New petrophysical models verified on laboratory data measured by an automatic acoustic test system

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***Abstract*—Understanding the relationship between pressure and rock physical parameters, such as acoustic velocities, absorption coefficients, elastic moduli, porosity is essential for exploring and exploiting of natural reserves. In this study petrophysical models are introduced which describe the relationship between acoustic longitudinal (P), transverse (S) wave velocities as well as quality factors and pressure. The governing equations of the models are based on the idea that the pore volume of a rock is decreasing with increasing pressure. To prove the applicability of the models, we measured P and S wave velocities in laboratory on sandstone samples by an automatic acoustic test system under uniaxial load. The loading of the samples were carried out by the Freely Programmable Interface module of the software DION 7. The velocities were measured automatically by the software GMuG/GL Test Systems PCUSpro. We applied 256-fold stacking to increase the signal/noise ratio. Acoustic velocities were measured by using the pulse transmission technique. By applying the programmed algorithm and the acoustic software the measurements have become completely automatic. On the other hand quality factor data set - published in literature - as a function of pressure was inverted too. After estimating the model parameters by joint inversion method, the velocities and quality factors can be calculated at any arbitrary stresses and the pressure dependent elastic moduli as well as loss angles can be derived. The quality checked joint inversion results showed that the misfits between measured and calculated data are small, the suggested petrophysical models can be applied well in practice.**

Keywords*—*acoustic velocity measurements; petrophysical models; pressure dependence; elastic moduli

I. INTRODUCTION

The knowledge of pressure dependence of rock physical parameters has a key role in the accurate interpretation of geophysical measurement data. Investigation of acoustic wave velocities and elastic properties has a great significance in seismic practice. Rocks response as perfectly elastic materials in case of rapidly changing stresses. With the assumption of the Hooke body, the elastic moduli describe how rocks resist different deformations. The higher the moduli are the stiffer the material is. It is observed that pressure has greater influence on velocities in the beginning phase of loading, later it lessens and the velocities tend to a limit value. Two main principles were published to explain this process. After Birch's consideration [1] the reason of the velocity increase is the decreasing pore volume with increasing pressure. Walsh and Brace [2] explain it with the closure of microcracks. A nonlinear relationship between velocity and pressure was proved by several empirical equations, however in these equations only the regression parameters are given, they do not provide the physical explanation of the process.

Beside the P and S wave velocities the pressure dependence of quality factors (Q_α, Q_β) or rather the attenuation (absorption coefficients) are often also investigated. There are several models in the international literature summarized by Barton [3] to explain the attenuation of elastic waves, among others the nonlinear friction model, the Biot model, the viscoelastic model and the elastic dispersion model. The theories for the pressure dependence of velocities are suitable for the description of the relationship between quality factor and pressure. Experiments denote, that the quality factors behave similarly to the velocities, a rapid nonlinear increase occurs at the beginning of loading [4] [5]. With the increasing pressure the pore volume decreases (or the microcracks close), the contacts between the grains become better and better thus the measurable absorption coefficient decreases and the quality factor increases.

During the development of the following rock physical models, the seismic/acoustic wave propagation phenomena is discussed with the application of the constant Q model, where the velocities and the quality factors - like phenomenological features - are rock stress dependent. After determining the velocities and quality factors at any pressure by inversion, the pressure dependent elastic parameters and loss angles can be deduced.

II. ROCK PHYSICAL MODELS

A. Model for the pressure dependence of acoustic velocities

The authors developed a rock physical model which explains the physical relationship between the applied stress and the acoustic P and S wave velocities [6]. The model is based on the idea formulated by Birch [1]. He assumed that the main reason for the increasing velocity under loading is the closure of pores. Hence the model law of our velocity model can be formulated by (1)

$$dV = -\lambda_V V d\sigma \ , \qquad (1)$$

where dV is the change of unit pore volume, $d\sigma$ is the applied stress increase and λ_V is the proportionality factor, a new rock physical parameter. The negative sign represents that the increasing stress causes decrease in the pore volume.

