

# Measuring and interpreting P and S wave velocity data – an application of a new petrophysical model

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## Summary

It is well known that acoustic wave propagation under pressure is very nonlinear and the elastic properties of rocks are hysteretic, which behavior is important for mechanical understanding of reservoirs during depletion. Pressure strongly influences the elastic parameters of rocks, thus wave velocities too. Therefore a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required. In this paper a petrophysical model is presented which describes the connection between the propagation velocity of acoustic waves (both P and S) and rock pressure both in case of loading and unloading phases as well as explains the mechanism of acoustic hysteresis. The developed model is based on the idea that the pores in rocks close under loading and reopen during unloading. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. P/S wave velocities were measured by a self-developed LabView software under pressure in laboratory on sandstone samples. The model was applied with success to measured data.

## Introduction

Studying the pressure dependence of acoustic wave velocities in laboratory provides important information to interpret seismic measurements in relation to petrophysical parameters. It has been observed that pressure has greater influence on velocities in the beginning phase of loading, and later it lessens and the velocities tend to a limit value. The most frequently used mechanisms for explaining this process are based on the closure of microcracks (Walsh and Brace 1964) or pores (Birch 1960) in rocks under pressure. For an analytical description of the nonlinear velocity vs. pressure relationship, exponential functions are most commonly used (Wepfer and Christensen 1991).

It is well known that the quasistatic elastic properties of rocks are hysteretic. The observable non-elastic response to pressure (namely acoustic hysteresis) may be caused by the irreversible closure of microcracks, irreversible compaction of pore spaces as well as improvement of contact conditions.

In literature several qualitative models are available to describe the pressure dependence of acoustic wave velocities but these empirical models do not explain the physical meaning of the process, they only give the regression function of the curve fitted to the measured data (Ji et al. 2007). To reasonably interpret laboratory measurements, a quantitative model - which provides the physical explanation - of the mechanism of pressure dependence is required.

### The pressure dependent velocity model

Following Birch's (1960) qualitative considerations we assume that the main factor determining the pressure dependence of propagation velocity is the closure of pores, i.e. decreasing of pore volume. Due to increasing pressure - from the unloaded state -, first the large pores are closed in the rock sample then after the slower compression process of smaller pores, approximately all pores are closed. Since the model is based on the pore volume or rather the change in pore volume, it is suitable to describe the pressure dependence of longitudinal and transverse wave velocity, respectively.

Let us introduce the parameter  $V$  as the unit pore volume of a rock. We assume that a  $d\sigma$  stress increase applied to the rock will generate a  $dV$  change in pore volume directly proportional to the change in stress. Eq. (1) summarize these assumptions in a differential equation and its solution

$$dV = -\lambda_V V d\sigma \rightarrow V = V_0 \exp(-\lambda_V \sigma), \quad (1)$$

where  $\lambda_V$  proportionality factor is a material dependent rock physical parameter,  $V_0$  is the pore volume at stress-free state ( $\sigma = 0$ ). The negative signs represent that the increasing stress decreases the pore volume. As the volume does not show anisotropy, Eq. (1) is the base of the model equations for both the P and S waves. We assume also a linear relationship between the infinitesimal change of the propagation wave velocities ( $d\alpha$  for P wave and  $d\beta$  for S wave) and the change in pore volume

$$d\alpha = -\kappa_\alpha dV, \quad d\beta = -\kappa_\beta dV, \quad (2)$$

where  $\kappa_\alpha$  and  $\kappa_\beta$  are proportionality factors, new material characteristics respectively for P and S waves. The negative signs represent that the velocity is increasing with decreasing pore volume. Combining the assumptions of Eqs. (1-2) and solve the differential equation one can obtain

$$\begin{aligned} d\alpha = \kappa_\alpha \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma &\rightarrow \alpha = K_1 - \kappa_\alpha V_0 \exp(-\lambda_V \sigma), \\ d\beta = \kappa_\beta \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma &\rightarrow \beta = K_2 - \kappa_\beta V_0 \exp(-\lambda_V \sigma), \end{aligned} \quad (3)$$

