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ELASTICALLY SUPPORTED CYLINDER IN TWO-DEGREE-OF-FREEDOM MOTION: A NUMERICAL STUDY

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ABSTRACT

This study deals with the numerical simulation of two-dimensional, Newtonian fluid flow around an elastically supported circular cylinder free to vibrate in two directions. The governing equations with the boundary and initial conditions are solved using the finite difference method. During the computations, mass ratio was chosen to be $m^*=10$ and structural damping was set to zero ($\zeta=0.0$) in order to obtain high amplitude oscillations. To validate the newly extended in-house code, first the effect of Reynolds number Re and reduced velocity U^* was studied independently. Then, in order to keep the natural frequency of the cylinder f_N at a constant value, Reynolds number was varied inversely with the reduced natural frequency F_N, where $F_N=1/U^*$. Computational results compare well with those available in the literature. A comparison between free and forced vibration is also carried out. Vorticity contours and drag-lift limit cycle curves for a small and a large amplitude oscillation case are shown in the paper. Vorticity contours compare well; agreement between limit cycles is better when the free vibration is damped.

INTRODUCTION

Flow around a cylinder is widely studied using both numerical and experimental means. The vortices shedding from a body can induce high amplitude oscillations which can cause serious damage to the structure. This phenomenon played an important role, for example, in the collapse of Tacoma-Narrows Bridge. Two basic models exist for the investigation of flow around an oscillating cylinder: *free vibration*, where the cylinder displacement is due to forces acting on the body, and *forced vibration* where the body is forced to follow in a predetermined path.

In this paper two-degree-of-freedom motion (2DoF) is studied, where the cylinder is allowed to move in both streamwise and transversely to the main stream. Non-dimensional parameters are introduced, namely the mass ratio m^*, the structural damping ζ and the reduced natural frequency F_N of the cylinder (their definitions can be found e.g. in [1]). Jauvtis and Williamson [2] and Raghavan and Bernitsas [3] carried out experimental studies at medium Reynolds numbers Re (Re=$U_\infty d/\nu$, where U_∞ is the free stream velocity, d is the cylinder diameter and ν is the kinematic viscosity of the fluid) for a cylinder undergoing 2DoF free vibration. Singh and Mittal [4] investigated the effect of Re and reduced velocity U^* separately, where $U^*=1/F_N$ and F_N is the reduced natural frequency. They carried out two sets of computations for (1) Re=100 while varying U^* and (2) $U^*=4.92$ while changing Re. Hysteresis appeared at the lower and higher range of the synchronization regime. Prasanth et al. [5] showed that hysteresis loops disappear by decreasing the blockage (the ratio of cylinder diameter and the height of the computational domain). It is well known that when keeping the natural frequency of the cylinder f_N at constant value, the reduced natural frequency of the body F_N changes inversely with the Reynolds number ($F_N=K/\mathrm{Re}$, where $K=f_N d^2/\nu=\mathrm{const}$). Prasanth and Mittal [6] investigated $K=16.6$ cases where f_N was the vortex shedding frequency for a stationary cylinder at Re=100.

Forced oscillation is also a widely used model for predicting aerodynamic forces. Vibrating the structure in 2DoF, different cylinder paths can be obtained depending on the dimensionless amplitudes (A_x, A_y), frequencies (f_x, f_y) and phase angle (θ) between the streamwise and transverse motion. Didier and Borges [7] investigated circular paths where both the oscillation amplitudes and frequencies were identical ($A_x=A_y$ and $f_x=f_y$). Baranyi [8] investigated elliptical paths at the Reynolds number range of Re=120-180. Baranyi [9] and Peppa et al. [10] carried out numerical studies on figure-8 paths where the frequency of cylinder oscillation in streamwise direction is double that of the transverse direction ($f_x=2f_y$).

In this study fluid flow around a circular cylinder in 2DoF free vibration is investigated using numerical approach. One objective of this paper is to validate the currently extended in-house code by comparing results obtained using the code and those available in the literature. There are only a few papers dealing with the comparison of free and forced vibrations. A second objective is to carry out comparisons between the results obtained for free and force oscillations in order to investigate how well forced vibration can model free vibration.

COMPUTATIONAL METHOD

The non-dimensional governing equations for the two-dimensional, laminar, constant property, Newtonian, incompressible fluid flow around a cylinder undergoing two-degree-of-freedom (2DoF) free vibration are the two-components of the Navier-Stokes equation – written in a non-inertial system fixed to the oscillating body, continuity equation and Poisson equation for pressure. Figure 1 shows the physical and computational domains, where R_1 is the cylinder radius and R_2 is the radius of the outer surface, where the flow is assumed to be undisturbed. For the velocity Dirichlet-type and for the pressure Neumann-type boundary conditions are applied both on the cylinder and outer surfaces.

