Theoretical Analysis of the Contact Area between Grinding Wheel Surface and Workpiece in Flat Face Grinding with Spindle Axis Inclination

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Theoretical analysis has been carried out for the determination of the analytical dependences connecting various parameters of contact area between wheel cutting surface and workpiece, such as length, width and arc length in the case of flat face grinding with preliminary inclination of spindle axis. The role of factors, such as angle of preliminary inclination of the spindle axis, grinding depth and grinding wheel diameter, in this process, are established. The capability to define the above mentioned parameters permits the calculation of the contact area between wheel cutting surface and workpiece. In addition, with the proposed methodology, it is possible to correctly

determine the value of cross-feed, in the case of multiple-pass processing scheme, which, as it is known, should be consistent with the value of contact width of wheel cutting surface with workpiece. It can be guaranteed that on the ground surface there will be no areas unaffected by the wheel. In the case of through-feed grinding the obtained theoretical dependences make it possible to determine the processing conditions, taking into account the allowable value of flatness deviation. Finally, the latter, contributes in improving flat face grinding process and thus expanding its technological capabilities.

Keywords: wheel cutting surface, contact area, spindle axis inclination, grinding depth, wheel diameter

1 Introduction

Grinding is regarded as a finishing manufacturing operation because of the high surface quality obtained in the finished product and its characteristic to be able to perform precision and high precision processing [1], even for hard-to-machine materials [2]. Furthermore, processing of flat surfaces is realized through grinding. The prospects of flat scheme of grinding are confirmed by the fact that a large number of parts in mechanical engineering have flat surfaces; the parts are subjected to various kinds of processing, i.e. snagging, pre-grinding, final grinding and precision grinding, on surface-grinding machines [3-6]. Flat face grinding holds a prominent position in this process, because it offers a number of advantages, compared to the scheme based on the use of peripheral grinding wheels [7].

It is known that grinding with end face of wheel is characterized by relatively large value of contact arc and interaction surface between wheel cutting surface (WCS) and workpiece, resulting in high temperature in the grinding area [4-6]. Therefore, contact area can be taken as a parameter used to control heat intensity of grinding process. One of the most effective processing methods in order to control contact area between WCS and workpiece is the preliminary inclination of spindle axis of the machine [8-10]. Wheel posture is known to influence surface profile of the workpiece and grinding forces [11]. In the relevant literature, wheel inclination has been employed in the cases of manufacturing aspheric [12], parabolic and toroidal [13], spherical convex [14], spherical [15] and conical [16] surfaces. Shiha and Lee [17] proposed mathematical models reflecting the impact of processing conditions on the flat face grinding using machines with vertical spindle, i.e. the spindle axis is pre-tilted in the direction of longitudinal feed, on form deviation of flat surfaces on the one hand and a trajectory left by abrasive grains on the ground surface on the other hand, in order to face the problem of sealing parts under high pressure in the fuel systems of diesel engines.

Spindle angle adjustments were used for the fine grinding of silicon wafers [18] and the avoidance of undercutting in precision grinding of threads, screws and gears [19]. Inclination of a cup wheel for flattening and thinning of silicon wafers in infeed grinding and the capability of the process to produce various non-flat shapes was analysed by Huo et al. [20]. Feng et al. [21] presented the calibration of the spindle tilt angle of a cup wheel in order to fit the shallow taper angle of a conical seal ring for its processing. Uhlmann et al. [22] used tilt surface grinding to improve roughness values produced by oscillating surface grinding. Tilt form grinding was employed in the manufacturing of the rotor of a dry vacuum pump to overcome the problem of concave profile grinding [23] and single point inclined axis grinding technique was studied to grind aspheric moulds of high profile accuracy [24]. The features of forming flat surfaces when using the scheme of through grinding and under conditions of multiple-pass grinding were considered in [25].

Analysis of literature sources reveals that there is no data regarding theoretical approaches for defining linear parameters of contact area between face grinding wheel and workpiece under conditions of preliminary inclination of spindle axis. This is a drawback that does not allow an engineer to take full advantage of the processing scheme of grinding with preliminary inclination of spindle axis. Therefore, the analysis presented in this paper intends to fill this gap in a grinding technique with many applications in contemporary industry. The aim of the research is to establish theoretical dependences for calculating linear parameters of contact area between WCS and workpiece at flat face grinding with preliminary inclination of spindle axis. In the following sections, the contact length, the contact width and the arc length contact between WCS and workpiece is theoretically studied and the contact area between WCS and workpiece is determined. The analysis is concluded with the presentation of a computer-aided geometric 3D model, validated through simulation and comparison of the obtained results with different methods.

2 Flat face grinding features

Flat face grinding can be realized both on machines with vertical and horizontal position of spindle. Depending on the direction of longitudinal feed the process can be carried out according to usual f_u or deep f_d scheme, see Fig. 1.