We assume also linear relationship between the infinitesimal change of the appropriate propagation wave velocity dv (substitutable with the longitudinal or shear wave) and dV

$$dv = -\kappa dV \ , \qquad (2)$$

where κ is a proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining (1-2) and solving the differential equations as well as applying the notation $\Delta v_0 = \kappa V_0$ one can obtain

$$v = v_0 + \Delta v_0 (1 - exp(-\lambda_V \sigma)) \ , \qquad (3)$$

where v_0 is the propagation velocity at stress-free state, while the quantity Δv_0 means the difference between the velocities measured at maximum and zero stresses i.e. $\Delta v_0 = v_{max} - v_0$, in other words it is the velocity-drop caused by the presence of pores at stress-free state [7]. The physical meaning of the parameter λ_V must be drawn up as well. The velocity-drop can be calculated at any arbitrary stresses as $\Delta v = v_{max} - v$. Substituting the formulas of Δv_0 and Δv (3) can be written in the form of

$$\Delta v = \Delta v_0 \, exp(-\lambda_V \sigma). \qquad (4)$$

At the characteristic stress σ^* the argument $\lambda_V \sigma^*$ equals 1 thus $\Delta v_0 = \Delta v / e$ i.e. the velocity-drop falls to $1/e$ of the "initial" velocity-drop. In addition the meaning of λ_V can be expressed as the logarithmic stress sensitivity of the velocity-drop [6] as

$$S(\sigma) = -\frac{1}{\Delta v}\frac{d\Delta v}{d\sigma} = -\frac{d\, ln(\Delta v)}{d\sigma} \rightarrow \lambda_V = -\frac{d\, ln(\Delta v)}{d\sigma} = S \ . \qquad (5)$$

By substituting the appropriate velocities the model equations describing the pressure dependence of longitudinal (α) and shear (β) waves can be obtained in the forms of (6)

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - exp(-\lambda_V \sigma))$$
$$\beta = \beta_0 + \Delta\beta_0 (1 - exp(-\lambda_V \sigma)) \ . \qquad (6)$$

Note that λ_V is the same for both types of waves therefore if both P and S wave velocity data are available they can be processed in joint inversion procedure.

B. Model for the pressure dependence of quality factors

Similarly to the velocity model, the quality factor model describing the longitudinal and transverse wave attenuation can be derived from varying pore volume [8]. The increasing stress causes compaction in the grain structure, e.g. the pore volume decreases. As a result increasing quality factors can be measured. Let us assume linear relationship between the change of pore volume (dV) and the change of quality factors (dQ_α and dQ_β) and introduce (7) as model laws

$$dQ_\alpha = -\chi_\alpha dV \ , \quad dQ_\beta = -\chi_\beta dV \ , \qquad (7)$$

where the Q_α and Q_β represent the quality factors for P and S waves respectively, χ_α and χ_β are proportionality factors and the negative signs represent that the decreasing pore volume results in increasing quality factor. Combining the (1) and (7) the following relations can be written

$$dQ_\alpha = \chi_\alpha \lambda_Q V_0 \, exp(-\lambda_Q \sigma) d\sigma \ ,$$
$$dQ_\beta = \chi_\beta \lambda_Q V_0 \, exp(-\lambda_Q \sigma) d\sigma \ . \qquad (8)$$

The quality factors at stress-free state ($Q_{\alpha 0}$ and $Q_{\beta 0}$) can be measured, thus the integration constants can be calculated (similarly to the velocity models). Introducing the notations $\Delta Q_{\alpha 0} = \chi_\alpha V_0$ and $\Delta Q_{\beta 0} = \chi_\beta V_0$, (8) take the forms

$$Q_\alpha = Q_{\alpha 0} + \Delta Q_{\alpha 0} (1 - exp(-\lambda_Q \sigma)) \ ,$$
$$Q_\beta = Q_{\beta 0} + \Delta Q_{\beta 0} (1 - exp(-\lambda_Q \sigma)) \ , \qquad (9)$$

where λ_Q is a common material dependent rock physical parameter. It can be seen from the model equations that the quality factors change also exponentially with pressure. $\Delta Q_{\alpha 0}$ and $\Delta Q_{\beta 0}$ mean the quality factor ranges for P and S waves, i.e. the differences between the quality factors at the stress-free state and at maximal stress ($Q_{\alpha\, max}$, $Q_{\beta\, max}$).

III. The Effect of Pressure on Elastic Moduli and Loss Angles

In case of rapidly changing stresses, the rocks response as perfectly elastic materials i.e. they suffer strain during loading but they perfectly recover their shapes after unloading. This is the assumption of the Hooke body, where the stresses are proportional to the deformations. The proportionality factors are called elastic moduli. One can distinguish static and dynamic moduli. Former ones are determined from stress-strain measurements, letter ones can rather be derived from acoustic measurements. This paper includes investigation of dynamic elastic moduli, the pressure dependence of these parameters can be deduced based on the previously introduced rock physical models.

