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GLOBAL JOINT INVERSION OF ACOUSTIC VELOCITY AND QUALITY FACTOR DATA USING ROCK PHYSICAL MODELS

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Abstract: In this study, we apply new rock physical models to describe the pressure dependence of seismic/acoustic velocity and quality factor data. The models are based on the previously discussed idea that microcracks in rocks are opened and closed with changes in pressure. Using the model equations as forward modeling formulae, a global optimization algorithm (Simulated Annealing) was applied to solve the joint inversion problem and to determine the petrophysical parameters of the models.

Keywords: *petrophysical model, longitudinal wave velocity, quality factor, pressure, joint inversion, Simulated Annealing*

INTRODUCTION

As a result of the increasing world market price of hydrocarbons, the utilization of non-conventional hydrocarbon resources and the associated research has become very intensive. This type of hydrocarbon occurrences can also be found in Hungary. The knowledge of petrophysical parameters (porosity, permeability) in typically pressurized, deep-seated and high-temperature reservoirs is very important. These non-conventional reservoirs have very low porosity and permeability. Thus it is expedient to create a reservoir geological model in which the petroacoustic characteristics (mostly the propagation velocity of the elastic wave and quality factor) and their pressure dependence play an important role because they have a direct connection to the porosity.

The propagation characteristics of acoustic waves contain information on important mechanical properties of rocks. Therefore the determination of velocity and attenuation of the seismic/acoustic wave is relevant in studying rock parameters, both in the laboratory and in-situ conditions. The velocity of acoustic waves propagating in different rocks under various confining pressures [1–3] and also under different pore pressures [4–7] have been investigated over several decades and by many researchers. The phenomenon in which the wave velocity increases with increasing pressure is well-known and has been discussed in several rock physical studies. One of the most frequently used mechanisms for explaining the phenomenon is based on the closure of microcracks in rocks under pressure [8–11]. Singh et al. created an empirical model for pressure-dependent wave velocity after observing measured P and S wave propagation velocities for several sandstone samples [12]. Prasad inferred the situation of gas-saturated and pressurized zones from the ratio of propagation velocities of P and S waves [13].

The attenuation of the acoustic wave decreases with increasing pressure, which can also be explained by the presence of microcracks in rocks [5, 14–15]. Toksöz et al. measured P and S wave velocities under pressure on dry samples, water, brine, or methane-saturated samples, and frozen samples and studied their attenuations [16]. It was found that the attenuation was higher for water-saturated and brine-saturated samples compared to methane-saturated or dry samples and the attenuation decreased with increasing pressure. The reason for this phenomenon can be the closure of microcracks according to Yu et al. [5], Best [10], and Johnston et al. [17].

Based on petrophysical models and experimental results, it is possible to infer the extent of emerging tensions in rocks, and perhaps even its dependence of direction, by means of measured longitudinal and transversal wave velocity data. Several petrophysical models are known in literature, e.g., the Biot model [18–19], Gassmann model [20], Contact radius model (CRM) [21], Friction model (FM) [22–23], etc. The Biot model describes wave propagation in a two-phase system, a porous elastic frame, and a viscous, incompressible pore fluid. The attenuation of the wave is ascribed to relative motion between the frame and its pore fluid. According to the Gassmann model, when the pore space of rock is saturated with fluid, there is a change in the apparent bulk modulus due to very small variations in pore pressure induced by the elastic wave. Thus in isotropic, porous rocks, the P wave velocity depends on the properties of the dry background and pore fluid as well as on porosity. CRM considers two elastic, homogeneous, identical spherical particles in contact. The radius of the area of contact between the spheres increases upon deformation due to a normal force. In the FM model, the frictional losses at shear strains are negligible in rock samples.

In this paper, we present a new approach for the description of the pressure dependence of phase velocity and the quality factor of acoustic waves propagating in a porous rock sample. Our considerations are based on the mechanism of closure of the microcracks in rocks under pressure.

