

A new petrophysical model for acoustic hysteresis based on transverse wave velocity measurements

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Summary

It is well known that acoustic wave propagation under pressure is very nonlinear and the elastic properties of rocks are hysteretic, which behaviour is important for mechanical understanding of reservoirs during depletion. Pressure strongly influences the elastic parameters of rocks, thus wave velocities too. Therefore a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which describes the connection between the propagation velocity of transverse wave and rock pressure both in case of loading and unloading phases as well as explains the mechanism of acoustic hysteresis. The developed model is based on the idea that the pores in rocks close under loading and reopen during unloading. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The model was applied with success to acoustic S wave velocity data sets measured under pressure in laboratory on sandstone samples by an automatic acoustic test system.

Introduction

To interpret seismic, acoustic borehole logging data and to relate laboratory measurement to in-situ parameters properly it is necessary to understand how pressure effects to e.g. acoustic velocity, porosity, permeability. Therefore the investigation of pressure dependence of propagation wave velocity in rocks is in focus of researchers for several decades. It has been observed that pressure has greater influence on S wave velocity in the beginning phase of loading, and later it lessens and the velocities tend to a limit value (Birch 1960). The most frequently used mechanisms for explaining this process are based on the closure of microcracks (Walsh and Brace 1964) or pores (Birch 1960) in rocks under pressure. For an analytical description of the nonlinear velocity vs. pressure relationship, exponential functions are most commonly used (Wepfer and Christensen 1991).

It is well known that the quasistatic elastic properties of rocks are hysteretic (Ji et al. 2007). Characterization of hysteretic behaviour is important for a mechanical understanding of reservoirs during depletion. The investigation of acoustic hysteresis is a common task in laboratory. The observable non-elastic response to pressure (namely acoustic hysteresis) may be caused by the irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions. The first theory assumes that the microcracks closed during loading do not reopen during subsequent unloading (Walsh and Brace 1964). After the conception of irreversible compaction of pore spaces, the pores which collapsed at higher pressures do not recover their original dimensions at lower pressures (Birch 1960). By idea of the improvement of contact conditions (Hill 1963) ductile minerals (e.g., chlorite, sericite or serpentine) can be embedded along grain boundaries and microcracks, modifying the elastic properties of the rock. In a rock, grains themselves act as perfectly elastic units, while the contacts between these grains often display non-linear elastic behaviour. As a result, the rock will show an overall elastically non-linear behaviour characterized by hysteresis.

In literature several qualitative models are available to describe the pressure dependence of acoustic wave velocity but these empirical models do not explain the physical meaning of the process they only give the regression function of the curve fitted to the measured data. To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which delivers the connection between the propagation velocity of transverse (S) wave and rock pressure both in case of loading and unloading periods as well as explains the mechanism of acoustic hysteresis.

Describing the loading phase

At the development of the physical based-model Birch's (1960) qualitative considerations were followed. We assume that the main factor determining the pressure dependence of transverse wave velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure -from the unloading state-, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Therefore we introduce the parameter V as the unit pore volume of a rock. Since the base of the model is the change of pore volume (which is independent of the direction of loading) the phenomenon of polarization is eliminated.

If a stress increase $d\sigma$ is created in a rock let us assume that the change of pore volume dV is directly proportional to the applied stress increase $d\sigma$ and also the pore volume V . One can describe the two assumptions with the following differential equation

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \quad (1)$$

where λ_V is new material quality dependent petrophysical parameter (Dobróka and Somogyi Molnár 2012) and V_0 is the pore volume at stress-free state ($\sigma = 0$). The negative sign represents that with increasing stress the pore volume decreases ($\lambda_V > 0$). We assume also a linear relationship between the infinitesimal change of the S wave propagation velocity $d\beta$ - due to stress increase - and dV

$$d = - \kappa dV, \quad (2)$$

where κ is a positive proportionality factor, a new material characteristic. The negative sign represents that the S wave velocity is increasing with decreasing pore volume. Combining Eqs. (1-2) and solving the differential equation one can obtain

$$d = V_0 \exp(-\lambda_V) d \rightarrow \beta = K - V_0 \exp(-\lambda_V), \quad (3)$$