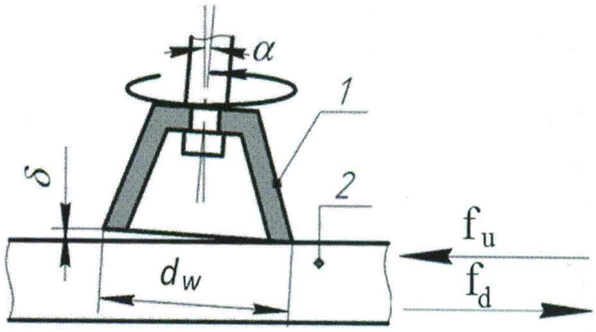


Fig. 1 *Scheme of flat face grinding with inclination of spindle axis (1 – face grinding wheel; 2 – workpiece)*

The inclination of spindle axis at angle α leads to changes in a number of parameters of the contact area between WCS and workpiece and affects the shaping of the surface to-be-machined as well. Length W′, arc length L and contact width B′ between WCS and workpiece can be considered as parameters of contact area in case of deep scheme, as is depicted in Fig. 2.

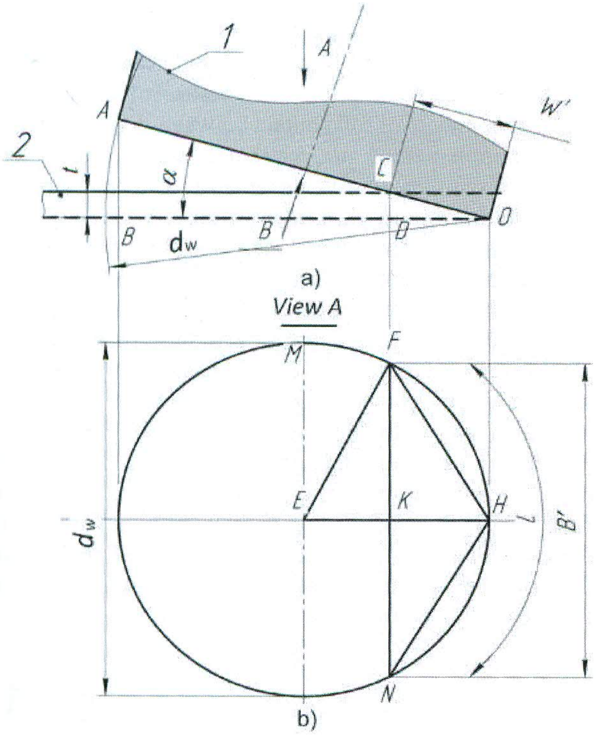
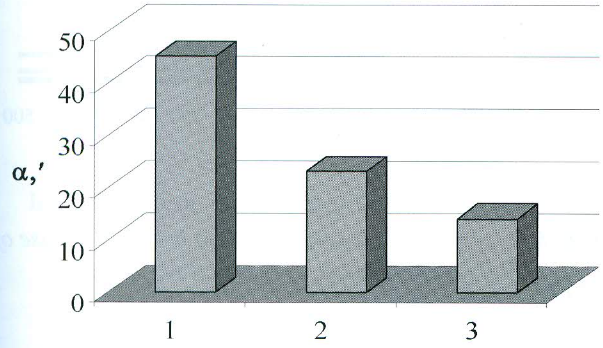


Fig. 2 *Scheme for calculating the contact area parameters of wheel cutting surface and workpiece*

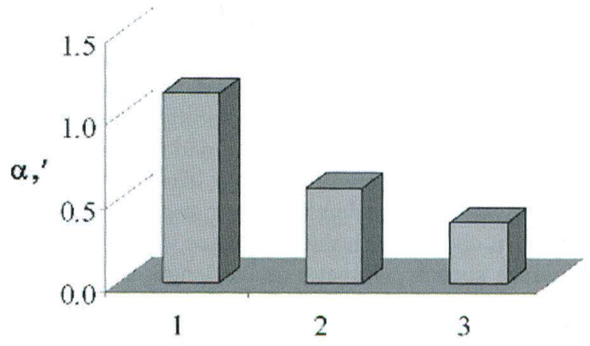
In this paper, the deep scheme of grinding is considered, although some findings also concern usual grinding scheme and even face milling.

2.1 Contact length between WCS and workpiece

It is recommended to incline spindle axis so that the gap value δ should be equal to 0.05 mm at finishing and 2 mm at roughing types of machining [6], i.e. the data is not tied to the wheel diameter. The latter injects some uncertainty in determining angle α, as is clearly illustrated by the data of Fig. 3 [8]. The diagrams indicate that the value of the angle does not exceed 1° in the ranges of values of gap δ and diameters d_w from 0.05 to 2 mm and 150 to 500 mm, respectively.