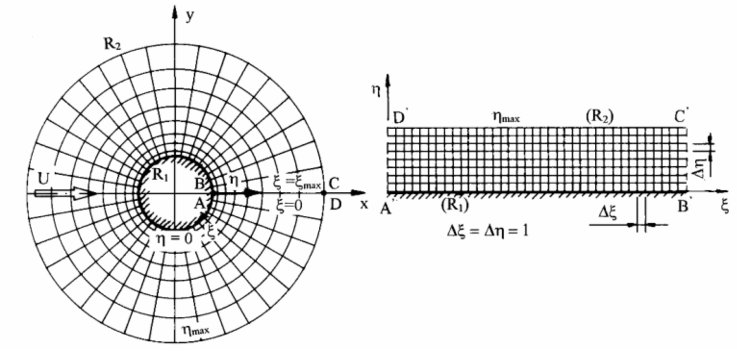


Figure 1
The physical and computational domains

The motion of the cylinder in streamwise and transverse directions are governed by the following two dimensionless equations, [1]:

$$\ddot{x}_0 + 4\pi F_N \zeta \dot{x}_0 + (4\pi F_N)^2 x_0 = \frac{2C_D(t)}{\pi m^*}, \tag{1}$$

$$\ddot{y}_0 + 4\pi F_N \zeta \dot{y}_0 + (4\pi F_N)^2 y_0 = \frac{2C_L(t)}{\pi m^*}. \tag{2}$$

In these equations dots and double dots mean first and second derivatives with respect to dimensionless time t (non-dimensionalized by d/U_∞). In Eqs. (1) and (2) $x_0, \dot{x}_0, \ddot{x}_0$ are the dimensionless streamwise displacement, velocity and acceleration of the cylinder, and $y_0, \dot{y}_0, \ddot{y}_0$ are the corresponding quantities for the transverse direction. In Eqs. (1) and (2) $m^*=4m/(d^2\pi\rho)$ is the mass ratio, where m is the mass of the cylinder, d is the cylinder diameter and ρ is the fluid density, $F_N=f_N d/U_\infty$ is the reduced velocity, where U_∞ is the free stream velocity and f_N is the natural frequency of the cylinder. Note that reduced velocity U^* is often used instead of F_N, defined as $U^*=1/F_N$. In these equations ζ is the structural damping ratio of the body and C_D and C_L are the drag and lift coefficients.

The unsteady lift and drag coefficients are defined as

$$C_L = \frac{2F_L}{\rho U_\infty^2 D}, \qquad C_D = \frac{2F_D}{\rho U_\infty^2 D}, \tag{3}$$

where F_L and F_D are the lift and drag per unit length of the cylinder, respectively.

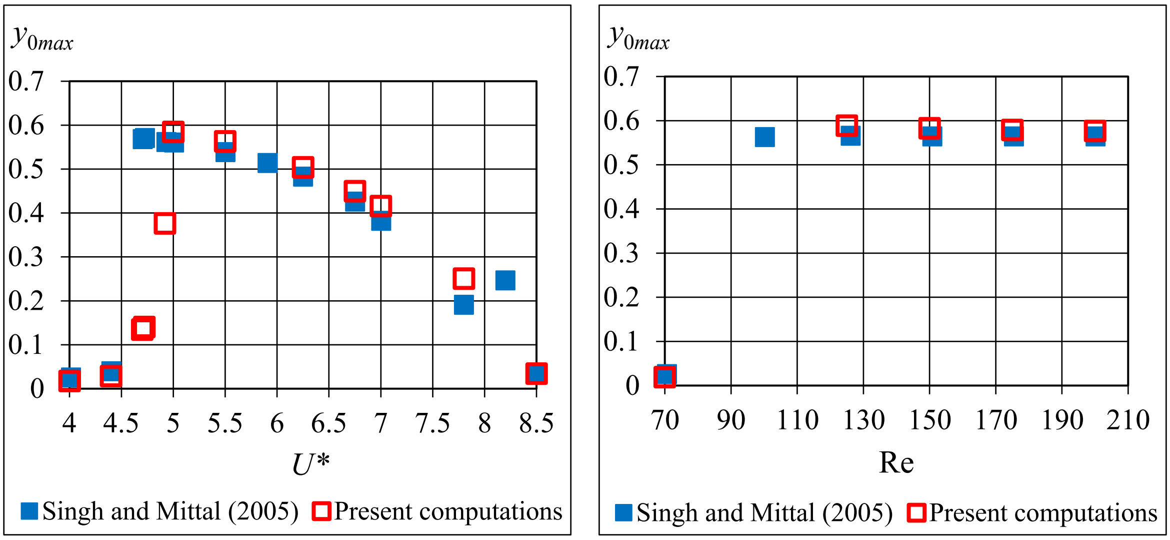
In order to impose boundary conditions accurately and to avoid numerical inaccuracies, boundary-fitted coordinates are used. The physical domain is mapped into a rectangular computational domain applying linear mapping functions [8]. The advantage of the chosen linear mapping functions is that the grid on the physical plane is very fine in vicinity of the cylinder and coarse in the far field, but equidistant over the computational domain. The transformed governing equations with the boundary conditions are solved using finite difference method [8]. The space derivatives are discretized using fourth order schemes except for the convective terms, which are approximated by a third order upwind difference scheme. The Poisson equation is solved using successive over-relaxation (SOR), the equations of motion and the structural equations are integrated explicitly and continuity equation is satisfied at each time step. The computational grid is generated only once.

