ANALYSIS OF THE REPRODUCTION OF THE OBJECT INCLUDING HELICES GAINED FROM PHOTOS TAKEN BY 2 CCD CAMERAS

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1. Introduction

A new possibility opened for the examination of tool surfaces when CCD cameras appeared. There is a possibility for in-process tool profile monitoring dimension inspection, which was a great step according to the measurement during operation [2,3]. The automated evaluation of the pictures is a further advantage [5]. In this way the production system can be assembled. This method gives the possibility of the feedback during machining.

The dramatic development of computer added image analysis opened a very wide possibility for the utilisation of CCD cameras and the application of image analysing and evaluation software.

During picture evaluation by machines the problem is that we have to reproduce spatial forms by two pictures done by two cameras. It is done by using three pictures in the classical case. There are those procedures, methods, which can reconstruct spatial form two pictures as well, in certain cases. When 2 CCD cameras are in perpendicular position, one procedure working in this way is shown in latest paper [1].

The representation of a point in Monge projection unambiguous provided, that the well-known conventions are fulfilled [4]. For example the representation of a circle of general position or a profile line is not bijective. For such and similar cases descriptive geometry found special methods to ensure bijectivity. We have given an other solution to this problem [].

The number of Monge projections to a given object can be described using 3 free parameters. Each Monge projection is corresponded to a point of a rectangular prism. The subset of the rectangular prism which indicates a Monge projection is named Monge cuboid. Each point of a Monge cuboid defines a triplet of real numbers which is in fact a triplet of angles providing us with a Monge projection with respect to a given object.

The points of a Monge cuboid can be divided into two subsets. There are points, which correspond to Monge mappings of the curve and another which correspond to the non-bijective mappings.

This work gives the solution to place CCD cameras to get the bijective Monge projection about the object including two helices.

2. The bijective part of Monge cubid With respect to the helix

Let us consider the pitch of the helix in that case when the axis is identical with coordinate axis *z*, and the start point is located in coordinate axis *x*.

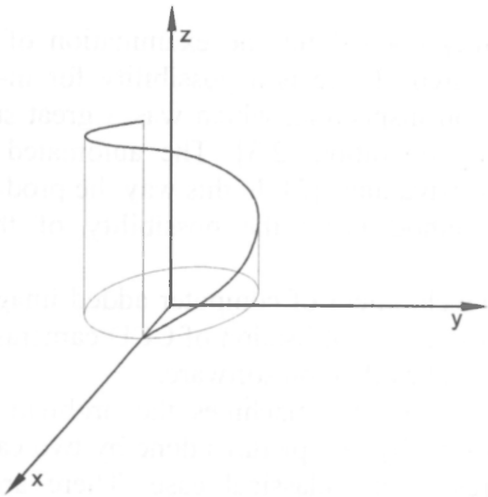

Figure 1. The helix in the given case

The equation of helix is

$$x = r \cos\varphi,\ y = r \sin\varphi,\ z = p\ \varphi,$$

where $p \in \mathbf{R} \setminus \{0\}$, $r \in \mathbf{R}^+$, $0 \leq \varphi < 2\pi$.

In this case tangent lines are

$$x = - r \sin\varphi,\ y = r \cos\varphi,\ z = p.$$

Tangents are collected in O, that is the tangential cone. ω is the angle between the axis *z* and tangents. The normal cone is created by the collecton of normal lines of tangent planes of tangential cone on O point. The normal cone of the tangential cone is shown in Figure 2.

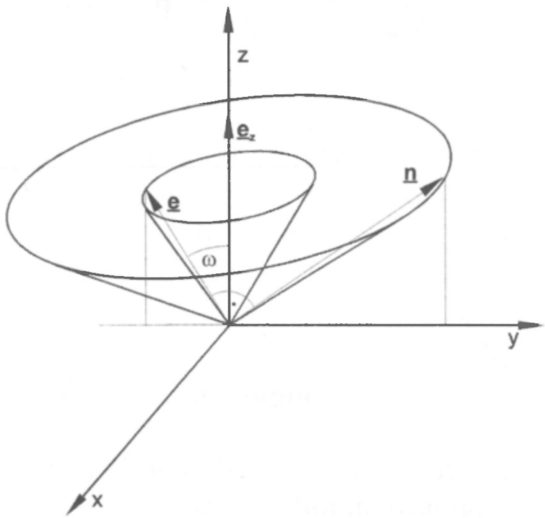

Figure 2. The normal cone of the tangential cone

The helix is projected perpendicularly on the image plane, which creates δ angle with the axis of the helix. If δ = 0, the projection is a circle. If δ = π/2, the projection is a curve of the sinus function. If $0<\delta<\pi/2$, the projection is a cycloid.