where  $K_1$  and  $K_2$  are integration constants which can be computed from Eqs. (3) as  $\alpha_0 = K_1 - \kappa_\alpha V_0$  and  $\beta_0 = K_2 - \kappa_\beta V_0$ , where  $\alpha_0$  and  $\beta_0$  are the propagation velocities at stress-free state which can be measured in laboratory. In the framework of the model, the velocities of acoustic waves increase from  $\alpha_0$  and  $\beta_0$  (at zero pressure) to  $\alpha_{max} = \alpha_0 + \Delta\alpha_0$  and  $\beta_{max} = \beta_0 + \Delta\beta_0$  (at high pressure, when all the pores are closed). So,  $\Delta\alpha_0$  and  $\Delta\beta_0$  can be considered the velocity-drops (relative to the fully compacted state where the pore volume equals zero) caused by the presence of pores at zero pressure (Ji et al. 2007). With introducing the notations  $\Delta\alpha_0 = \kappa_\alpha V_0$ ,  $\Delta\beta_0 = \kappa_\beta V_0$  Eqs. (3) can be rewritten in the forms

$$\alpha = \alpha_0 + \Delta\alpha_0 [1 - \exp(-\lambda_V \sigma)], \quad \beta = \beta_0 + \Delta\beta_0 [1 - \exp(-\lambda_V \sigma)]. \quad (4)$$

Eqs. (4) provide a theoretical connection between the P and S wave velocities and rock pressure in case of loading. In these equations  $\lambda_V$  is the logarithmic stress sensitivity of the propagation velocity (Dobróka and Somogyi Molnár 2012). Note that in the range of high pressures, reaching a critical pressure (Anselmetti and Eberli 1997) the reversible range is exceeded and destruction of the sample may occur thus decreasing velocities can be observed. This effect is outside of our present investigations.

To characterize the unloading phase,  $v=V_0-V$  as the closed pore volume of a rock is required to be introduced. If we decrease the pressure (from a maximum pressure value  $\sigma_m$ ) the closed pores start to open again, so decreasing propagation velocity can be measured. Therefore we assume  $dv$  (the change of the closed pore volume) being proportional with closed pore volume and the stress decrease  $d\sigma$

$$dv = \lambda'_V v d\sigma \rightarrow v = v_m \exp[-\lambda'_V (\sigma_m - \sigma)], \quad (5)$$

where  $\lambda'_V$  is another new material characteristic constant (which differs from the previously introduced parameter  $\lambda_V$ ) and  $v_m$  is the closed pore volume at maximum pressure value  $\sigma_m$ . After Birch (1960) there is always a certain amount of irreversibility in the closure-reopen of pores, i.e. pores closed during loading do not reopen completely during unloading. This irreversibility is denoted by these different parameters  $\lambda_V$  and  $\lambda'_V$  in our model. Combining Eqs. (2) and Eq. (5) by using the formulas  $dV=-dv$ ,  $\kappa_P v_m = \Delta\alpha_m$  and  $\kappa_V v_m = \Delta\beta_m$  one can find

$$\alpha = \alpha_m - \Delta\alpha_m \{1 - \exp[-\lambda'_V (\sigma_m - \sigma)]\}, \quad \beta = \beta_m - \Delta\beta_m \{1 - \exp[-\lambda'_V (\sigma_m - \sigma)]\}. \quad (6)$$