THE PRESSURE DEPENDENT VELOCITY MODEL

Elastic wave velocity in rocks depends on many rock properties, such as the type of rock-forming minerals and cementing material (rock matrix), porosity, and pore-filling fluid, as well as microcracking. Following Walsh and Brace's idea [24], we assume that the main factor determining the stress dependence of the wave propagation velocity is the closure of the microcracks. In our considerations, we restrict ourselves only to the uniaxial stress state and longitudinal acoustic waves propagating along the direction of load. We assume further that only the projection of the area of the microcrack perpendicular to the load direction has an influence on the change in the propagation characteristics. For this reason, we introduce the concept of the effective area $A_\perp$ as the sum of the areas of the open individual microcracks projected to a plane perpendicular to the direction of the uniaxial load.

If we create a dσ stress increase in the rock, we find that $dA_\perp$ (the change in the effective area of the open microcracks) is directly proportional to the applied dσ stress increase. At the same time $dA_\perp$ is directly proportional to $A_\perp$, which is the total effective area of the open microcracks. We can unify both assumptions in the following differential equation as

$$dA_\perp = -\lambda A_\perp \, d\sigma \,, \tag{1}$$

where λ is a proportionality factor (material dependent rock physical parameter). In *Equation (1)* the negative sign represents the fact that with increasing stress – with closing microcracks – the total effective area of the open microcracks decreases. The solution of the equation is

$$A_\perp = A_\perp^{(0)} \exp(-\lambda\sigma) \,, \tag{2}$$

where $A_\perp^{(0)}$ is the total effective area of the open microcracks in a stress-free state (σ = 0). Another assumption is the linear relationship between the longitudinal wave velocity change dv and the change in the effective area $dA_\perp$

$$dv = -\alpha \, dA_\perp \,, \tag{3}$$

where α is a proportionality factor (a material quality dependent constant). The negative sign represents the fact that the velocity increases with decreasing effective area of the open microcracks. Combining *Equation (3)* with *Equation (1)* and *Equation (2)* one can obtain

$$dv = \alpha\lambda A_\perp^{(0)} \exp(-\lambda\sigma) \, d\sigma \,. \tag{4}$$

After integrating *Equation (4)* we have

$$v = K - \alpha A_\perp^{(0)} \exp(-\lambda\sigma) \,. \tag{5}$$

In a stress-free state (σ = 0) the propagation velocity v_0 can be measured, which can be computed from *Equation (5)* as $v_0 = K - \alpha A_\perp^{(0)}$. From here we get the integration constant $K = v_0 + \alpha A_\perp^{(0)}$. With this result and the introduction of $\Delta v = \alpha A_\perp^{(0)}$, *Equation (5)* can be rewritten in the following form

$$v = v_0 + \Delta v (1 - \exp(-\lambda\sigma)) \,, \tag{6}$$

where $\lambda=1/\sigma^*$ and σ^* is that stress value at which the velocity (starting from v_0) approximates the value v_{max} with 1/e accuracy.

Equation (6) provides a theoretical connection between the longitudinal wave velocity and rock pressure. It contains three model parameters (v_0, Δv, λ) which are to be determined by using the stress-dependent velocity data measured on the investigated rock sample. *Equation (6)* shows that the propagation velocity – as a function of stress – starts from v_0 and increases up to the value $v_{max} = v_0 + \Delta v$ according to the function $1\text{-}exp(\lambda\sigma)$. Consequently, the velocity reaches its limit v_{max} at high stress values. The value $\Delta v = v_{max}–v_0$ is a velocity range in which the propagation velocity can vary from the stress-free state up to the state characterized by high rock pressure. (Certainly, it is only valid in the framework of the model assumptions because in the range of high stresses new microcracks can open in the rock. It should be mentioned that the description of the latter was not a goal in our approach.)