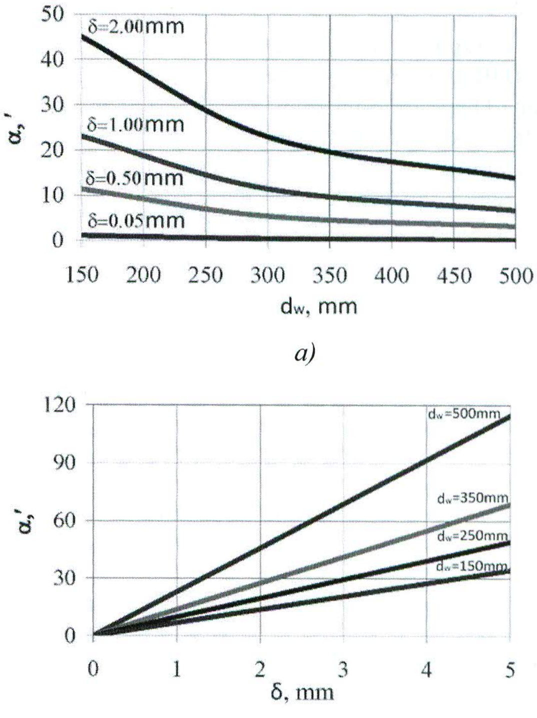
a)



b)

Fig. 3 *Inclination angle α versus wheel diameter for (a) δ=2 mm and (b) δ=0.05 mm (1 – d_w=150 mm; 2 – d_w=300 mm; 3 – d_w=500 mm)*

Furthermore, it can be seen that the dependences $\alpha=f(d_w)$ at constant δ are nonlinear and the dependences $\alpha=f(\delta)$ for constant d_w are linear, see Fig. 4.



a)

b)

Fig. 4 *Dependence (a) $\alpha=f(d_w)$ and (b) $\alpha=f(\delta)$.*

It should be noted that these data are not in agreement with the recommendations given in [4], which proposed the inclination angle α to be between 2° and 4°. Therefore, the recommendations regarding the values of the angles that are cited in literature sources of face grinding process with preliminary inclination of spindle axis show that the range of the angles should be reviewed taking into account the influence of other parameters, e.g. wheel diameter and depth of grinding. This will make it possible to develop scientifically-based recommendations for improving grinding process on machines with vertical or horizontal spindle, e.g. for sharpening operations in industry.

The analysis shows that parameters of contact area between grinding wheel and workpiece are mainly determined by three factors, namely, wheel inclination angle α, depth of grinding t and wheel diameter d_W. In Fig. 2 the scheme of contact between face grinding wheel 1 and the workpiece 2, in the case of the inclination of spindle axis to angle α, is presented. With the increase in angle α, providing a constant depth of cut t, a part of WCS that will be in contact with the workpiece, decreases. Let us introduce such a parameter as the length of contact between WCS and workpiece (W′), which is an important characteristic of the deep scheme processing. As shown in Fig. 2, the length W′ depends on depth of grinding t and angle α. With the increase in angle α, providing a constant depth of cut t, a part of WCS that will be in contact with the workpiece, decreases, according to the law:

$$W' = \frac{t}{\sin\alpha} \qquad (1)$$

In preliminary processing, when the value of angles of spindle axis inclination are, for example, in the range of 1° to 4°, only a part of WCS width will be in contact with the workpiece. According to eq. (1), in the range of depth t=0.01-0.09 mm, W′ will vary between 0.15 and 5.17 mm. There are obviously specific combinations of parameters α, d_w and t at which WCS will be in full contact with workpiece provided that the width of workpiece to-be-ground is not smaller than the wheel diameter, i.e. $B \geq d_W$; in this case the condition $W' = d_W$ should be met.

In case of symmetric grinding the formula for determining angle α at which there will be a full contact between WCS and workpiece is determined, as shown in Fig. 5, as:

$$a = \arcsin\left(\frac{AB}{BC}\right) = \arcsin\left(\frac{t}{d_w}\right) \qquad (2)$$