In the general form of the Hooke body the two constants describing the stress-deformation relationship are the Lamé coefficients

$$\sigma_{ik} = 2\mu\varepsilon_{ik} + \lambda\Theta\delta_{ik} ,$$

where ε_{ik} are the elements of the deformation tensor, σ_{ik} are the elements of stress tensor, Θ is the volumetric strain, δ_{ik} is the unit tensor ($\delta_{ik} = 1 \;\; if\; i = k,\;\; \delta_{ik} = 0 \;\; if\; i \neq k$), μ and λ are the first and second Lamé coefficients. They can be calculated from velocities as

$$\mu = \beta^2\rho , \qquad \lambda = \alpha^2\rho - 2\mu , \qquad (10)$$

where ρ is the density of the medium. The pressure dependence of density is assumed negligible in comparison of the pressure dependent velocity. Therefore one requires accurate velocity measurements to obtain pressure dependent elastic moduli. Based on the direction of applying and measuring stress and strain, one can define further elastic moduli. The compression/bulk modulus (K) is assigned as the ratio of the hydraulic stress to the volumetric strain. The shear modulus (G) is the ratio of the shear stress to the shear strain and it is equal to the first Lamé coefficient. Young's modulus (E) is considered the ratio of extensional stress to extensional strain. They can be calculated as

$$K = \rho\left(\alpha^2 - \frac{4}{3}\beta^2\right),\; G = \beta^2\rho ,\; E = \beta^2\rho\frac{3\alpha^2 - 4\beta^2}{\alpha^2 - \beta^2} . \qquad (11)$$

If measured data of quality factors besides velocities are also available the pressure dependent dissipative parameters can be also deduced. With the assumption of constant Q model the Lamé coefficients are complexes

$$\mu = \mu^*(1 + i\varepsilon) , \qquad \lambda = \lambda^*(1 + i\varepsilon') ,$$

where μ^*, λ^* are the real part of the Lamé coefficients, ε, ε' are the so-called loss angles for which

$$tg\delta = \frac{Im\{\mu\}}{Re\{\mu\}} = \varepsilon , \qquad tg\delta' = \frac{Im\{\lambda\}}{Re\{\lambda\}}) = \varepsilon' .$$

(For small dissipations $tg\delta \approx \delta$). Solving the wave equations for body waves, the quality factors can be calculated as

$$Q_\beta = \frac{1}{\varepsilon} , \qquad Q_\alpha = \frac{\lambda + 2\mu}{\lambda\varepsilon' + 2\mu\varepsilon} .$$

Although the velocities and quality factors are determined during the measurements, the "real" material characteristics are μ, λ, ε, and ε'. The pressure dependent loss angles can be expressed as

$$\varepsilon = \frac{1}{Q_\beta} , \qquad \varepsilon' = \frac{\lambda + 2\mu}{\lambda Q_\alpha} - \frac{2\mu}{\lambda Q_\beta} . \qquad (12)$$

To prove the applicability of the presented models, laboratory measured data were processed, finally the pressure dependent elastic (compression, shear, Young's moduli, Lamé coefficients) and dissipative (loss angles) parameters were calculated.

IV. Measurement of Acoustic Velocities

Acoustic velocities are measured in the laboratory mostly by using the pulse transmission technique [4]. The detection of transverse wave arrival is a greater challenge than that of the longitudinal one. The reason is that at small transmitter-receiver distances the differentiation of P and S waves is difficult, however, if the distances are increased then also the attenuation increases and the signal-to-noise ratio decreases.

We performed measurements on many different cylindrical sandstone samples by an automatic acoustic test system under uniaxial stresses (results of sample A is presented in the paper, density 2620 g/cm^3). The test system includes a load frame, a pressure cell and an ultrasonic 2-channel testing device. The experimental setup is shown in Fig. 1.