where K is an integration constant. At stress-free state ($\sigma=0$) the propagation velocity β_0 can be measured which is computed from Eq. (3) as $\beta_0 = K - V_0$. With this result and introducing the notation $\Delta\beta_0 = \kappa V_0$ Eq. (3) can be rewritten in the following form

$$\beta = \beta_0 + \Delta\beta_0 [1 - \exp(-\lambda_V)]. \quad (4)$$

Eq. (4) provides a theoretical connection between the transverse wave velocity and rock pressure for loading. The S wave velocity increases from β_0 to β_{max} (at high pressure, when approximately all the pores are closed). So, $\Delta\beta_0$ can be considered the velocity-drop caused by the presence of pores at zero pressure. Petrophysical characteristic λ_V is the logarithmic stress sensitivity of the velocity-drop (Dobróka and Somogyi Molnár 2012). Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded, hence decreasing velocity is observed. This effect is outside of our present investigations.

Describing the unloading phase

To characterize the unloading phase, $v=V_0-V$ as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value σ_m) the closed pores start to open again, so decreasing velocity can be measured. Therefore we assume dv (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease $d\sigma$

$$dv = - \lambda'_V v d\sigma \rightarrow v = v_m \exp[-\lambda'_V (\sigma_m - \sigma)], \quad (5)$$

where λ'_V is another new material characteristic constant and v_m is the closed pore volume at maximum pressure value σ_m . After Birch (1960) there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during loading do not reopen completely during unloading. This irreversibility is denoted by these different parameters λ_V and λ'_V in our model. Combining Eq. (2) and Eq. (5) by using the formulas $dV=-dv$ and $\kappa v_m = \Delta\beta_m$ one can find

$$\beta = \beta_m - \Delta\beta_m \{1 - \exp[-\lambda'_V (\sigma_m - \sigma)]\}. \quad (6)$$

Eq. (6) shows the propagation velocity – pressure function of unloading phase. In the two limiting cases (at pressure value $\sigma=\sigma_m$ and $\sigma=0$) Eq. (6) gives β_m and $\beta_1 = \beta_m - \Delta\beta_m [1 - \exp(-\lambda'_V \sigma_m)]$, respectively (notation $\beta(0) = \beta_1$ was used), thus the following formula can be formed (similar to Eq. (4))

$$\beta = \beta_1 + \Delta\beta_1 [1 - \exp(-\lambda'_V \sigma)], \quad (7)$$

where $\Delta\beta_1 = - \Delta\beta_m \exp(-\lambda'_V \sigma_m)$.

Laboratory measurements

To confirm the reliability of the model it was tested on velocity data sets. Acoustic transverse wave velocity was measured in laboratory by using the pulse transmission technique. We performed measurements on many different cylindrical sandstone samples with an automatic acoustic test system under uniaxial stresses. Wave velocities - as a function of pressure - were measured at adjoining pressures during loading and unloading phases. A 256-fold stacking was applied to increase the signal/noise ratio as well as honey was used for coupling. Since the loading then unloading of the samples were carried out by a Freely Programmable Interface module the measurements become completely automatic. To avoid the failure of the samples we loaded them only up to one third of the critical uniaxial strength.

Our measurements showed that in the lower pressure range, the increase in velocities with increasing pressure is steep and nonlinear. This is due to the closure of pore volume, which significantly affects the elastic properties of rock and thereby the velocities. In the higher pressure range, the increase in velocities (with increasing pressure) become moderate as the closeable pore volume lessens. A slight difference can be found between the characteristics of the loading and unloading curves which can be explained by the phenomenon of acoustic hysteresis. Two typical test results (Sample A: fine-, medium-grained sandstone, depth: 2820 m and Sample B: fine-grained sandstone, depth: 3490 m) are presented in the paper.

Inversion results

Proving the validity and applicability of the introduced velocity model, we present the interpretation of measurement data. The parameters appearing in the model equations (loading: $\beta_0, \Delta\beta_0, v$, unloading: $\beta_1, \Delta\beta_1, v$) have to be estimated by processing measurement data based on joint inversion method (Menke 1984). During data processing the developed model equations (Eqs. (4) and (7)) were used as response functions. The inversion problem was overdetermined therefore the Gaussian Damped Least Squares Method was used. The estimated model parameters are summarized in Table 1. For the characterization of the accuracy of inversion estimates the relative distance in data space, i.e. the root mean square (RMS) is also provided. The table contains the mean spread values as well, which indicate that the parameters are in moderate correlation.