If two tangets of the helix are included by the profile plane of the Monge-projection, there exists a piece of the helix, which cannot be reproduced by two perpendicular image.

If **n** is the direct vector of the generating line of the normal cone, and |**n**| =1, **z** (0, 0, 1), using formulas

$\underline{n}\,\underline{z} = |\underline{n}|\,|\underline{z}|\cos(90^\circ - \omega) = \sin\omega$

and

$\underline{n}\,\underline{z} = n_x 0 + n_y 0 + n_z 1 = n_z,$

so

$n_z = \sin\omega.$

The |**n**| = 1 gives out that

$n_x^2 + n_y^2 + n_z^2 = 1.$

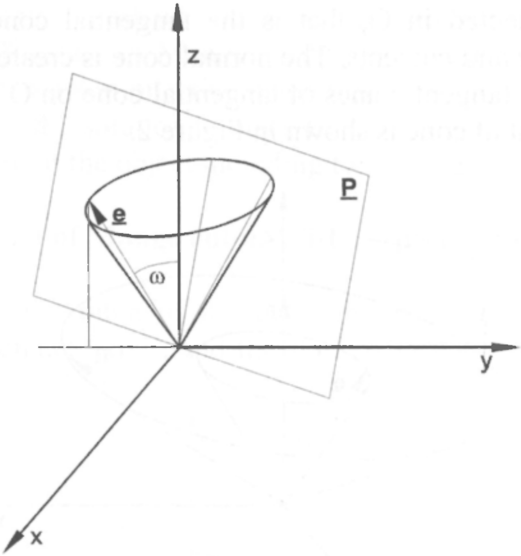

Figure 3.

From resulting the fact that the normal cone is intersected by the profile plane of the Monge-projection, follows that
$n_x^2 + n_y^2 > \cos^2\omega$.
$\mathbf{v_1}, \mathbf{v_2}$ direct vectors of projecting line of the Monge-projection satisfy the next equation

$(\mathbf{v_1}\mathbf{v_2}) = 0,$

and $\mathbf{n} = \mathbf{v_1} \times \mathbf{v_2}.$

If the ω is the angle between the axis and tangents, then (α,β,γ) satisfies the following conditions, which give the bijective parts of the Monge cubid witht respect to the given helix:

- in case $\alpha,\beta,\gamma \neq 0, \pi/2, \pi$,

$$((\mathrm{tg}\alpha\,\mathrm{ctg}\gamma + \mathrm{tg}\beta + \mathrm{tg}\alpha\,\mathrm{tg}^2\beta\,\mathrm{ctg}\gamma)/(-\mathrm{ctg}\alpha - \mathrm{tg}\beta\,\mathrm{ctg}\gamma - \mathrm{tg}\alpha))^2 + ((\mathrm{tg}\alpha\,\mathrm{tg}\beta - \mathrm{ctg}\gamma)/(-\mathrm{ctg}\alpha - \mathrm{tg}\beta\,\mathrm{ctg}\gamma - \mathrm{tg}\alpha))^2 < \mathrm{ctg}^2\omega,$$

- $0<\alpha<\pi,\ \beta=\pi,\ \gamma=\pi/2$,
- in case $0<\alpha<\pi,\ \beta=\pi,\ 0<\gamma<\pi/2\ \pi/2<\gamma<\pi$,

$$\sin^4\alpha\,\mathrm{tg}^2\gamma + \cos^2\alpha\sin^2\alpha\,\mathrm{tg}^2\gamma < \mathrm{ctg}^2\omega,$$

- in case $0<\alpha<\pi/2,\ \pi/2<\alpha<\pi,\ 0<\beta<\pi/2\ ,\pi/2<\beta<\pi,\ \gamma=\pi$,

$$(-\mathrm{tg}\beta - \mathrm{ctg}\beta)^2/\mathrm{ctg}^2\alpha + \mathrm{ctg}^2\beta \leq \mathrm{ctg}^2\omega,$$

- in case $\alpha=\pi/2,\ 0<\beta<\pi/2\ ,\pi/2<\beta<\pi,\ \gamma=\pi/2$,

$$tg^2\beta \leq ctg^2\omega,$$

- in case $\alpha=\pi/2$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $0<\gamma<\pi/2$ $\pi/2<\gamma<\pi$,

$$((ctg\beta + tg\beta) + tg^2\gamma)^2/ctg^2\beta tg^2\gamma \leq ctg^2\omega,$$

- in case $0<\alpha<\pi/2$, $\pi/2<\alpha<\pi$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $\gamma=\pi/2$,

$$tg^2\beta(1 + tg^2\alpha)/(-tg\beta - tg\alpha) \leq ctg^2\omega.$$

3. Description of two helices with perpendicular axes

The axis identical with axis z, the radius is r_1, and the parameter is $\sqrt{3}\, r_1$ of the first helix. Then angle $\omega_1$between the axis and tangents is $\pi/6$. The angle between the axis and generating lines of the k_1 normal cone of the tangential cone equals $\pi/3$. The axis identical with axis y, the radius is r_2, and the parameter is $\sqrt{3}\, r_1$ of the second helix. Then angle ω_2 between the axis and the tangents equals $\pi/4$. Then the angle between the axis and the generating lines of the k_2 normal cone of the tangential cone equals $\pi/4$. It is shown in Figure 4.

