

Joint Inversion Based P/S Wave Velocity Data Processing to Test a New Rock Physical Model Describing Acoustic Hysteresis

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SUMMARY

Understanding the relationship between pressure and rock physical parameters, such as acoustic velocities, elastic moduli, porosity is essential for exploring and exploiting of natural reserves. In this paper we introduce a rock model which describes the acoustic P, S wave velocities-pressure relationship for loading and unloading phases as well as explains the mechanism of acoustic hysteresis. After Birch we assume that the main factor determining the pressure dependence is the closure of pores. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three parameter exponential equations. Laboratory measured acoustic P and S wave velocities as a function of pressure were inverted to prove the applicability of the model and to obtain that of parameters. The uniaxial loading of the samples was carried out by an automatic acoustic test system. The quality checked joint inversion results showed that the calculated data matched accurately with measured data and also proved that the suggested petrophysical model performs well in practice.

Introduction

Due to the increasing demand for hydrocarbons and the depletion of the known hydrocarbon fields there is a growing claim to predict rock physical parameters more accurate at non-conventional conditions also. It is well known that acoustic velocity in rocks strongly depends on pressure which influences the mechanical and elastic properties of rocks. The wave propagation under pressure is very nonlinear and the elastic properties of rocks are hysteretic. Characterization of hysteretic behaviour is important for mechanical understanding of reservoirs during depletion.

This observable non-elastic response to pressure may be caused by the processes: irreversible closure of micro cracks, irreversible compaction of pore spaces as well as improvement of contact conditions. According to the theory of irreversible closure of micro cracks, the micro cracks closed during loading do not reopen during subsequent unloading (Walsh and Brace 1964). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original dimensions at lower pressures (Birch 1960). By idea of the improvement of contact conditions (Hashin and Shtrikman 1963) in a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

The idea that the pressure-acoustic velocity connection can be characterized by exponential function is well-known but the developed empirical models are based on mathematical curve fitting, however the physical meaning is partly understood. Therefore in this paper we present a quantitative petrophysical model, which explains the mechanism of pressure dependence of P and S wave velocities as well as describes well the acoustic hysteresis.

The new rock physical model

Following Birch's (1960) qualitative considerations we assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure -from the unloaded state-, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Since the model is based on the pore volume or rather the change in pore volume, it is suitable to describe the pressure dependence of longitudinal and transverse wave velocities, respectively.

Let us introduce the parameter V as the unit pore volume of a rock. We assume that a $d\sigma$ stress increase applied to the rock will generate a dV change in pore volume directly proportional to the change in stress. Eq. (1) summarizes these assumptions in a differential equation and its solution

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \quad (1)$$

where λ_V proportionality factor is a material dependent rock physical parameter, V_0 is the pore volume at stress-free state ($\sigma = 0$). The negative signs represent that the increasing stress decreases the pore volume. As the volume does not show anisotropy, Eq. (1) is the base of the model equations for both the P and S waves. We assume also a linear relationship between the infinitesimal change of the propagation wave velocities ($d\alpha$ for P wave and $d\beta$ for S wave) and the change in pore volume

$$d\alpha = -\kappa_\alpha dV, \quad d\beta = -\kappa_\beta dV, \quad (2)$$

where κ_α and κ_β are proportionality factors, new material characteristics respectively for P and S waves. The negative signs represent that the velocities are increasing with decreasing pore volume. Combining the assumptions of Eqs. (1-2) and solve the differential equations one can obtain

$$d\alpha = \kappa_\alpha \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \rightarrow \alpha = K_I - \kappa_\alpha V_0 \exp(-\lambda_V \sigma),$$

$$d\beta = \kappa_{\beta} \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \rightarrow \beta = K_2 - \kappa_{\beta} V_0 \exp(-\lambda_V \sigma), \quad (3)$$

where K_1 and K_2 are integration constants which can be computed from Eqs. (3) as $\alpha_0 = K_1 - \kappa_{\alpha} V_0$ and $\beta_0 = K_2 - \kappa_{\beta} V_0$, where α_0 and β_0 are the propagation velocities at stress-free state which can be measured in laboratory. In the framework of the model, the velocities of acoustic waves increase from α_0 and β_0 (at zero pressure) to $\alpha_{max} = \alpha_0 + \Delta\alpha_0$ and $\beta_{max} = \beta_0 + \Delta\beta_0$ (at high pressure, when all the pores are closed). So, $\Delta\alpha_0$ and $\Delta\beta_0$ can be considered the velocity-drops (relative to the fully compacted state where the pore volume equals zero) caused by the presence of pores at zero pressure (Ji et al. 2007). With introducing the notations $\Delta\alpha_0 = \kappa_{\alpha} V_0$, $\Delta\beta_0 = \kappa_{\beta} V_0$ Eqs. (3) can be rewritten in the forms

$$\alpha = \alpha_0 + \Delta\alpha_0 [1 - \exp(-\lambda_V \sigma)], \quad \beta = \beta_0 + \Delta\beta_0 [1 - \exp(-\lambda_V \sigma)]. \quad (4)$$