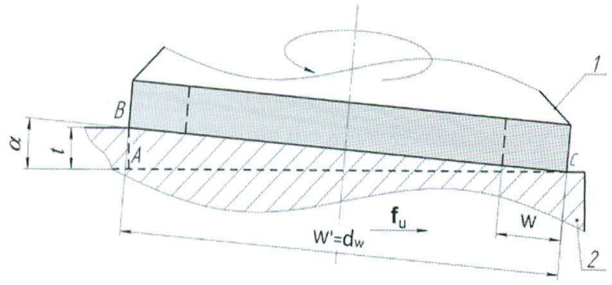
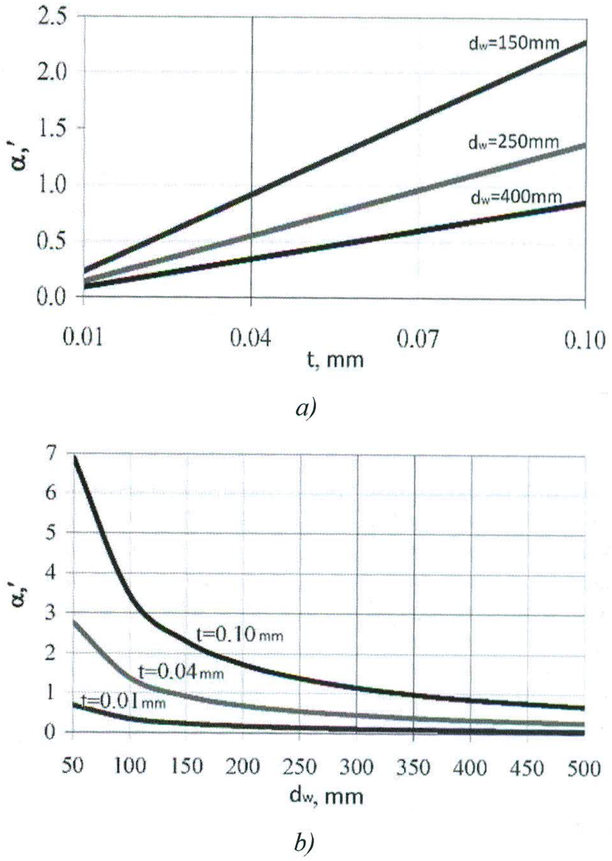


***Fig. 5** Case of full contact between WCS and workpiece (1 – wheel; 2 – workpiece)*

In case of usual grinding scheme, the full contact between WCS and workpiece takes place when α=0. However, the main allowance is removed only with a part of WCS, on which the taper lead is formed. Another part of WCS, namely the end part, acts as if it cleans the surface. In grinding with inclination of spindle axis, when WCS is deepened into the body of workpiece to a depth t, the full contact between WCS and the surface of workpiece will take place at α>0, as well. The graphs presented in Fig. 6(a) and Fig. 6(b) are in agreement with such combination of parameters α, d_w and t, at which WCS is in full contact with workpiece, assuming that the width of workpiece to-be-ground is not smaller than wheel diameter, i.e. $B \geq d_W$. Thus, unlike usual scheme, all WCS will be loaded more evenly, which will have a positive impact on parameters such as the stability of wheel macro-profile.

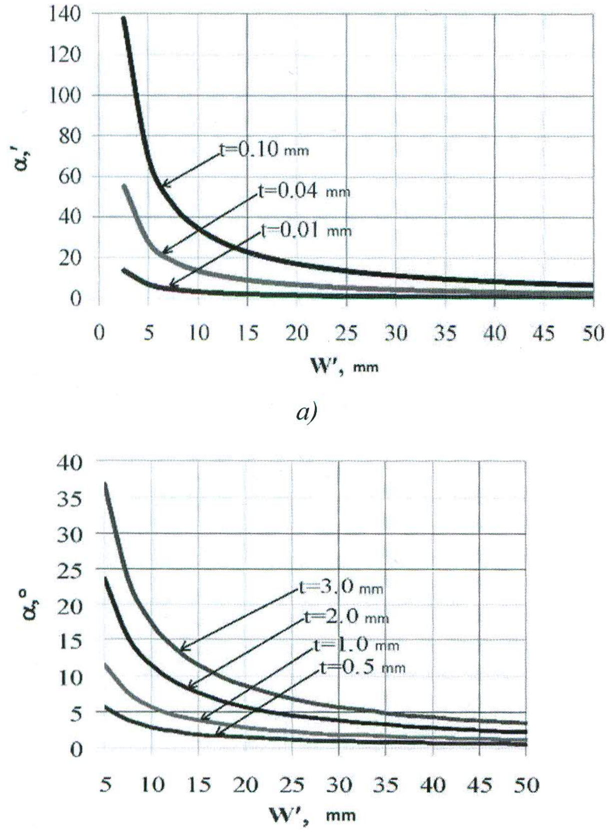
From the graphs of Fig. 6 it can be seen that the deeper the cut of grinding and the smaller the wheel diameter, the greater the angle of preliminary inclination of spindle axis at which there will be full contact between WCS and workpiece and vice versa. The values of angles α, at which the scheme of full contact between WCS and workpiece is implemented, are very small, starting from a few to tens of minutes. If it is necessary to use only a part of face grinding wheel, within the width of WCS, e.g. when sharpening tools, the dependences α=*f*(W′, t) are of interest in theory and practice and can be found in Fig. 7. They indicate that in order to achieve the same value of parameter W′, depending on depth of grinding, inclination angles α vary significantly. Thus, in case of application of usual values of grinding depth, i.e. t≤0.1 mm, the angles of preliminary inclination of spindle axis do not exceed a few degrees, see Fig. 7(a). At the same time, they shall be tens of degrees at increased depth, as shown in Fig. 7(b). Thus, depth of grinding is one of the defining parameters, the value of which, coupled with the value of angle α, determines the length W′ of contact between WCS and workpiece.



a)

b)

***Fig. 6** Dependence (a) α=f(t) and (b) α=f(d_w) in case of full contact between WCS and workpiece*



a)

b)

***Fig.** 7 Dependence α=f(W, t') at (a) usual and (b) increased depth of grinding*

2.2Contact width between WCS and workpiece

Width B′ is essentially a technological parameter of contact area between WCS and workpiece. As shown in [9, 25], it is a determining factor for establishing the required value of cross-feed at multiple-pass face grinding with preliminary inclination of spindle axis and in other cases. Fig. 2 will aid to establish analytical dependences that link parameter B′ to processing conditions. The calculations are made assuming that the value of contact length W′ does not exceed half of wheel diameter, i.e. W′≤(d_w/2).