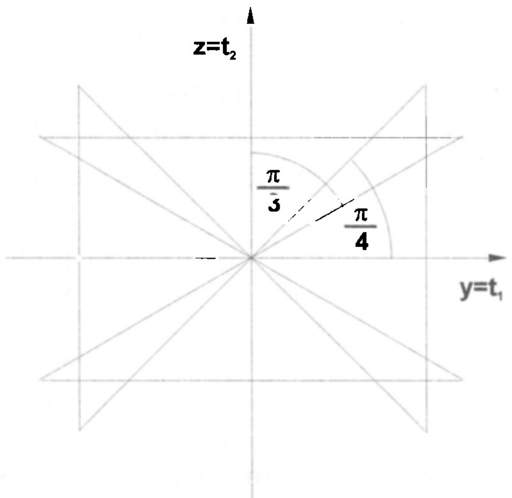

Figure 4. The k_1 and k_2 normal cones

Normal lines of profile planes of bijective Monge projections to given helices are simultaneously inside k_1 and k_2 normal cones. The bijective part of the Monge cubid to the given helices can be determine by the following:

- in case $\alpha,\beta,\gamma \neq 0, \pi/2, \pi$,

$$((tg\alpha ctg\gamma + tg\beta + tg\alpha tg^2\beta ctg\gamma)/(-ctg\alpha - tg\beta ctg\gamma - tg\alpha))^2 +$$
$$((tg\alpha tg\beta - ctg\gamma)/(-ctg\alpha - tg\beta\, ctg\gamma - tg\alpha))^2 \leq ctg^2\omega_1=3,$$

and the

$$(\mathrm{ctg}\beta + \mathrm{ctg}\alpha\mathrm{tg}\gamma + \mathrm{tg}\beta)^2/(\mathrm{tg}\gamma - \mathrm{ctg}\alpha\mathrm{ctg}\beta)^2 +$$
$$(-\mathrm{ctg}\alpha - \mathrm{tg}\beta\ \mathrm{ctg}\gamma - \mathrm{tg}\alpha)^2/(\mathrm{tg}\gamma - \mathrm{ctg}\alpha\mathrm{ctg}\beta)^2 \le \mathrm{ctg}^2\omega_2 = 1,$$

- in case $0<\alpha<\pi$, $\beta=\pi$, $0<\gamma<\pi/2$ $\pi/2<\gamma<\pi$,

$$\sin^4\alpha\mathrm{tg}^2\gamma + \cos^2\alpha\sin^2\alpha\mathrm{tg}^2\gamma \le \mathrm{ctg}^2\omega_1=3,$$

and

$$\mathrm{tg}^2\alpha + \mathrm{ctg}^2\gamma/\cos^2\alpha\sin^2\alpha \le \mathrm{ctg}^2\omega_2 = 1,$$

- in case $0<\alpha<\pi/2$, $\pi/2<\alpha<\pi$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $\gamma=\pi$

$$(-\mathrm{tg}\beta - \mathrm{ctg}\beta)^2/\mathrm{ctg}^2\alpha + \mathrm{ctg}^2\beta \le \mathrm{ctg}^2\omega_1=3,$$

and

$$(-\mathrm{tg}\beta - \mathrm{ctg}\beta)^2/\mathrm{ctg}^2\alpha\ \mathrm{ctg}^2\beta + \mathrm{tg}^2\beta \le \mathrm{ctg}^2\omega_2 = 1,$$

- in case $\alpha=\pi/2$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $\gamma=\pi/2$

$$\mathrm{tg}^2\beta \le \mathrm{ctg}^2\omega_1=3,$$

and

$$\mathrm{ctg}^2\beta \le \mathrm{ctg}^2\omega_2 = 1,$$

- in case $\alpha=\pi/2$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $0<\gamma<\pi/2$ $\pi/2<\gamma<\pi$,

$$((\mathrm{ctg}\beta + \mathrm{tg}\beta) + \mathrm{tg}^2\gamma)^2/\mathrm{ctg}^2\beta\mathrm{tg}^2\gamma \le \mathrm{ctg}^2\omega_1=3,$$

and

$$((\mathrm{ctg}\beta + \mathrm{tg}\beta) + \mathrm{tg}^2\gamma)^2/\mathrm{ctg}^2\beta\mathrm{tg}^2\gamma \le \mathrm{ctg}^2\omega_2 = 1,$$