Eqs. (4) provide a theoretical connection between the P and S wave velocities and rock pressure in case of loading. Note that in the range of high pressures, reaching a critical pressure (Anselmetti and Eberli 1997) the reversible range is exceeded and destruction of the sample may occur thus decreasing velocities can be observed. This effect is outside of our present investigations.

To characterize the unloading phase, $v = V_0 - V$ as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed pores start to open again, so decreasing propagation velocities can be measured. Therefore we assume dv (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease $d\sigma$

$$dv = \lambda'_V v d\sigma \rightarrow v = v_m \exp[-\lambda'_V (\sigma_m - \sigma)], \quad (5)$$

where λ'_V is another new material characteristic constant (which differs from the previously introduced parameter λ_V) and v_m is the closed pore volume at maximum pressure value σ_m . After Birch (1960) there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during loading do not reopen completely during unloading. This irreversibility is denoted by these different parameters λ_V and λ'_V in our model. Combining Eqs. (2) and Eq. (5) by using the formulas $dV = -dv$, $\kappa_P v_m = \Delta\alpha_m$ and $\kappa_S v_m = \Delta\beta_m$ one can find

$$\alpha = \alpha_m - \Delta\alpha_m \{1 - \exp[-\lambda'_V (\sigma_m - \sigma)]\}, \quad \beta = \beta_m - \Delta\beta_m \{1 - \exp[-\lambda'_V (\sigma_m - \sigma)]\}. \quad (6)$$

Eqs. (6) show the P/S wave velocity-pressure function of unloading phase. In the two limiting cases (at pressure value $\sigma = \sigma_m$ and $\sigma = 0$) Eqs. (6) give α_m/β_m and $\alpha_1 = \alpha_m - \kappa_P v_m [1 - \exp(-\lambda'_V \sigma_m)]$, $\beta_1 = \beta_m - \kappa_S v_m [1 - \exp(-\lambda'_V \sigma_m)]$ respectively, (notation $\alpha(0) = \alpha_1$ and $\beta(0) = \beta_1$ were used). This gives the formulas for unloading phase (similar to Eqs. (4))

$$\alpha = \alpha_1 + \Delta\alpha_1 [1 - \exp(-\lambda'_V \sigma)], \quad \beta = \beta_1 + \Delta\beta_1 [1 - \exp(-\lambda'_V \sigma)], \quad (7)$$

where $\Delta\alpha_1 = -\kappa_P v_m \exp(-\lambda'_V \sigma_m)$ and $\Delta\beta_1 = -\kappa_S v_m \exp(-\lambda'_V \sigma_m)$.

Joint inversion based measurement data processing

To confirm the reliability of the model it was tested on velocity data sets. Acoustic P and S wave velocities were measured in laboratory by using the pulse transmission technique. We performed measurements on many different cylindrical sandstone samples with an automatic acoustic test system under uniaxial stresses. Wave velocities - as a function of pressure - were measured at adjoining pressures during loading and unloading phases. Since the loading then unloading of the samples were carried out by a Freely Programmable Interface module the measurements become completely automatic. A 256-fold stacking was applied to increase the signal/noise ratio as well as honey was used for coupling. Our measurements showed that in the lower pressure range, the increase in velocities with increasing pressure is steep and nonlinear. This is due to the closure of pore volume, which significantly affects the elastic properties of rock and thereby the velocities. In the higher pressure range, the increase in velocities (with increasing pressure) become moderate as the closable pore volume lessens. A slight difference can be found between the characteristics of the loading and unloading curves which can be explained by the phenomenon of acoustic hysteresis. To avoid the failure of the samples we loaded them only up to one third of the critical uniaxial strength. One typical test results (Sample A: fine-grained sandstone, depth: 3490 m) is presented in the paper.