Fig. 2(b) shows an ellipse as the projection of a circle on the horizontal plane inclined at angle α, see Fig. 2(a)-view A. The fragments FN and FK are the width (B′) and half-width (B′/2) of contact between WCS and workpiece, respectively. From the right-angled triangle EFK, it is:

$$FK = B'/2 = \sqrt{EF^2 - EK^2} \tag{3}$$

In order to determine FK, the canonical equation of ellipse [26], i.e. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is used, where x and y are the coordinates of the point F, see Fig. 2(b). In this case, x=FK= (B′/2) and y=EK. Then eq. (3) will be as follows:

$$\frac{FK^2}{EM^2}+\frac{EK^2}{EH^2}=1 \tag{4}$$

As shown in Fig. 2(b), the larger axis of the ellipse EM has a length equal to a half of wheel diameter, i.e. EM=(d_w/2). At the same time EH represents the smaller semi-axis of the ellipse and can be defined from Fig. 2(a) right-angled triangle AOB as:

$$EH = \frac{OB}{2} = \frac{d_w}{2}\cos\alpha \tag{5}$$

Based on eq. (4), taking into account eq. (5), it is obtained:

$$\frac{B'^2}{d_w^2}+\frac{4\cdot EK^2}{d_w^2\cos^2\alpha}=1 \tag{6}$$

Then, EK can be determined as EK=EH-KH. In turn, KH can be defined from the right-angled triangle COD, as:

$$KH = OD = W'\cos\alpha = \frac{t}{\sin\alpha}\cos\alpha = \frac{t}{\tan\alpha} \tag{7}$$

By combining eq. (5) and eq. (7), then:

$$EK = \frac{d_w}{2}\cos\alpha - \frac{t}{\tan\alpha} \tag{8}$$

Taking into account the last equation, eq. (6) can be written as follows:

$$\frac{B'^2}{d_w^2}+\frac{4\left(\frac{d_w}{2}\cos\alpha-\frac{t}{\tan\alpha}\right)^2}{d_w^2\cos^2\alpha}=1 \tag{9}$$

By expanding the previous equation it may be obtained:

$$B'^2 = \left[1-\frac{4\left(\frac{d_w}{2}\cos\alpha-\frac{t}{\tan\alpha}\right)^2}{d_w^2\cos^2\alpha}\right]d_w^2 \Rightarrow B' = \sqrt{d_w^2-\frac{4\left(\frac{d_w}{2}\cos\alpha-\frac{t}{\tan\alpha}\right)^2}{\cos^2\alpha}} \tag{10}$$

In its final form, eq. (10) is as follows:

$$B' = \sqrt{d_w^2-4\left(\frac{d_w}{2}-\frac{t}{\sin\alpha}\right)^2} \tag{11}$$

The calculations of values of contact width B′ by eq. (11) are shown that they coincide with those obtained by computer-aided geometric simulation [25]. Table 1 presents a fragment of comparative data on different methods of determination of contact width between WCS and workpiece. These data, firstly, corroborate the correctness of analytical formulas, in this case the results of eq. (11), and secondly, show reasonable high accuracy of power dependencies obtained by means of mathematical processing of computer-aided geometric simulation findings [9, 25]; the error does not exceed 3%.

***Tab. 1** Comparative data on different ways of determining the width B′ for d_w=400 mm and t=0.05 mm, where B'_{cs}, B'_{pd} and B'_{ad} are results from computer-aided simulation, power dependence [25] and analytical eq. (11), respectively*

α (°)	0.05	0.25	1	1.5	2.75
B'_{cs} (mm)	280.25	133.45	67.46	55.15	40.78
B'_{pd} (mm)	281.05	129.80	66.72	54.92	41.06
B'_{ad} (mm)	280.25	133.45	67.46	55.15	40.78

2.3 Length of contact arc between WCS and workpiece

Length L of contact arc of WCS with workpiece should be considered as a physical parameter of grinding process. Parameters L and W′ determine the contact area between WCS and workpiece to-be-ground [10]. By using the approximation formula of Huygens [27] in order to establish the analytical dependence of length L of arc FN, see Fig. 2(b), of contact between WCS and workpiece, the following equation may be obtained:

$$L = F\widehat{N} = 2FH + \tfrac{1}{3}\left(2FH - FN\right) \qquad (12)$$

Here, FN is a chord, the value of which is equal to the contact width between WCS and workpiece, i.e. FN=B′. From the right triangle FKH, it can be written: $FH^2=FK^2+KH^2$. Furthermore, FK=(B′/2) and KH=OD=(t/tanα). From these it may be obtained that:

$$FH^2 = \left(\frac{B'}{2}\right)^2 + \left(\frac{t}{\tan\alpha}\right)^2 \Rightarrow FH = \sqrt{\frac{B'^2}{4} + \frac{t^2}{\tan^2\alpha}} \qquad (13)$$