- in case $0<\alpha<\pi/2$, $\pi/2<\alpha<\pi$, $0<\beta<\pi/2$,$\pi/2<\beta<\pi$, $\gamma=\pi/2$,

$$\mathrm{tg}^2\beta(1 + \mathrm{tg}^2\alpha)/(-\mathrm{tg}\beta - \mathrm{tg}\alpha) \le \mathrm{ctg}^2\omega_1=3,$$

and

$$\mathrm{tg}^2\beta + (-\mathrm{tg}\beta - \mathrm{tg}\alpha)^2/\mathrm{tg}^2\beta\mathrm{tg}^2\alpha \le \mathrm{ctg}^2\omega_2 = 1.$$

4.Examples

The $\alpha=\pi/3$, $\beta=\pi/6$, $\gamma=\pi/18$ do not satisfy the above conditions. The Monge projection, which is determined by the given triplet of number is not bijective. There exist some pieces of helices that are non-bijective. It is shown Figure 5.

The $\alpha=\pi/2$, $\beta=\pi/4$, $\gamma=\pi/2$ triplet of real numbers satisfies the conditions. In that Monge projection, which is determined by them, all pieces of helices can only be reproduced by two perpendicular images. See Figure 6.

The $\alpha=\pi/3$, $\beta=\pi/6$, $\gamma=\pi/2$ triplet of real numbers satisfies the first condition and does not satisfy the second condition. In that Monge projection, which is determined by them, all pieces of the first helix is bijective and a non-bijective piece of the second helix exists. Figure 7.

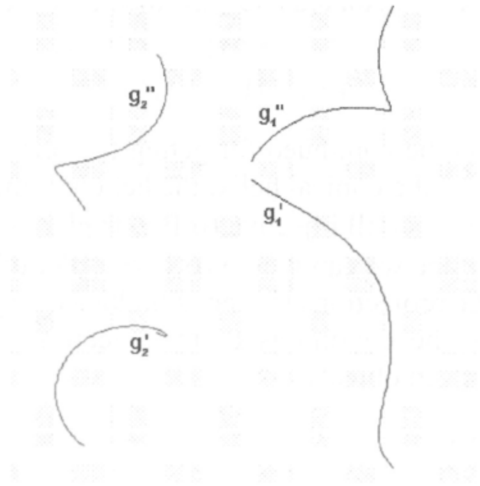

Figure 5. The helices in non-bijective position

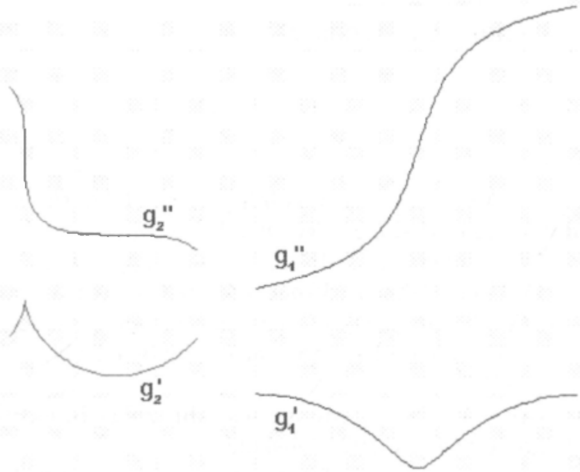

Figure 6. The helices in bijective position

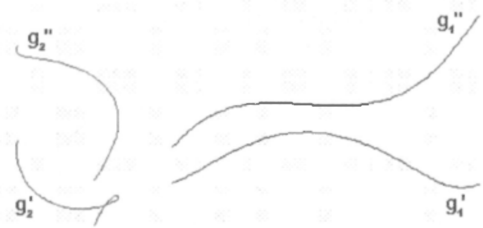

Figure 7. The helices in non-bijective and bijective position

The Figure 5, 6, 7 are calculated and described by C computer language.

5. Conclusion

The analysis can be continued for geting the bijetive part of the Monge cubid with respect to the conical helix, the helicoid, the conical helicoid and the worm gear. If we get all bijective (α,β,γ) triplet of numbers with respect to any curve or surface we can search (α,β,γ) triplet of numbers, which give the bijective Monge projections to them simultaneously.

We can study the conditions of the bijectivity in the given Monge projections to the given objects.

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