Based on measurement data the petrophysical parameters (loading: $\alpha_0, \Delta\alpha_0, \lambda_v$ and $\beta_0, \Delta\beta_0, \lambda'_v$, unloading: $\alpha_1, \Delta\alpha_1, \lambda'_v$ and $\beta_1, \Delta\beta_1, \lambda'_v$) appearing in the model equations were determined by means of quality checked joint inversion method (Menke 1984). The inversion results can be seen in Table 1. For the characterization of the accuracy of inversion estimates, the RMS value and mean spread (S) were calculated at the end of the inversion procedure. It can be seen that the data misfits were small and the mean spread values indicate that the parameters are in moderate correlation, but the inversion results are reliable. These results confirm the accuracy of the inversion estimates and the feasibility of the developed petrophysical model.

Table 1 Model parameters, data misfits and mean spreads estimated by joint inversion using the developed model.

P wave							
Loading			Unloading			RMS (%)	S
α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ_v (1/MPa)	α_1 (km/s)	$\Delta\alpha_1$ (km/s)	λ'_v (1/MPa)		
4,69	0,37	0,0404	4,72	0,29	0,1927	0,12	0,55
S wave							
Loading			Unloading			RMS (%)	S
β_0 (km/s)	$\Delta\beta_0$ (km/s)	λ_v (1/MPa)	β_1 (km/s)	$\Delta\beta_1$ (km/s)	λ'_v (1/MPa)		
2,71	0,17	0,0456	2,72	0,16	0,1944	0,1	0,59

With the estimated parameters, the velocities can be calculated at any pressure by substituting them into the model equations Eqs. (4) and (7). The results are shown in Fig. 1, where the solid line shows the calculated velocity-pressure function produced by the velocity model, while symbols represent the measured data. The figures show that the calculated curves are in good accordance with the measured data proving that the petrophysical model describing the acoustic hysteresis applies well in practice in case of also P and S waves.

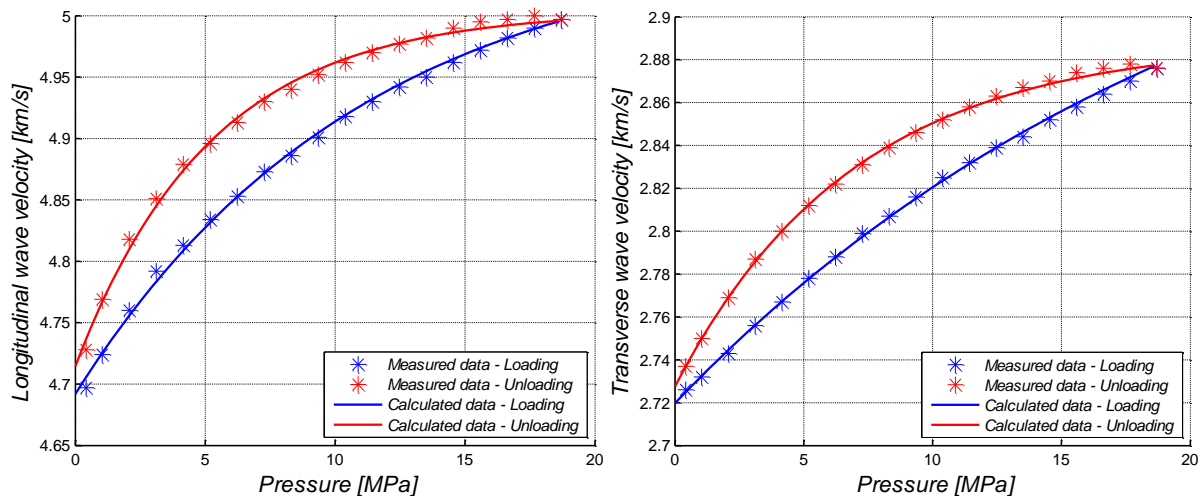


Figure 1 P and S wave velocities as a function of pressure of Sample A in case of loading and unloading.

Conclusions

We presented a new quantitative petrophysical model describing the acoustic hysteresis, i.e. provides the P and S wave velocity-pressure connection both in case of loading and unloading phases. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. After Birch we assume that the main factor determining the pressure dependence is the closure of pores. Since the base of the model is the change of pore volume (which is independent of the direction of loading) it can be applied also in case of S waves. Based on the model the acoustic hysteresis can be expressed by two different parameters (λ_V, λ'_V) because the closed pores do not reopen entirely during unloading. The model is valid only in reversible/elastic range. To confirm the reliability of the model it was applied to acoustic velocity data measured on sandstone samples by an automatic acoustic test system. By means of joint inversion-based data processing, the model parameters were determined from measurement data. Calculated data could be produced by using the petrophysical model and a very good fit between measured and calculated data (in case of P and S waves too) was found thus inversion results confirmed the accuracy and feasibility of the petrophysical model.

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