After substitution of eq. (13) into eq. (12) it is derived that:

$$L = 2\sqrt{\frac{B'^2}{4} + \frac{t^2}{\tan^2\alpha}} + \frac{2}{3}\sqrt{\frac{B'^2}{4} + \frac{t^2}{\tan^2\alpha}} - \frac{1}{3}B' \qquad (14)$$

The obtained theoretical dependencies for determining the linear parameters of contact zone of WCS and workpiece can be used for analytical determination of the contact area between WCS with workpiece under grinding conditions with preliminary inclination of spindle axis.

3 Determination of the contact area between WCS and workpiece

Contact area between WCS and workpiece depends on several factors [10]. First of all, those would include inclination angle α, wheel diameter d_w and depth of processing t that define the linear parameters of contact zone of WCS and workpiece, namely W′, B′ and L. For example, width of WCS W, width of workpiece B and processing scheme, i.e. symmetric or asymmetric, can be considered as limiting factors under certain conditions. As an example, the contact area between WCS and workpiece may be determined, assuming that the contact length does not exceed the width of WCS, or from Fig. 8 that it is W′≤W, B′≤B and d_{min}≤B≤d_w. In this case, the contact area represents a portion of a circle or a segment bounded by the arc FN and the chord FN. As one can see from Fig. 8, chord FN=B′ and arc FN=L.

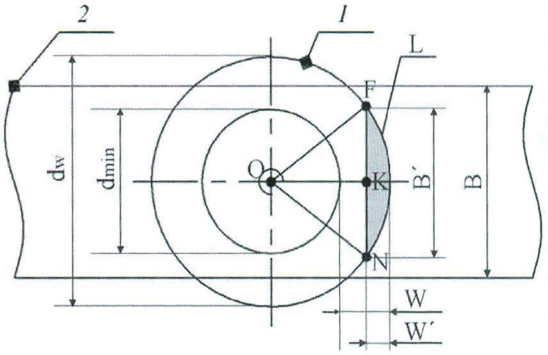


***Fig. 8** Analytical scheme for defining contact area between WCS (1) and workpiece (2) on condition that W′≤W, B′≤B and d_{min}≤B≤d_w.*

As known, there are several ways to determine the area of a segment. Considering the area of a segment A_S as part of the area of a sector, it can be written that $A_S=A_{sector}-A_{triangle}$, where A_{sector} is the area of sector OFN and $A_{triangle}$ is the area of triangle OFN. Then based on Fig. 8 it is derived that:

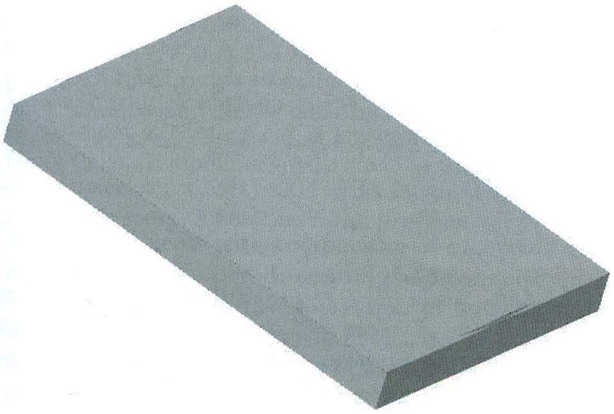
$$A_S = \frac{1}{2}L\frac{d_w}{2} - \frac{1}{2}\cdot FN\cdot OK \qquad (15)$$

Or after the transformations:

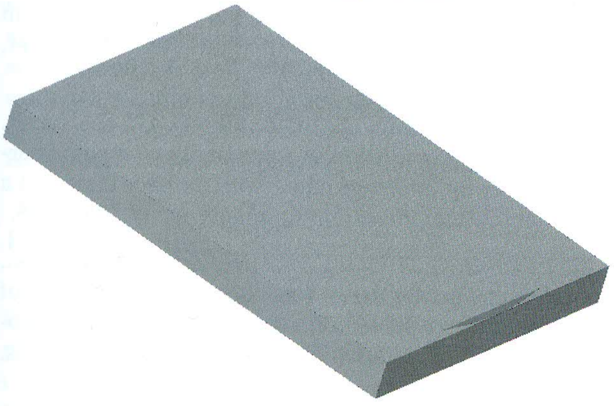
$$A_S = \frac{1}{2}\left[L\frac{d_w}{2} - B'\left(\frac{d_w}{2} - W'\right)\right] \qquad (16)$$

Parameters W′, B′ and L that are included in eq. (16) are calculated according to the eqs (1), (11) and (14), respectively. The scheme presented in Fig. 8 holds true both in the case of multiple-pass grinding and through-feed grinding when with increasing number of passes, in the implementation of vertical feed, condition B′<B remains valid.