During the computations the radius ratio $R_2/R_1=160$ and the computational grid is characterized by grid points 360×292 (peripheral × radial) and the dimensionless time step Δt is 0.0005.

COMPUTATIONAL RESULTS

In this study flow around a cylinder undergoing 2DoF free vibration is investigated numerically. The mass ratio was chosen to be $m^*=10.0$ and in order to encourage high amplitude oscillations the structural damping was set to zero ($\zeta=0.0$). The maximum oscillation amplitude (often called cylinder response) in transverse direction y_{0max} and vorticity contours are shown. Mechanically oscillated cylinder model is widely used to predict aerodynamic forces acting on the body (e.g. [7-10]). The second objective of this paper is to investigate any similarities between the results obtained from free and forced vibration models. For this aim instantaneous vorticity contours and limit cycle curves (C_D,C_L) are compared.

First the effect of Reynolds number and reduced velocity U^* is investigated. In correspondence with [4], two sets of computation are carried out: (1) for Re=100 while varying U^* and (2) for U^*=4.92 while varying Re. The maximum transverse oscillation amplitude $y_{0\max}$ is shown in Fig. 2. As seen in the figure, good agreement was found between the current results and values in [4].



(a) Re=100 (b) U^*=4.92

Figure 2

The maximum oscillation amplitude $y_{0\max}$ in transverse direction for 2DoF free vibration; a comparison with [4]

Few studies (e.g. [6]) have dealt with the investigation of constant natural frequency f_N cases. In order to keep f_N at a fixed value, Re is varied inversely with F_N as follows:

$$\mathrm{Re} = \frac{K}{F_N}, \tag{4}$$

where $K=f_N d^2/\nu$ is constant value. Prasanth and Mittal [6] carried out computations for K=16.6, where f_N matched the vortex shedding frequency from a stationary circular cylinder at Re=100. In Fig. 3 the maximum oscillation amplitude $y_{0\max}$ in transverse direction is compared with values obtained in [6]. As can be seen in the figure, excellent agreement is obtained.

Snapshots of vorticity contours (where vortices marked in red are positive and those in blue are negative) are shown in Fig. 4 for Reynolds numbers Re=90 and 180 at dimensionless time t=800. In both cases two single vortices are shed in each motion cycle. This configuration is called the 2S mode of vortex shedding (see Fig. 4b). Williamson and Roshko [11] found that for high-amplitude transverse vibrations the vortices in the cylinder wake coalesce, which is referred as C(2S) mode. In our case high-amplitude motion is observed at Re=90 (see Fig. 4a) and the oscillation amplitude was low at Re=180 (see Fig. 4b). Significant decrease in vortex shedding frequency – the distance between the vortices with the same sign – can also be seen in the contour plots (see Fig. 4).

