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Economic orthogonally welded stiffening of a uniaxially compressed steel plate

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Abstract

Stiffened plates and shells are the most characteristic structural types for optimization, since the number of stiffeners influences the cost significantly. A previous study has shown that a plate stiffened on one side with open section longitudinal ribs subject to uniaxial compression is not so economic than a cellular one. In the present article a plate orthogonally stiffened on one side is optimized and compared to the plate with longitudinal stiffeners only. The orthogonal grid of ribs is more economic, since the transverse stiffeners increase significantly the overall buckling strength of the plate. For the comparison a cost function is used which includes the material and welding costs.

Keywords: plate buckling, stiffened plates, welded structures, structural optimization, fabrication cost, minimum cost design

Introduction

The main requirements of a modern engineering structure are the safety, fitness for production and economy. In the optimum design process the safety and producibility are fulfilled by design and fabrication constraints as well as the economy is achieved by the minimization of a cost function.

We have developed a cost calculation method mainly for welded structures, thus, we are able to determine the economy of a structural version and to compare the costs of these versions to each other [1]. Welded stiffened plates are applied in many steel structures. Our aim is to determine the most economic stiffening of a uniaxially compressed plate. Our structural model is a rectangular steel plate with simply supported edges, stiffened orthogonally by halved rolled I-section stiffeners welded to the base plate by double fillet welds.

In our other study we have compared the costs of a plate stiffened on one side and a cellular plate both stiffened longitudinally and loaded by uniaxial compression [2]. Economic stiffening has been determined for an orthogonally stiffened plate loaded by bending [3].

In the optimization process the base plate thickness, as well as the number and height of stiffeners in both directions are sought, which fulfil the buckling constraints and minimize the cost function.

The applied mathematical method is the particle swarm algorithm.

The classic buckling stress is derived from the Huber's differential equation [4]. This stress is modified taking into account the effect of residual welding stresses and initial imperfections.

The cost function includes the material and fabrication (welding) costs and is formulated according to the fabrication sequence. A series of rolled I-section stiffeners is selected according to the ARCELOR catalogue [5]. The flange width and thickness, as well as the web thickness are expressed by the section height using approximate formulae, thus, in the optimization only five unknowns should be determined.

The result is compared to the cost of the plate stiffened only in the direction of the compression force. This comparison shows that the orthogonal stiffening is much more economic than the longitudinal one.

Problem formulation

Determine the economic orthogonal stiffening of a rectangular plate with given main dimensions a_0 and b_0, subject to a uniformly distributed uniaxial compression of intensity N_x (Figure 1), which fulfils the design and fabrication constraints and minimizes the cost function. Halved rolled I-section stiffeners are welded to the base plate by double fillet welds.