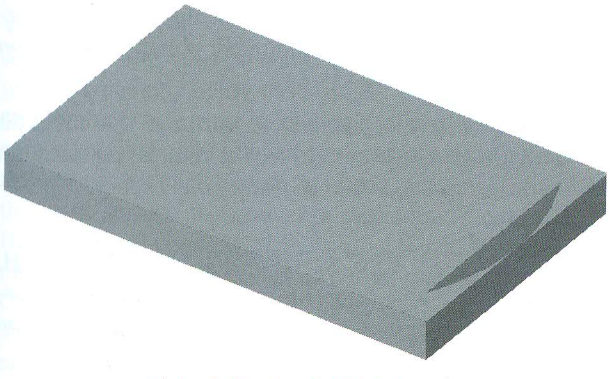
Computer-aided 3D simulation of contact area between WCS and workpiece permits in the environment «COMPASS» not only to verify the theoretical calculations, but also clearly illustrate the ability to control the processing area, as it can be seen from Fig. 9 and Table 2. In Fig. 9, the change in contact area between WCS and workpiece, as inclination angle α is reduced, may be observed.



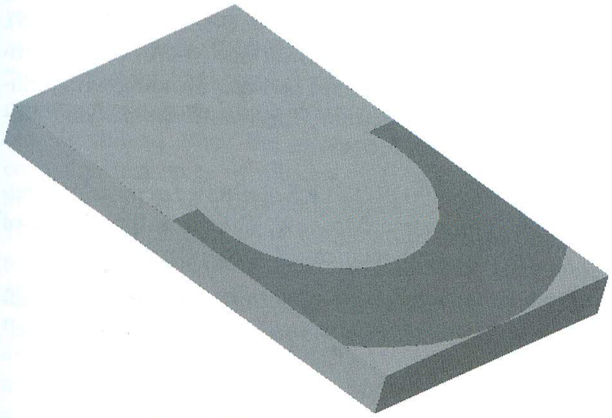
a) α=2.75°; AS=226.08 mm^2



b) α=1.5°; A_S=559.69 mm^2



c) α=0.5°; A_S=2872.78 mm^2



d) α=0.05°; A_S=30859.82 mm^2

***Fig. 9** 3D models representing the influence of inclination of spindle axis on contact area between WCS and workpiece for d_w=400 mm; d_{min}=200 mm; B=250 mm; t=0,2 mm)*

***Tab. 2** Comparative data on different ways of determining the area A_S for d_w=400 mm and t=0.2 mm, where A_{Scs} and A_{Sad} are results from computer-aided simulation and analytical eq. (16), respectively*

α (°)	0.5	1.0	1.5	2.75
A_{Scs} (mm^2)	2875.02	1026.10	559.93	226.25
A_{Sad} (mm^2)	2872.78	1025.11	559.69	226.08

4 Conclusions

Parameters of contact area between grinding wheel and workpiece are mainly determined by three factors, namely, the wheel inclination angle α, grinding depth and wheel diameter. As the angle increases and depth of cut reduces, a part of WCS, which will be in contact with workpiece, decreases. The obtained analytical dependences, which were confirmed by results from computer-aided geometric simulation, can be used in order to calculate length, width, length of arc and contact area between WCS and workpiece. The researches have confirmed the relevancy of the use in practice of the empirical power dependencies obtained earlier [9, 10, 25], but compared to them the analytical dependences give more accurate results in the range of small angles of preliminary inclination of spindle axis, i.e. for α≤0.5°. A considerable part of the results obtained in this paper, can also be applied to the process of flat face grinding with preliminary inclination of spindle axis in accordance with usual scheme of processing, and in case of face milling with preliminary inclination of spindle axis that is known to be used to reduce wear of cutting elements of milling cutters.

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References

[1] NOVAK, M. (2012). Surfaces with high precision of roughness after grinding. *Manufacturing Technology*. 12, 66-70.

[2] LATTNER, R., HOLEŠOVSKÝ, F., KAREL, T., LATTNER, M. (2015). Abrasive Machining of Ti6Al4V Alloy. *Manufacturing Technology*. 15(4), 571-575.

[3] TOENSHOFF, H.K., DENKENA, B. (2013). *Basics of Cutting and Abrasive Processes*. Berlin Heidelberg: Springer-Verlag.

[4] LURIE, G.B., KOMISSARZHEVSKAYA, V.N. (1972). *Grinding machines and their setting-up*. Moscow: Higher school (In Russian).

[5] LOSKUTOV, V.V. (1976). *Grinding machines*. Moscow: Mechanical Engineering (In Russian).