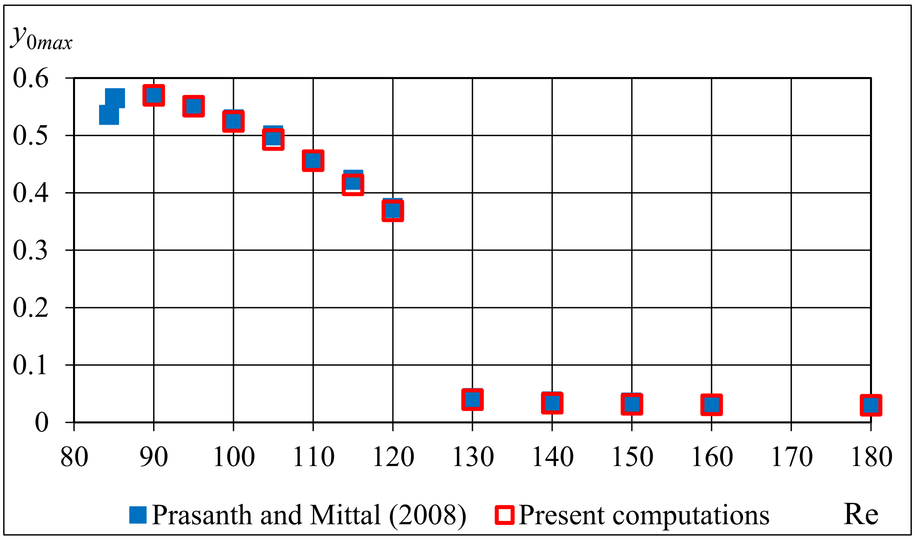


Figure 3
The maximum oscillation amplitude $y_{0\max}$ in transverse direction against Re at m^*=10.0 and ζ=0.0; a comparison with [6]

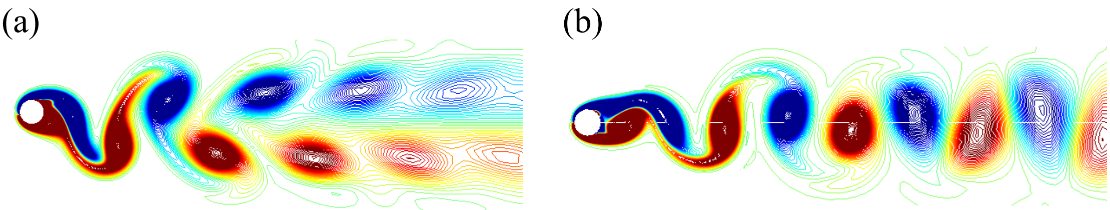


Figure 4
Vorticity contours for Re=90 (a) and 180 (b), m^*=10.0 and ζ=0.0 at t=800

Aerodynamic forces are often predicted using a forced vibration model. In this case the cylinder is mechanically moved and follows a predetermined path. The displacement of the cylinder in x and y directions can be written as

$$x_0(t) = x_{0max}\cos(2\pi f_x t + \theta), \quad y_0(t) = y_{0max}\sin(2\pi f_y t), \tag{5}$$

where $x_{0\max}$, f_x are the non-dimensional oscillation amplitude and frequency values in streamwise direction; $y_{0\max}$, f_y are the corresponding quantities in transverse direction. In Eq. (5) θ is the phase shift between streamwise and transverse motions. The computational methods for free and forced cylinder motions are very similar. The main difference is that for forced motion the cylinder displacement is described by Eq. 5, while for free vibration the cylinder displacement is determined by Eqs. (1) and (2) and caused by fluid forces.

The question arises how well forced vibration can model free vibration. To examine this, computations are carried out using free cylinder oscillation model.

From the time histories of the cylinder displacement the dimensionless amplitudes, frequencies and phase shift between in-line and transverse displacement are computed. Several cases were investigated, but only two of them are presented here (Table 1): low and high amplitude cylinder oscillations. Using the parameters contained in the table, forced vibration computations are carried out.

Table 1
Cylinder responses obtained from free cylinder oscillation at Re=160 and m^*=10,where St_0=0.1882 is the dimensionless vortex shedding frequency at Re=160 for stationary cylinder

F_N	$\zeta=\zeta_x=\zeta_y$	x_{0max}	f_x	y_{0max}	f_y	θ [°]
$0.8St_0$	0.01	0.0003851	0.3531	0.0583	0.1765	107.7961
St_0	0.0	0.007735	0.3715	0.5454	0.1857	46.1575

Figure 5 shows the instantaneous vorticity contours for free and forced vibrations. In both oscillation models (free and forced) the 2S mode of vortex shedding occurs, as expected. It was shown previously in Fig. 4 that at high oscillation amplitudes the vortices coalesce in the wake of the cylinder, in contrast to low-amplitude cylinder oscillation. This corresponds to the currently investigated cases, where Fig. 5b shows a C(2S) mode of vortex shedding – where y_{0max} is high – and in Fig. 5a – where y_{0max} is low – the typical 2S vortex shedding mode can be seen.