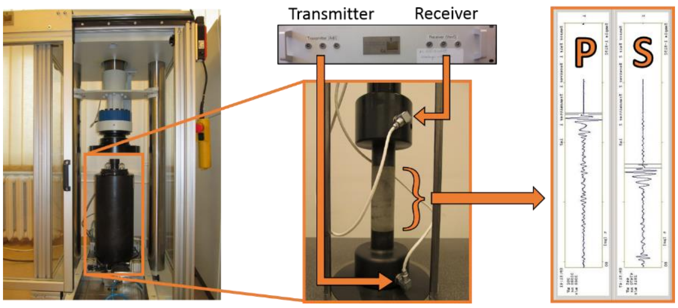

Fig. 1. Experimental setup. Left: load frame and pressure cell. Middle: ultrasonic device, sandstone sample between transmitter and receiver built in the pressure stamps. Right: P and S wave arrivals.

The accuracy of the load frame (max. 300 kN) according to ISO 7500-1 is grade 1 up to 30 kN and grade 0.5 above. The linearity of the load cell is 0.05 %. The loading of the samples were carried out by the Freely Programmable Interface module of the software DION 7. Fig. 2. shows the test algorithm. As it can be seen the samples were loaded linearly with 0.05 kN/s by a ramp function. After each stress step a 340 s duration break is embedded because of the relaxation time of the samples. To avoid the failure of the samples we loaded them only up to the 1/3 of uniaxial strength, for sample A it was 20.79 MPa.

The ultrasonic 2-channel testing device enables the possibility to measure the travel times of both compressional (P) and shear (S) waves on 35 mm diameter cylindrical, maximum 100 mm length specimens. The device consists of two pairs of transmitters and receivers, input amplifier, ADC, measuring memory, signal processing unit, computer interface. Therefore the complete system is suitable for generating, receiving, converting and visualizing (real-time) ultrasonic waves, as well as evaluating that of propagation properties by

PC (data export in ASCII). The pulse generator of the ultrasonic 2-channel testing device transmits a voltage pulse (run time 10 ns) which starts acoustic wave in the rock sample. (Transducers in the stamps have 1 MHz eigenfrequency and they are sealed against the confining pressure.) The receiver transforms this acoustic signal to voltage pulse. One can detect the arrival times of the pulse by the software GMuG/GL Test Systems PCUSpro, i.e. by knowing the length of the sample one can calculate the P and S wave velocities. This is the so-called pulse transmission technique. The velocities can measured automatically by the software GMuG/GL Test Systems PCUSpro. We applied 256-fold stacking to increase the signal/noise ratio and 32 dB gain. By applying the programmed algorithm and the acoustic software the measurements have become completely automatic.

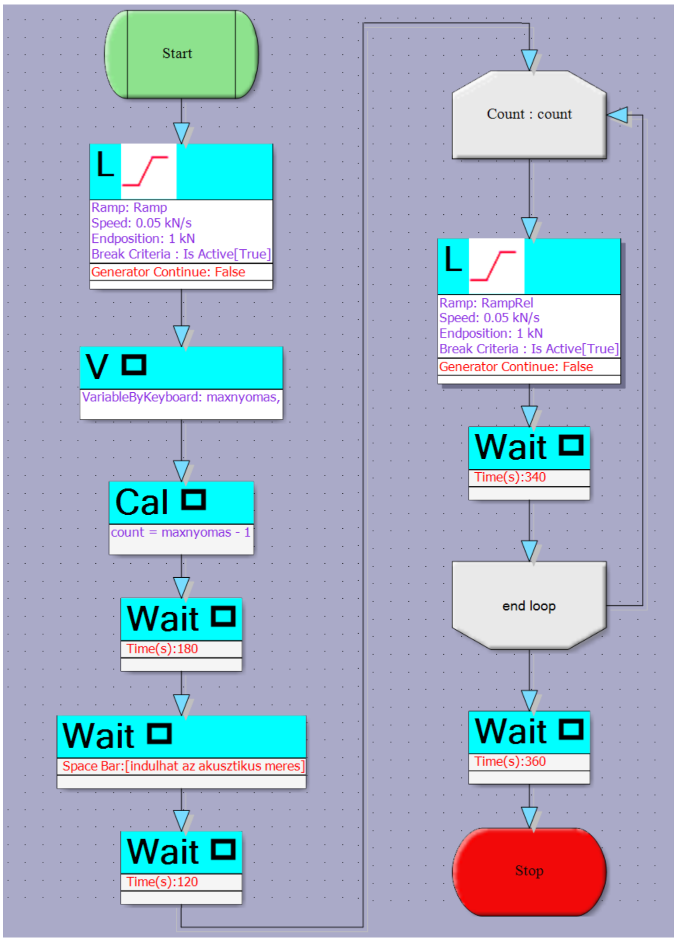

Fig. 2. Test algorithm of the automatic measurement.