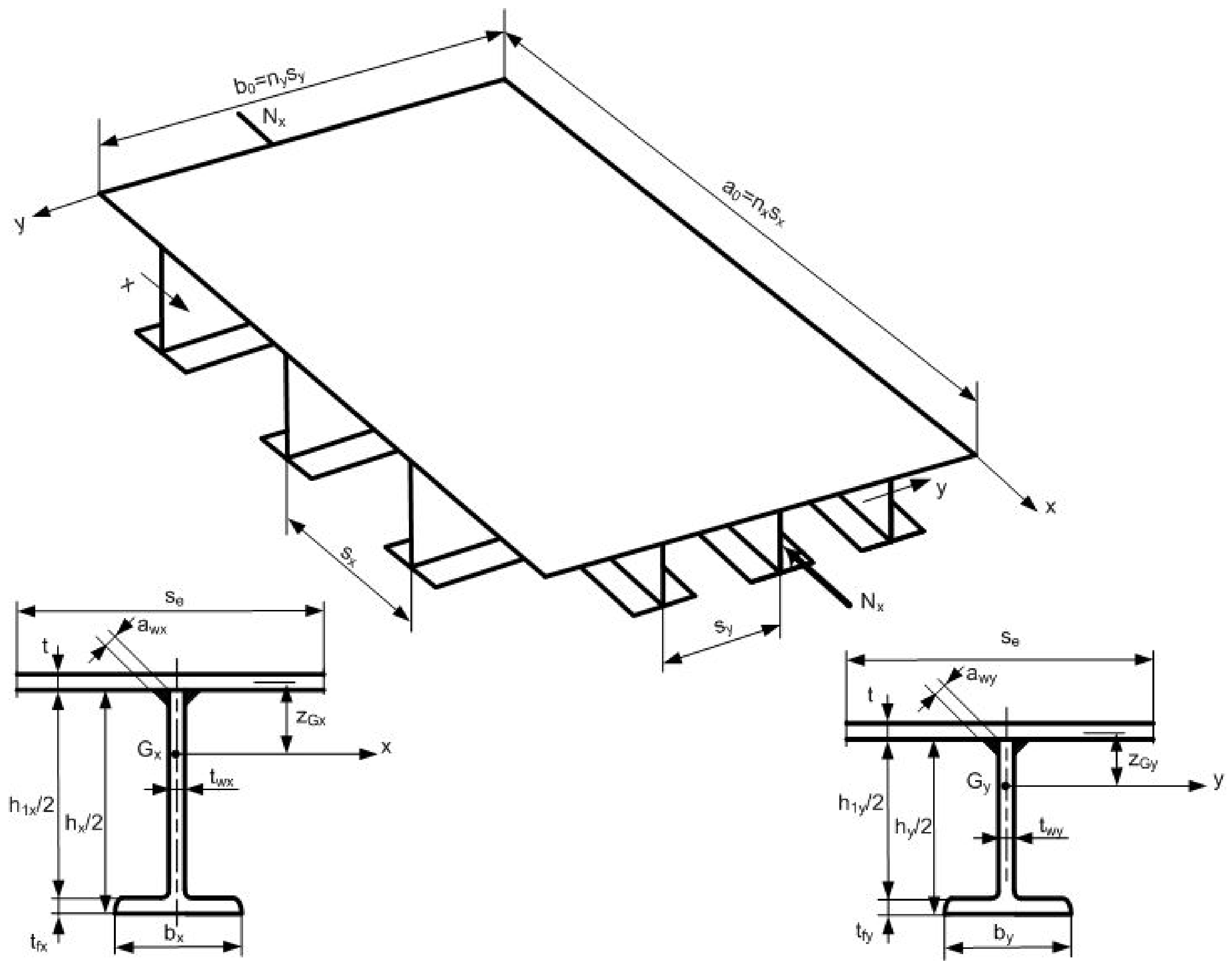

Figure 1. Orthogonally stiffened plate loaded by uniaxial compression

Numerical data (Figure 1): a_0 = 24000, b_0 = 8000 mm, N_x = 3x10^7 MPa, steel yield stress f_y = 355 MPa, elastic modulus E = 2.1x10^5 MPa, density ρ = 7.85x10^{-6} kg/mm^3, selected rolled I-sections UB profiles.

Unknowns to be optimized: base plate thickness t, sizes and number of stiffeners in both directions: h_y, h_x, n_y, n_x. Ranges of unknowns: $4 < t < 20$ mm, $152 < h < 1016$ mm, $4<n<n_{max}$, n_{max} are determined by the following fabrication constraints:

$$\frac{b_0}{n_y} - b_y \geq 300\,\text{mm}, \qquad \frac{a_0}{n_x} - b_x \geq 300\,\text{mm}. \qquad (1)$$

The other dimensions of a halved rolled I-section are expressed by the main height h as follows:

$t_f = \sqrt{33.20533808 + 0.0006701288\,h^2}$,

$b = \sqrt{5851.784768098 + 0.01671843845 h^2 \ln(h)}$,

$t_w = \sqrt{15.62577015376 + 4.358946969\text{x}10^{-5} h^2 \ln(h)}$,

$h_1 = h - 2t_f$.

The discrete values of h are as follows: 152.4, 177.8, 203.2, 257.2, 308.7, 353.4, 403.2, 454.6, 533.1, 607.6, 683.5, 762.2, 840.7, 910.4 mm.

The maximum values of n_i is given by the fabrication constraints Eq. (1).

The n_{max} values are given in the Table 1.

Table 1. n_{max}- values for rolled I-sections – dimensions in mm

h	152.4	177.8	203.2	257.2	308.7	353.4	403.2	454.6	533.1	607.6	683.5	762.2	840.7	910.4
b	88.7	101.2	133.2	101.9	101.8	126.0	142.2	152.9	209.3	228.2	253.7	266.7	292.4	304.1
n	20	19	18	19	19	18	18	17	15	15	14	14	13	13

Geometric characteristics of stiffeners

Effective cross-sectional areas ($i = x,y$)

$$A_{ei} = \frac{h_{1i} t_{wi}}{2} + b_i t_{fi} + s_{ei} t, s_y = \frac{b_0}{n_y}, s_x = \frac{a_0}{n_x}; s_E = 1.9t\sqrt{\frac{E}{f_y}} . \qquad (2)$$

when $s_E < s_i$ $\quad s_{ei} = s_E$,
when $s_E > s_i$ $\quad s_{ei} = s_i$.