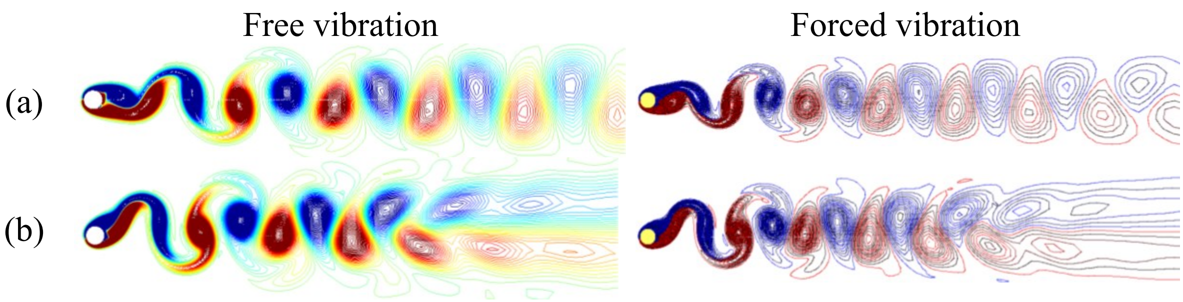
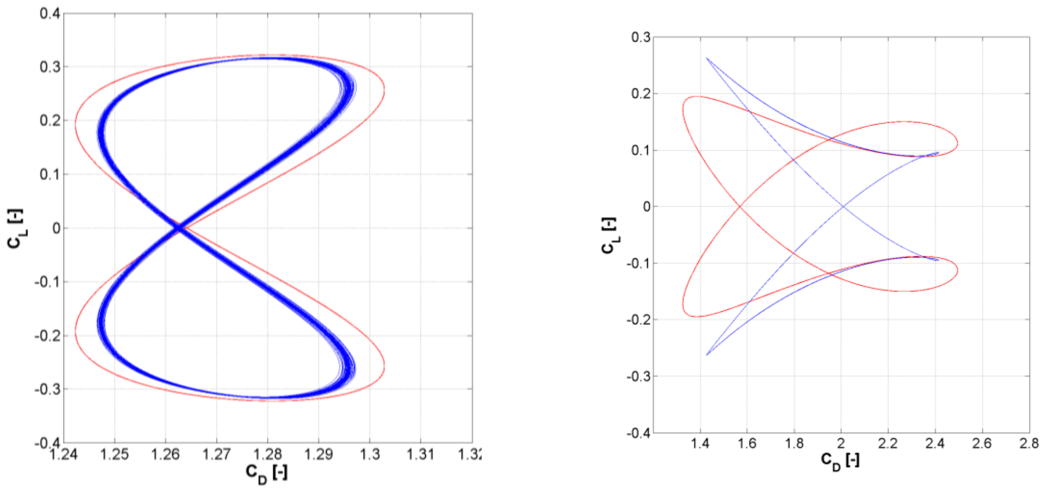


Figure 5
Vorticity contours for free and forced vibration at t=800
(a) F_N=0.15056, ζ=0.01; (b) F_N=0.1882, ζ=0.0

Limit cycle curves (C_D,C_L) for free and forced vibration can be seen in Fig. 6. The agreement between the two (C_D,C_L) plots is better for damped (ζ=0.01) cylinder oscillation, shown in Fig. 6a. For undamped (ζ=0.0) vibrations the agreement is poorer (see Fig. 6b). These preliminary results are somewhat promising, but the identification of conditions under which the two models show good agreement requires further investigation.



(a) F_N=0.15056, ζ=0.01 (b) F_N=0.1881, ζ=0.0

Figure 6
Limit cycle curves for free and forced vibration.
Red: forced vibration; Blue: free vibration

CONCLUSIONS

This two-dimensional numerical study deals with the investigation of flow around a cylinder undergoing two-degree-of freedom free vibration and placed in an otherwise uniform stream of incompressible constant property Newtonian fluid. The newly extended in-house code is tested against results available in the literature. During the computations the mass ratio and the structural damping was set to m^*=10 and ζ=0.0, respectively. First the reduced natural frequency and Reynolds number are varied independently. Then, in order to keep the natural frequency of the cylinder f_N at a constant value, Re was varied inversely with reduced natural frequency F_N. The results compare well with those in the literature. Free and forced vibration models are also investigated and results obtained are compared with each other. It was found that the vorticity contours agree well with each other, while limit cycle curves match better for the damped case. The identification of conditions under which the two models show good agreement requires further investigation.

ACKNOWLEDGEMENTS

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