Table 1 Model parameters, data misfits and mean spreads estimated by joint inversion using the developed model.

Sample	Loading			Unloading			RMS (%)	S
	β_0 (km/s)	$\Delta\beta_0$ (km/s)	v (1/MPa)	β_1 (km/s)	$\Delta\beta_1$ (km/s)	v (1/MPa)		
A	2,29	0,51	0,0212	2,31	0,46	0,0395	0,26	0,5
B	2,71	0,17	0,0456	2,72	0,16	0,1944	0,1	0,59

After determining the model parameters in the inversion procedure, the S wave velocities can be calculated by the model equations for any arbitrary stresses. The inversion results after 20 iteration steps for each sample are shown in Fig. 1. Asterisks mean the measured values while lines represent the values calculated by inversion. The figure shows that the calculated curves are in good accordance with measured data proving that the petrophysical model describing the acoustic hysteresis of transverse wave velocity applies well in practice. It is confirmed by the small RMS values (Table 1) also. It can be also seen that the model characterizes well both loading and unloading phases.

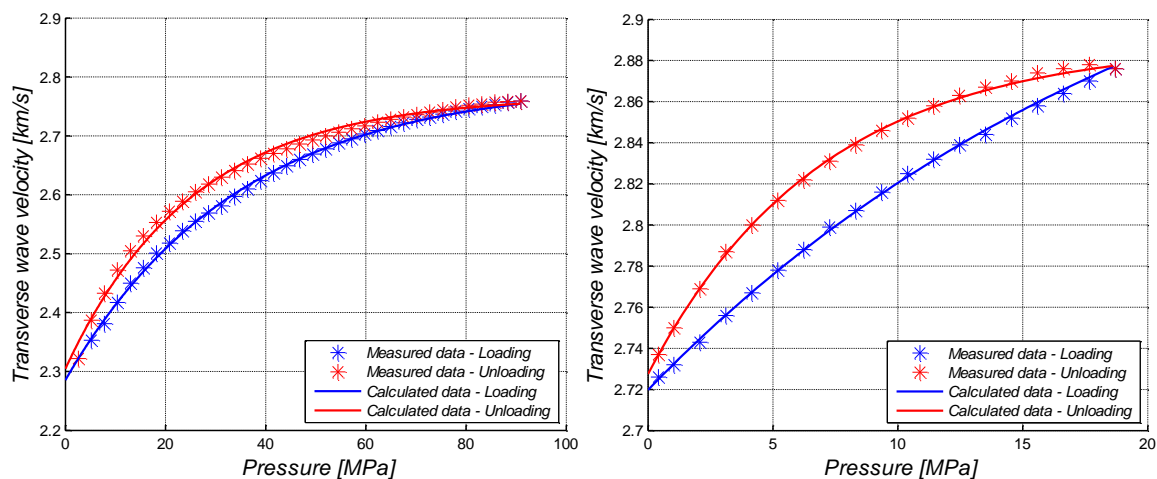


Figure 1 S wave velocity as a function of pressure for Sample A and B

Conclusions

A new quantitative model describing the mechanism of pressure dependence of acoustic S wave velocity was presented. It gives the velocity-rock pressure connection both in case of loading and unloading phases, i.e. it also describes well and explains the mechanism of acoustic hysteresis. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The model (valid only in reversible/elastic range) is based on the idea that pore volume changes with pressure. It was found that there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during loading do not reopen completely during unloading. This phenomenon is denoted by two different parameters λ in the model which was introduced as a new material quality dependent petrophysical constant, the logarithmic stress sensitivity of the propagation velocity.

To confirm the reliability of the suggested model it was tested on transverse wave velocity data. Measurements were carried out on many different sandstone samples by an automatic acoustic test system. Measured data were processed by joint inversion technique. Inversion results showed that the misfits between measured and calculated data are small, proving that the proposed petrophysical model can be applied well in practice.

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