Note that the formula of s_E is given by ECCS rules [6].

The distances of the gravity centres G_i

$$z_{Gi} = \frac{1}{A_{ei}} \left[\frac{h_{1i} t_{wi}}{2} \left(\frac{h_{1i}}{4} + \frac{t}{2} \right) + b_i t_{fi} \left(\frac{h_i + t - t_{fi}}{2} \right) \right], \qquad (3)$$

.

The moments of inertia

$$I_i = s_{ei} t z_{Gi}^2 + \frac{h_{1i}^3 t_{wi}}{96} + \frac{h_{1i} t_{wi}}{2} \left(\frac{h_{1i}}{4} + \frac{t}{2} - z_{Gi} \right)^2 + b_i t_{fi} \left(\frac{h_i + t - t_{fi}}{2} - z_{Gi} \right)^2 . \qquad (4)$$

The bending stiffnesses

$$B_x = \frac{EI_y}{s_y}; B_y = \frac{EI_x}{s_x} . \qquad (5)$$

Design constraints

Overall buckling constraint according to DNV [7]

$$\sigma = \frac{N_x}{n_y A_{ey}} \le \sigma_{cr} = \frac{f_{y1}}{\sqrt{1+\lambda^4}}, f_{y1} = \frac{f_y}{1.1} \qquad (6)$$

$$\lambda = \sqrt{\frac{f_{y1}}{\sigma_E}}, \sigma_E = \frac{N_E s_y}{A_{ey}}, N_E = \frac{\pi^2}{b_0^2}\left(B_x \frac{b_0^2}{a_0^2} + B_y \frac{a_0^2}{b_0^2}\right) \qquad (7)$$

It can be seen from the load-carrying capacity formula N_E that, when $a_0 > b_0$, to have a larger N_E, B_x (h_x) should be larger than B_y (h_y).

Instead of local buckling constraint for base plate s_e is calculated in both directions.

Cost function

The cost function includes the cost of material, assembly, welding as well as painting and is formulated according to the fabrication sequence.

The cost of material

$$K_M = k_M \rho V_2; k_M = 1.0 \text{ \$/kg.} \qquad (8)$$

Welding of the base plate from butt welds (3 in direction of a_0 and 3 in direction of b_0) (SAW - submerged arc welding) [1]:

The fabrication cost factor is taken as k_F = 1.0 \$/min, the factor of complexity of the assembly $\Theta_W = 2$:

$$K_0 = k_F \left[\Theta_W \sqrt{16\rho V_0} + 1.3 C_W t^n (3a_0 + 3b_0)\right], \qquad (9)$$

$$V_0 = a_0 b_0 t, \qquad (10)$$

$$\text{for } t < 11 \qquad C_W = 0.1346x10^{-3}; n = 2, \qquad (11a)$$

$$\text{for } t \ge 11 \qquad C_W = 0.1033x10^{-3}; n = 1.904. \qquad (11b)$$

Welding (n_x-1) stiffeners to the base plate in x direction with double fillet welds (GMAW-C - gas metal arc welding with CO_2):

$$K_{W1} = k_F \left[\Theta_W \sqrt{n_x \rho V_1} + 1.3x0.3394x10^{-3} a_{wx}^2 2b_0 (n_x - 1)\right], \qquad (12)$$

$a_{Wx} = 0.4\, t_{wx}$ but $a_{wx.min} = 3$ mm.

Welding of (n_y – 1) stiffeners to the base plate in y direction with double fillet welds. These stiffeners should be interrupted and welded with fillet welds to the stiffeners in the x direction.

$$K_{W2} = k_F \left[\Theta_W \sqrt{(n_y n_x - n_y + 1)\rho V_2} + 1.3x0.3394x10^{-3} a_{wy}^2 2a_0 (n_x - 1) + T_1\right], \qquad (13)$$

$$T_1 = 1.3x0.3394x10^{-3} a_{wy}^2 4(n_y - 1)(n_x - 1)\left(\frac{h_{1y}}{2} + b_y\right), \quad (14)$$

$a_{Wy} = 0.4\, t_{wy}$ but $a_{Wy.min} = 3$ mm.