V. CASE STUDIES

Based on measurement data the petrophysical parameters appearing in the model equations were determined by means of quality checked joint inversion method.

A. Case study for the P and S wave velocity model

Since λ_V is a common parameter, the 5 model parameters $(\alpha_0, \Delta\alpha_0, \lambda_V, \beta_0, \Delta\beta_0)$ of the two response equations formulated in (6) were determined by joint inversion process (Table I).

TABLE I. ESTIMATED MODEL PARAMETERS BY JOINT INVERSION METHOD

P wave		Common	S wave	
α_0 (m/s)	$\Delta\alpha_0$ (m/s)	λ_V (1/MPa)	β_0 (m/s)	$\Delta\beta_0$ (m/s)
4695.6 (±2.9)	379.6 (±7.8)	0.0844 (±0.0040)	2711.1 (±2.3)	198.6 (±5.2)

In a joint inversion procedure we integrate all of the measurement data into one combined data vector and we give an estimate for the P and S wave velocity data in an inversion algorithm, where λ_V connects the two data sets. The relative estimation errors of the model parameters (which are in parenthesis after each parameter) are given as well by the formula

$$\sigma_i = \sqrt{cov(m)_{ii}} \, ,$$

which implies the elements of the main diagonal of the covariance matrix in parameter space (the number of model parameters i=1,...,5 in the given problem).

The inversion problem was overdetermined, therefore the Gaussian Least Squares Method was used. After determining the model parameters the velocities were calculated by (6) for the stresses in the region of interest. Based on these velocities by applying (11) the pressure dependent elastic moduli (K, $G=\mu$, E, λ) were derived. The inversion results after 20 iteration steps for sample A are shown in Fig. 3. In the figure the upper two graphs present the velocities, the other ones show the elastic moduli. Dots mean the measured values (in case of elastic moduli they are calculated from the measured velocities), the lines represent the values calculated by inversion (in case of elastic moduli they are calculated from the velocities determined by inversion). For the characterization of the accuracy of inversion estimates the RMS value was calculated according to the following formula given by [9]

$$RMS = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{d_k^{(m)} - d_k^{(c)}}{d_k^{(c)}}\right)^2} \cdot 100\ [\%]\, ,$$

where $d_k^{(m)}$ is the measured data at the k-th pressure and $d_k^{(c)}$ is the k-th calculated data which can be computed by the model equations and N is the number of data. The mean spread which shows the reliability of the suggested petrophysical model was also determined by the formula

$$S = \sqrt{\frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j=1}^{M}\left(corr(m)_{ij} - \delta_{ij}\right)^2}\, ,$$

where δ is a Kronecker-delta symbol, M is the number of model parameters and $corr(m)$ is the correlation matrix in parameter space, which provides the strength of linear relationships between each pair of model parameters. The RMS and mean spread values are summarized in Table II. It can be seen that the data misfits (RMS) are small (0.11-0.94 %) and the mean spread (S) is 0.52 which means that the parameters are in moderate correlation.

