# One possible numerical solution for some contact pressure optimization problems

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For designer, it is always important to avoid singularities within the contact regions in order to keep stresses at a low level. This requirement leads to optimal design of contact surface shape and proper material selection, thus generating a class of contact optimization problems. The design parameters in structural optimization are usually defined as material moduli, structure size and shape, characteristic dimensions, supports, loads, inner links, reinforcement and topology, cf. books by Banichuk and Neittaanmaki [1] and Banichuk [2].

The paper [3] has a survey of literature according to different contact optimization problems: The contact pressure optimization was analysed for an elastic punch on a rigid substrate assuming the linear elasticity relations. A nearly constant contact pressure distribution was achieved by appropriate shape optimization for axially symmetric bodies. Contact optimization problems were analysed with account for frictional contact, for multiple load cases and for incomplete external loading data, for kinematical constraints etc.

Our works [4-6] provide a new type of solution for 2D and 3D problems, in which the contact pressure distribution is partially controlled by minimizing the maximal contact pressure. In [7] several classes of optimization problems have been considered with account of wear process.

In our analysis, it is assumed that the bodies are in contact on the whole subdomain $\Omega_c$ of the contact zone $S_c = \Omega$. The subdomain $\Omega_c$ is given by us. Introduce the surface coordinates $s$, $t$ and assume that the following pressure distribution is reached due to shape optimization [4] on the $\Omega_c$

$$p_n(\mathbf{x}) = c(\mathbf{x}) p_{n,\max} \tag{1}$$

where the assumed control function $c(\mathbf{x})$ must satisfy the condition $0 \le c(\mathbf{x}) \le 1$, and

$$p_{n,\max} = \max p_n(\mathbf{x}), \quad \mathbf{x} = [s, t] \tag{2}$$

In the subdomain $\Omega_{nc} (\Omega = \Omega_c \bigcup \Omega_{nc})$ the contact pressure is not controlled and does not exceed the values specified by (1), so that

$$\chi(\mathbf{x}) = c(\mathbf{x}) p_{\max} - p_n(\mathbf{x}) \ge 0 \quad \mathbf{x} \in \Omega_{nc} \tag{3}$$

Usually control function $c(\mathbf{x})$ depends on some geometrical parameters. Some of the parameters are fixed while the others are determined in the optimization process.

The lecture gives some examples for controlled contact pressure optimization for different beam structures and cylindrical bodies. It is assumed that strains are small and the materials of the contacting bodies are linearly elastic. An effective method is given in numerical calculation for determination of the optimal distribution of the contact load.

For some problems the Green influence functions for Signorini contact conditions [8,9] can be applied

$$d = (u_n^{(2)}(p_n) + u_{n,load}^{(2)}) - (u_n^{(1)}(p_n) + u_{n,load}^{(1)} + \lambda) + g^{(0)} \ge 0, \quad p_n \ge 0, \quad p_n d = 0 \tag{4}$$

where $u_n^{(i)}(p_n) = \int\limits_{S_c} (H_{n,n}^{(i)}(x,s) + \mu H_{n,t}^{(i)}) p_n(s) \, ds$ normal displacement, $H_{n,n}^{(i)}(x,s), H_{n,t}^{(i)}(x,s)$ are the

Green functions for normal and tangential tractions, $u_{n,load}^{(i)}$ is the normal displacement from given

load, $g^{(0)}$ is the initial gap , $\lambda$ is the rigid body displacement for the first body, that is $d = d(p_n, \lambda)$. The contact problem is solved by principle of modified complementary energy or by displacement formulation using minimum principle of the total potential energy in the penalty form. In the last case, we use *h* or *p*-version finite element method for discretization [10].

By the prescribed distribution of contact pressure the optimization problem is solved in the discretized form. In these contact optimization problems, the initial gap (shape form of the contact surface) is the unknown function. The calculation of the gap can be made with special iteration [3,4]. The discretized equation for determination of the discretized gap is

$$\mathbf{d} = \mathbf{H}^{(2)}\mathbf{p} + \mathbf{u}_{n,load}^{(2)} - {}^{(iter)}\mathbf{u}_n^{(1)}(\mathbf{p}) - \mathbf{u}_{n,load}^{(1)} - \mathbf{e}\,{}^{(iter)}\lambda + {}^{(iter)}\mathbf{g}^{(0)} = \mathbf{0}$$

$$\mathbf{H}^{(2)}\mathbf{p} + \mathbf{u}_{n,load}^{(2)} - {}^{(iter)}\mathbf{u}_n^{(1)}(\mathbf{p}) - \mathbf{u}_{n,load}^{(1)} = \mathbf{e}\,{}^{(iter)}\lambda - {}^{(iter)}\mathbf{g}^{(0)} = -{}^{(iter)}\mathbf{g}^{(0)*} \qquad (5)$$

$${}^{(iter)}\mathbf{g}_{min}^* = \min\,{}^{(iter)}\mathbf{g}^{(0)*}, \quad \mathbf{g}^{(0)} = {}^{(iter)}\mathbf{g}^{(0)*} - \mathbf{g}_{min}^*$$

Usually after some iterations the solution is converged. Also, must be said, that the distribution of the contact pressure is very sensitive to the form of contact surface shape.

The numerical examples demonstrate the efficiency of the solution algorithms.

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