Painting

$$K_P = k_P \Theta_P S_P \quad (15)$$

$k_P = 14.4x10^{-6}$ \$/mm^2 , $\Theta_P = 2$,

Surface to be painted

$$S_P = 2a_0b_0 + a_0\,(n_y - 1)(h_{1y} + 2b_y) + b_0\,(n_x - 1)(h_{1x} + 2b_x) \quad (16)$$

The total cost

$$K = K_M + K_0 + K_{W1} + K_{W2} + K_P \quad (17)$$

For a comparison the cost without K_0 and K_P is also calculated

$$K_1 = K_M + K_{W1} + K_{W2} \quad (18)$$

The Particle Swarm Optimization (PSO) algorithm

The general optimization problem to be considered here is therefore:

$$\underset{w.r.t.\, \boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x}),\ \boldsymbol{x} = [x_1, x_2, \ldots x_n]^T \in R^n, \quad (19)$$

subject to the inequality and equality constraints: (20)

$$\begin{aligned} g_j(\boldsymbol{x}) &\le 0, \quad j = 1,2,\ldots,m \\ h_j(\boldsymbol{x}) &= 0, \quad j = 1,2,\ldots,r \end{aligned},$$

and side constraints:

$$x_i^l \le x_i \le x_i^u, \quad i = 1,2,...,n,$$

where $f(\boldsymbol{x})$, $g_j(\boldsymbol{x})$ and $h_j(\boldsymbol{x})$ are scalar functions of the real column vector $\boldsymbol{x}$. For generality equality constraints, $h_j(\boldsymbol{x})=0$, $j=1,2,\ldots,r$ are also specified, although they are not explicitly imposed in this study. The optimum solution is denoted by $\boldsymbol{x}^*$ with associate optimum function value $f(\boldsymbol{x}^*)$.

Particle Swarm Optimization (PSO) techniques belong to a relatively new class of evolutionary based search procedures that may be used to find the optimum solution x^* of the general optimization problem. The original PSO algorithm, proposed by Kennedy and Eberhardt [8], was inspired by the modelling of the social behaviour patterns of organisms that live and interact within large groups. In particular, PSO incorporates swarming behaviours observed in flocks of birds, schools of fish, or swarms of bees. A PSO algorithm is easy to implement in most programming languages, since the core of the program can be written in a few lines of code. It has been proven to be both fast and effective, when applied to a diverse set of optimization problems. PSO algorithms are especially useful for parameter optimization in continuous, multi-dimensional search spaces [9].

In performing a search in the multi-dimensional space associated with the optimization problem of the form (19, 20), the PSO technique assigns direction vectors and velocities to each member (particle) of the swarm at their current positions. Each particle then "moves" or "flies" through the search space according to the particle's assigned velocity vector, which may be influenced by the directions and velocities of other particles in its neighbourhood. These localized interactions with neighbouring particles, propagate through the entire "swarm" of particles and results in the swarm as a whole moving to regions of the space closer to the solution of problem (19, 20). The extent to which a particular particle influences other particles is determined by its so-called "fitness" along its trajectory of candidate solution points. The "fitness" is a measure assigned to each potential solution, and it indicates how good a particular candidate solution is relative to all other solution points. Hence, an evolutionary idea of "survival of the fittest" (in the sense of Darwinian evolution) comes into play, as well as a social behaviour component through a "follow the local leader" effect and emergent pattern formation [10].

Optimization and results

Table 2. shows the optima for two cost functions. K includes the cost of welding of the base plate K_0 and the cost of painting K_P. K_1 expresses the material and welding costs only and is calculated in order to compare the minimum cost of orthogonally stiffened plate with that of longitudinally stiffened one, which has been investigated in our previous study [2].

Table 2. Optimization results. Dimensions in mm

Cost function	h_y	h_x	n_y	n_x	t	minimum cost $
K_1	152.4	683.5	18	4	10	26358
K	152.4	683.5	15	4	11	43760

The plate having the same main dimensions, loaded by the same uniaxial compressive force with longitudinal stiffeners only and optimized in [2] has the minimum cost of K_1 = 52970 $, thus, the orthogonally stiffened plate is by 50% cheaper.

It can also be seen that the cost function influences the optima, since n_y and t is changed with the change of the cost function.

Conclusions

Orthogonally stiffened plates are important elements of welded structures, thus their minimum cost design influences the economy of these structures significantly. The basic formula for overall buckling strength shows that the transverse stiffening increases the plate strength in a great measure.

In the optimization process the height and number of halved rolled I-section stiffeners as well as the base plate thickness is sought, which fulfil the design constraints and minimize the cost function.

The efficient particle swarm algorithm is used to find the optima. The cost comparison between plates stiffened longitudinally and orthogonally shows that the later is much more economic.

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