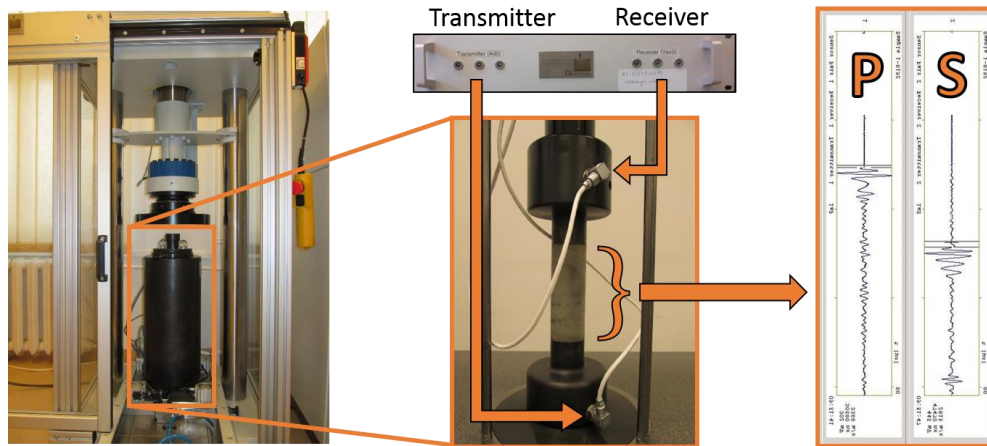
Eqs. (6) show the P/S wave velocity-pressure function of unloading phase. In the two limiting cases (at pressure value  $\sigma=\sigma_m$  and  $\sigma=0$ ) Eqs. (6) give  $\alpha_m/\beta_m$  and  $\alpha_1 = \alpha_m - \Delta\alpha_m [1 - \exp(-\lambda'_V \sigma_m)]$ ,  $\beta_1 = \beta_m - \Delta\beta_m [1 - \exp(-\lambda'_V \sigma_m)]$  respectively, (notation  $\alpha(0) = \alpha_1$  and  $\beta(0) = \beta_1$  were used). This gives the formulas for unloading phase (similar to Eqs. (4))

$$\alpha = \alpha_1 + \Delta\alpha_1 [1 - \exp(-\lambda'_V \sigma)], \quad \beta = \beta_1 + \Delta\beta_1 [1 - \exp(-\lambda'_V \sigma)], \quad (7)$$

where  $\Delta\alpha_1 = -\Delta\alpha_m \exp(-\lambda'_V \sigma_m)$  and  $\Delta\beta_1 = -\Delta\beta_m \exp(-\lambda'_V \sigma_m)$ .

### Measurement of P/S wave velocity and inversion based data processing

To confirm the reliability of the model it was tested on velocity data sets. We performed measurements on many different cylindrical sandstone samples under uniaxial stresses. The pressure cell and a load frame can be seen is Fig.1.



**Figure 1** Experimental setup. Left: load frame and pressure cell. Middle: ultrasonic device, sandstone sample between transmitter and receiver. Right: P and S wave arrivals.

The acoustic measurement was realized in LabView based on the pulse transmission technique. A 256-fold summary was applied in the software, which means a running average of 256 consecutive measurements. With this method the amplitudes of external noises in the displayed wave were significantly reduced, consequently the first arrivals can be determined accurately. Our measurements showed that in the lower pressure range, the increase in velocities with increasing pressure is steep and nonlinear. This is due to the closure of pore volume, which significantly affects the elastic properties of rock and thereby the velocities. In the higher pressure range, the increase in velocities (with increasing pressure) become moderate as the closable pore volume lessens. A slight difference can be found between the characteristics of the loading and unloading curves which can be explained by the phenomenon of acoustic hysteresis.

