Those results for which the place of stiffeners coincides with the circumferential welds of the shell segments are not applicable for fabrication reasons (*n* = 9, 19).

Table 2. Computational results: the number of stiffeners, thickness of the stiffeners, material and total costs in the case of optimum shell thickness t = 7 mm. The optimum solution is marked by bold letters.

n	t_r	K_M	K
6	21	39291	76041
7	**19**	**39211**	**75870**
8	18	39266	76296
9	17	39278	76531
10	16	39252	76595
11	16	39448	77640
12	15	39365	77446
13	15	39538	78384
14	14	39404	77965
15	14	39555	78803
16	13	39379	78191
17	13	39509	78935
18	13	39640	79679
19	12	39409	78819
20	12	39520	79476
21	12	39632	80132
22	12	39744	80787
23	11	39451	79646
24	11	39545	80222
25	11	39639	80796
26	11	39733	81370
27	11	39827	81943
28	10	39470	80505
29	10	39547	81005
30	10	39625	81505

Table 3. Cost distribution for the optimum solution

n	t_r	20 K_{F0}	20 K_{F1}	K_{F2}	K_{F3}	K_{F4}	K_M	K
7	19	19991	4707	3459	1076	7425	39211	75870

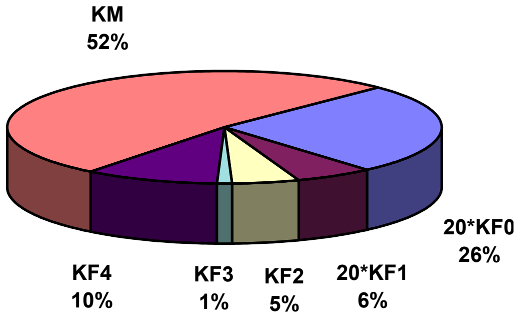


Figure 5. Cost distribution for the optimum solution (t = 7, t_r = 19, n = 7).

Table 3 shows the value of the different cost elements and Fig. 5 gives the percentage of them.

Conclusions

The shell thickness is determined by the constraints on local shell buckling as well as on deflection. Since the number of ring-stiffeners does not influence these constraints, in order to assure a stable circular shell shape, a certain number of rings should be used. Since the design rules do not give any prescriptions for the minimum number of ring-stiffeners, for the investigated case we have selected a ring number domain of $n = 6 – 30$ and have performed the optimization in this domain.

The Det Norske Veritas design rules give suitable formulae for the design of rings, the dimensions of which decrease with the increase of the number of rings.

The initial radial deformation of the shell caused by the shrinkage of circumferential welds affects the local shell buckling strength significantly. Cost calculation methods are proposed for the forming of shell elements into circular shape and for the cutting of flat plate ring-stiffeners. The cost function is formulated according to the fabrication sequence.

The optimization results (Table 2) show that, due to the cutting and welding costs of stiffeners, the smaller number of stiffeners is more economic. The optimum ring number is 7, which minimizes the total mass (material cost) and the total cost. Material cost is about half of the total one and is insensitive to the variation of ring numbers. The forming cost of the shell elements (K_{F0}) is significant. The difference between the best and worst optima indicated in Table 2 is 7 %, thus it is worth to use an optimization process in the design stage. The result is greatly dependent on local situation, parameters, but this numerical evaluation and comparison show the benefit of optimum design.

Acknowledgements

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