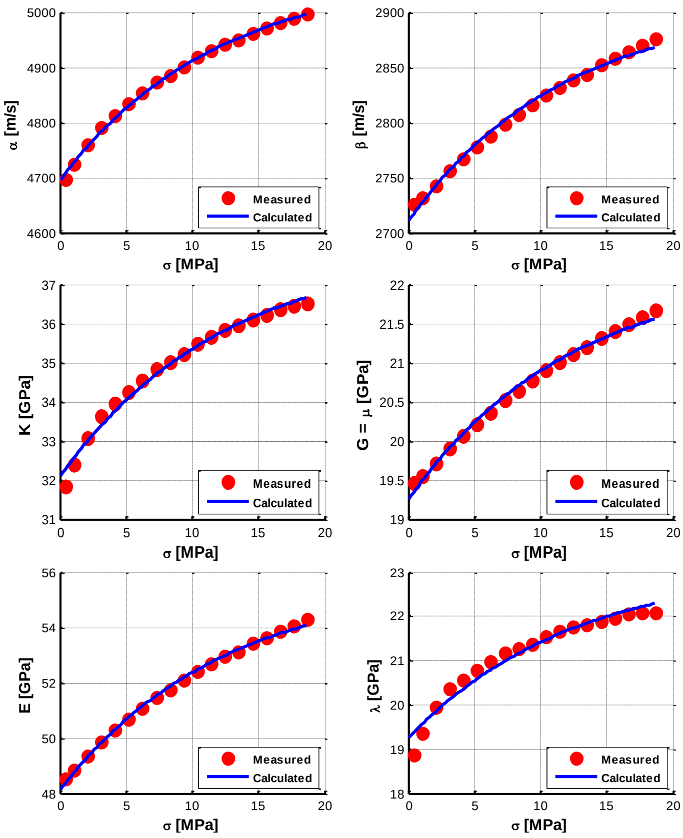

Fig. 3 The pressure (σ) dependence of velocities (α: P wave, β: S wave velocity) and elastic moduli (K: compressional, G: shear / μ: first Lamé coefficient, E: Young's modulus, λ: second Lamé coefficient) of sample A.

TABLE II. Estimated RMS and mean spread values

RMS (%)					S (-)
v	*K*	*G=μ*	*E*	*λ*	
0.11	0.47	0.27	0.17	0.94	0.52

B. *Case study for the velocity and quality factor models*

To test the suggested quality factor model measurement data published in literature were processed from which one is presented in the paper. The velocity and quality factor data set of coal nr. 16 sample was measured by [10]. The Upper Permian black coal sample originated from the Bulli Seam was homogeneous and microbanded in the central locality. The authors applied the pulse transmission technique to measure the P and S wave velocities and the spectral ratio technique [4] to determine quality factors. Similarly to the previous section measurement data were inverted by means of joint inversion processing. The calculated parameters together with their estimation errors can be seen in Table III.

TABLE III. Estimated model parameters and RMS values for coal nr.16 by joint inversion method

Velocities				**Common parameter**
P wave		***S wave***		
α_0 (km/s)	$\Delta\alpha_0$ (km/s)	β_0 (km/s)	$\Delta\beta_0$ (km/s)	λ_V (1/MPa)
2.23 (±0.02)	0.35 (±0.02)	1.02 (±0.01)	0.17 (±0.01)	0.1494 (±0.0123)
RMS = 0.54 %				
Quality factors				**Common parameter**
P wave		***S wave***		
$Q_{\alpha 0}$ (-)	$\Delta Q_{\alpha 0}$ (-)	$Q_{\beta 0}$ (-)	$\Delta Q_{\beta 0}$ (-)	λ_Q (1/MPa)
10.92 (±1.12)	53.66 (±4.31)	14.09 (±1.02)	66.58 (±4.50)	0.0293 (±0.0043)
RMS = 7.18 %				

With the estimated parameters the velocities and quality factors can be determined at any pressure by means of the developed model equations (6) and (9). Fig. 4-5. represent the results, the calculated Lamé coefficients and loss angles are produced by (10) and (12). The calculated curves are in good accordance with the measured data, which is strengthened by the calculated low RMS values (Table III). In case of quality factors RMS values are higher than those at the velocities which can be explained by the difficulty of quality factor measurements. Even so the noise in data space is small-scale, which confirms the accuracy of the inversion results and the feasibility of the suggested petrophysical models for the explanation of the exponential relationship between the P and S wave velocities/quality factors and rock pressure. The moderate (S=0.48) mean spread value confirms also that the inversion results are reliable.

VI. Conclusions

The goal of our present investigation was to determine the pressure dependent elastic moduli and loss angles. They can be calculated if accurate P and S wave velocity/quality factor data are available for any pressures. Therefore we suggested petrophysical models for describing the connection between the velocity/quality factor of P, S waves and rock pressure. The models are valid only in the reversible range and are based on the idea that pore volume of the rock is decreasing with increasing pressure. To prove the applicability of our models they were tested on laboratory data measured by an automatic acoustic test system. The programmed test algorithm and the acoustic software enables us to measure P and S wave velocities completely automatically. After estimating the model parameters by joint inversion procedure and calculating the velocities/quality factors, the pressure dependence of elastic moduli and loss angles can be deduced. The accuracy of the inversion estimates and the reliability of the suggested petrophysical model was proved.