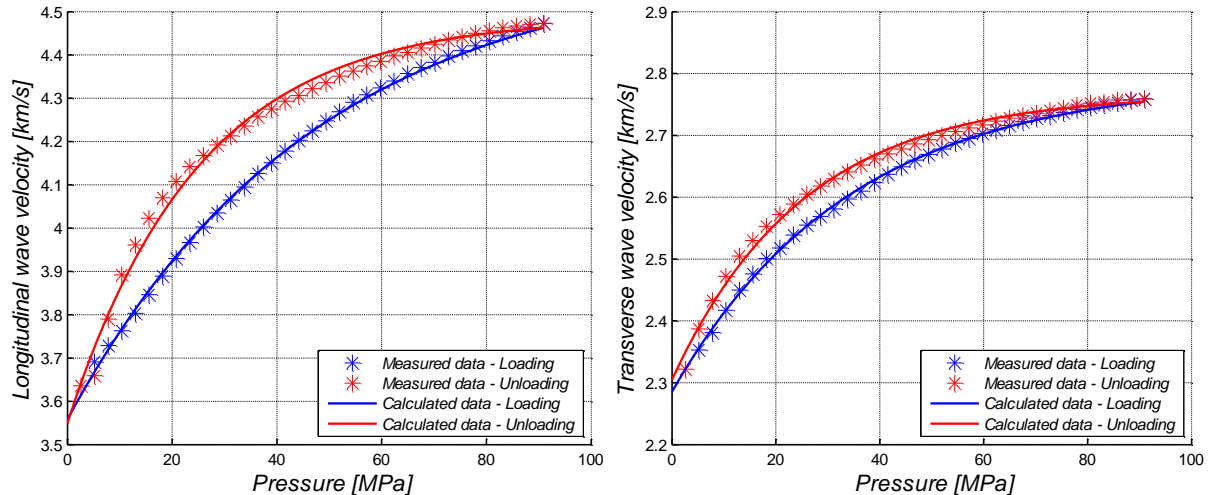
Saving the measured velocity-pressure data into a matrix in LabView enables immediate data processing. The measured velocity data were processed by joint inversion method. P/S wave velocity-pressure data (loading and unloading, too) belong to the first arrivals at each pressure measured by the developed software provided the input data of the inversion program written in MATLAB development environment. During programming the least squares method was used (Menke 1984). The output data of the data processing were the estimated model parameters and the quantities which characterise the accuracy of inversion estimates.

The estimated model parameters for a typical test result (sandstone sample) are summarized in Table 1. The table contains the relative distances in data space, - i.e. the root mean square (RMS) - and the mean spread values as well, which indicate that the parameters are in moderate correlation.

**Table 1** Model parameters, RMS and mean spread values estimated by joint inversion using the developed model.

P wave							
Loading			Unloading			RMS (%)	S
$\alpha_0$ (km/s)	$\Delta\alpha_0$ (km/s)	$\lambda_v$ (1/MPa)	$\alpha_1$ (km/s)	$\Delta\alpha_1$ (km/s)	$\lambda_v$ (1/MPa)		
3,56	1,06	0,0212	3,56	0,94	0,0401	0,39	0,59
S wave							
Loading			Unloading			RMS (%)	S
$\beta_0$ (km/s)	$\Delta\beta_0$ (km/s)	$\lambda_v$ (1/MPa)	$\beta_1$ (km/s)	$\Delta\beta_1$ (km/s)	$\lambda_v$ (1/MPa)		
2,29	0,51	0,0212	2,31	0,46	0,0395	0,26	0,5

By the help of the model equations Eqs. (4) and (7) the velocity values at any pressure can be calculated, which can be seen in Fig. 2. Asterisks mean the measured values while the lines represent the calculated ones. The figure shows that the calculated curves are in good accordance with the measured data proving that the petrophysical model describing the acoustic hysteresis applies well in practice. It can be also seen that the model characterizes well both loading and unloading phases.



**Figure 2** *P* and *S* wave velocities as a function of pressure in case of loading and unloading.

## Conclusions

In this paper a rock physical model based on quantitative explanation of the pressure dependence of seismic/acoustic velocities is introduced. The advantage of the model is that it is not based on simple curve fitting, but gives physical explanation for the process with three-parameter exponential equations. The model (valid only in reversible/elastic range) is based on the idea that pore volume changes with pressure. It was showed that the model was tested with success on laboratory measured data. P/S wave velocity was measured by a self-developed measuring-data acquisition software and processed by a joint inversion method.

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