[6] NAERMAN, M.S. (1985). *Handbook of Young Grinder*. Moscow: Higher School (in Russian).

[7] KLOCKE, F. (2009). *Manufacturing Processes 2: Grinding, Honing, Lapping*. Berlin Heidelberg: Springer-Verlag.

[8] KLIMENKO, V.G., PYZHOV, I.M. (2013). Improvement of flat face grinding process. *High Technologies in Mechanical Engineering: Collected Scientific Papers*. 23, 68-79 (In Ukrainian).

[9] PYZHOV, I.M., KLIMENKO, V.G. (2015). Research of contact width between wheel cutting surface and workpiece at flat face grinding with inclination of spindle axis. *Journal of Engineering Sciences: Scientific Journal*. 2(1), A10-A15 (In Ukrainian).

[10] PYZHOV, I.M., KLIMENKO, V.G. (2015). Research of contact area between wheel cutting surface and workpiece at flat face grinding with preliminary inclination of spindle axis. *Journal of Engineering Sciences: Scientific Journal*. 2(2), A1-A6 (In Ukrainian).

[11] NAKAO, M., HATAMURA, Y. (1996). Development of an Intelligent Face Grinding Machine to Fabricate Ultraflat Surfaces on Thin, Brittle Substrates. *Annals of the CIRP*. 45(1), 397-400.

[12] ZHONG, Z. (1992). New Grinding Methods for Aspheric Mirrors with Large Curvature Radii. *Annals of the CIRP*. 41(1), 335-338.

[13] ZHONG, Z., VENKATESH, V. C. (1994). Generation of Parabolic and Toroidal Surfaces on Silicon and Silicon-Eased Compounds Using Diamond Cup Grinding Wheels. *Annals of the CIRP*. 43(1), 323-326.

[14] CHEN, W.K., HUANG, H. (2003). Ultra precision grinding of spherical convex surfaces on combination brittle materials using resin and metal bond cup wheels. *Journal of Materials Processing Technology*. 140, 217–223.

[15] YIN, S., WANG, J., CHEN, F., JIANWU, Y., WANG, Y., ZHAO, Q., LI, H. (2011). Inclined Axis Ultra-Precision Grinding for Spherical Surface. *Solid State Phenomena*. 175, 145-149.

[16] HUO, F.W., GUO, D.M., FENG, G., KANG, R.K., WANG, R.L. (2012). A new kinematics for ultra precision grinding of conical surfaces using a rotary table and a cup wheel. *International Journal of Machine Tools & Manufacture*. 59, 34–45.

[17] SHIHA, A.J., LEE, N.L. (1999). Precision cylindrical face grinding. *Precision Engineering*. 23(3), 177-184.

[18] SUN, W., PEI, Z.J., FISHER, G.R. (2005). Fine grinding of silicon wafers: machine configurations for spindle angle adjustments. *International Journal of Machine Tools & Manufacture*. 45, 51–61.

[19] CHIANG, C.-J., FONG, Z.-H. (2009) Undercutting and interference for thread form grinding with a tilt angle. *Mechanism and Machine Theory*. 44, 2066–2078.

[20] HUO, F. GUO, D., LI, Z., FENG, G., KANG, R. (2013). Generation of rotationally symmetric surfaces by infeed grinding with a rotary table and a cup wheel. *Precision Engineering*, 37, 286– 298.

[21] FENG, G., HUO, F.W., JIN, Z.J., KANG, R.K., GUO, D.M. (2013). High-accuracy Calibration of the Wheel Spindle Tilt Angle for Grinding Hydrostatic Seal Rings Used in Reactor Coolant Pumps. *Advanced Materials Research*. 797, 140-145.

[22] UHLMANN, E., BORSOI KLEIN, T., KOPROWSKI, S. (2014). Tilt angle effects in surface grinding with mounted points. *Production Engineering*. 8, 431-442.

[23] ZHANG, Z.-X., FONG, Z.-H. (2015). A novel tilt form grinding method for the rotor of dry vacuum pump. *Mechanism and Machine Theory*. 90, 47–58.

[24] CHEN, F., YIN, S., HUANG, H., OHMORI, H. (2015). Fabrication of small aspheric moulds using single point inclined axis grinding. *Precision Engineering*. 39, 107–115.

[25] KUNDRÁK, J. FEDOROVICH, V., PYZHOV, I., MARKOPOULOS, A., KLIMENKO, V. (2015). Some Features of the Surface Micro- and Macroprofile Formation at Flat Face Grinding with Spindle Axis Inclination. *Applied Mechanics and Materials*. 809-810, 45-50.

[26] ALEKSANDROV, P.S. (1968). *Lectures on analytic geometry*. Moscow: The publishing house of phisico-mathematical literature (In Russian).

[27] VYGODSKY, M. (2006). *Mathematical handbook: Elementary mathematics*. Moscow: Astrel' (In Russian).

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