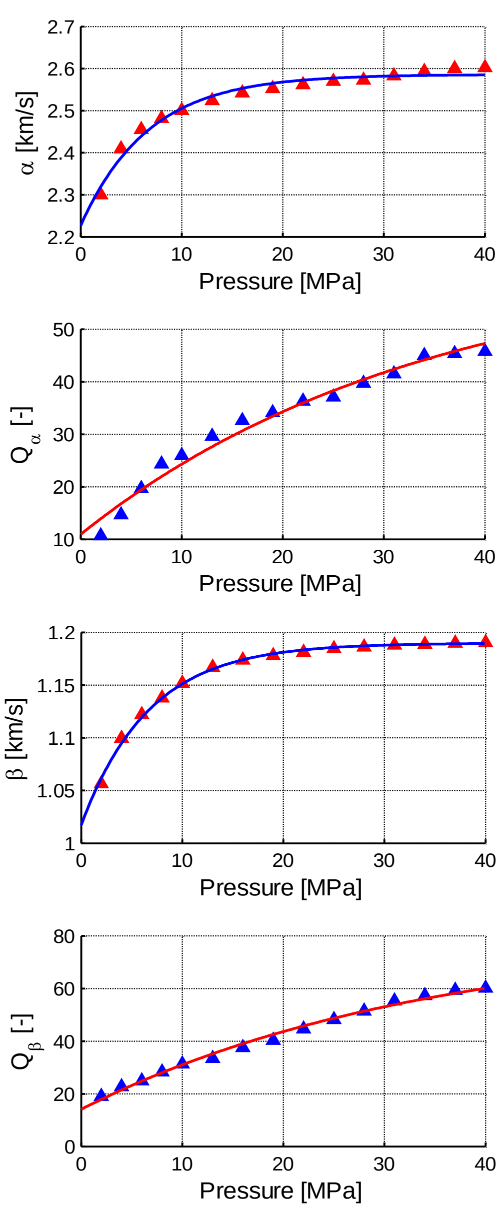

Fig. 4 Velocities and quality factors of P and S waves vs. uniaxial pressure of sample coal nr.16 (solid line – data calculated by the developed models, symbols – measured data). Data obtained by [10].

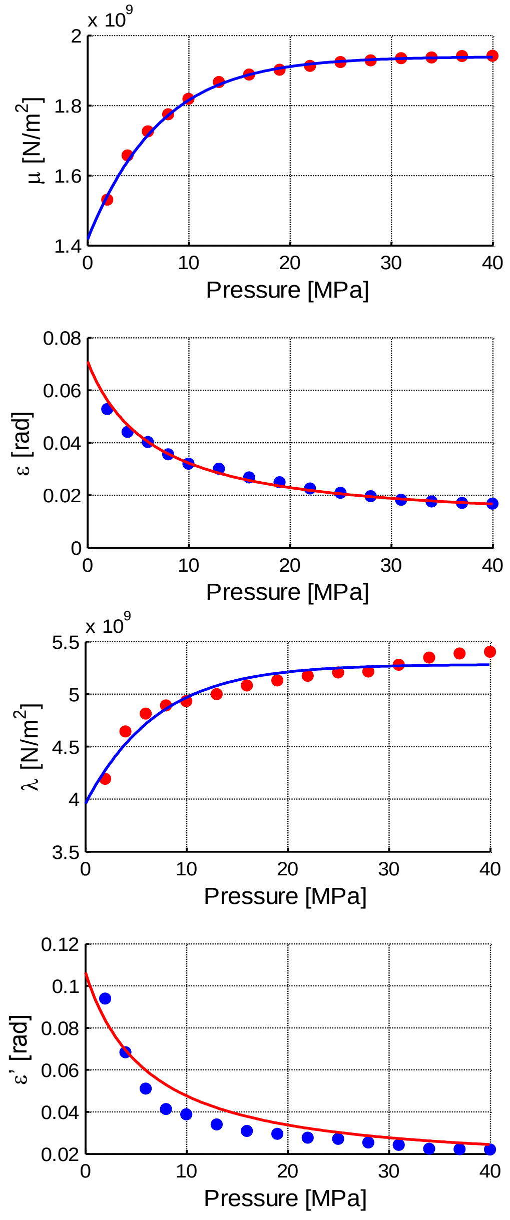

Fig. 5 Lamé coefficients (μ, λ) and loss angles (ε, ε') vs. uniaxial pressure of sample coal nr.16. Data obtained by [10].

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