THE PRESSURE DEPENDENT QUALITY FACTOR MODEL

The effect of microcracks for the quality factor – like in the case of propagation velocity – can be studied based on qualitative considerations without detailed analysis of the structure and the physical mechanisms. It is obvious that the $dA_\perp$ change in the effective area of the open microcracks leads to a dQ change in the quality factor of the longitudinal wave. We assume again a linear relationship between them and write

$$dQ = -\beta dA_\perp , \tag{7}$$

where β is another proportionality factor (a material quality dependent constant). The negative sign represents the fact that the quality factor increases with closing microcracks (thus decreasing the effective area of the open microcracks). Combining *Equation (7)* with *Equation (1)* and *Equation (2)* the following equation can be obtained

$$dQ = \beta\lambda A_\perp^{(0)} \exp(-\lambda\sigma)d\sigma . \tag{8}$$

Solving *Equation (8)* and introducing the notation $\Delta Q = \beta A_\perp^{(0)}$, one can get the following model equation, which is similar to *Equation (6)*,

$$Q = Q_o + \Delta Q\left(1 - \exp(-\lambda\sigma)\right), \tag{9}$$

where Q_0 is the quality factor in a stress-free state ($\sigma = 0$) and λ is the common proportionality factor (material characteristic) in the two model equations *[Equations (6)* and *(9)]*. It is well-known that there can be several reasons for absorption. With *Equation (9)* we describe the attenuation caused by only the change in the effective area of the open microcracks. *Equation (9)* contains three model parameters (Q_0, ΔQ, λ) that can be determined by using the stress-dependent quality factor data measured on the investigated rock sample.

THE GLOBAL INVERSION METHOD MSA

The established petrophysical models *[Equations (6)* and *(9)]* create the possibility to calculate the propagation velocity and quality factor at any pressure based on known parameters (v_0, Δv, Q_0, ΔQ, λ) of rock (this is the so-called forward problem). In contrary, the parameters appearing in the above model equations can be derived from the observed data by using a geophysical (joint) inversion method [25]. In the joint inversion procedure, we give an estimate for the velocity and quality factor data in a single algorithm, where λ is the common petrophysical parameter connecting the two data sets in this case. So we integrate the measurement data into the combined data vector

$$\mathbf{d}^{obs} = \{v_1^{obs},...,v_{N_v}^{obs}, Q_1^{obs},...,Q_{N_Q}^{obs}\}, \quad (10)$$

where N_v and N_Q are the numbers of the measured velocity and quality factor data, respectively. Introducing the combined vector **m** of model parameters as

$$\mathbf{m} = (v_0, \Delta v, Q_0, \Delta Q, \lambda) \quad (11)$$

we can write the (joint) response function in the form

$$\mathbf{d}^{calc} = \mathbf{g}(\mathbf{m}), \quad (12)$$

where

$$\mathbf{d}^{calc} = \{v_1^{calc},...,v_{Nv}^{calc}, Q_1^{calc},...,Q_{N_Q}^{calc}\} \quad (13)$$

with

$$d_k^{calc} = g_k(\mathbf{m}) = \begin{cases} v^{calc}(\mathbf{m}_v, \sigma_k), \; k = 1,..., N_v \\ Q^{calc}(\mathbf{m}_Q, \sigma_k), \; k = N_v + 1,..., N_v + N_Q \end{cases} \quad (14)$$

and

$$\mathbf{m}_v = (v_0, \Delta v, \lambda), \quad \mathbf{m}_Q = (Q_0, \Delta Q, \lambda). \quad (15)$$

In the inversion procedure, the parameter vector **m** is determined usually by means of minimizing the weighted norm

$$E = (\mathbf{e}, \mathbf{W}\,\mathbf{e}) \quad (16)$$

of the

$$\mathbf{e} = \mathbf{d}^{obs} - \mathbf{d}^{calc} \quad (17)$$

deviation vector (the W_{kk} weights are usually given a priori).

As the forward modeling formula *Equation (12)* is nonlinear, linearization is often used. It is well-known that linearized inversion procedures can trap in a local minimum of the *Equation (16)* function. To ensure finding the absolute (global) minimum, the methods of global optimization are to be used. In this paper, the Metropolis algorithm (a member of the family of Simulated Annealing procedures, abbreviated as MSA) [26–27] is used. In this method, an adequate objective function is required, which is minimized during the optimization procedure. In most cases, it is based on the L2-norm (or its weighted version in *Equation 16*) of the difference between the measured and calculated data (called Energy in the terminology of MSA). The SA methods are based on the random modification of the elements of the model vector in an iteration procedure. The modification of the *j*-th parameter can be written as

$$m_j^{new} = m_j^{old} + b_j^{\max} \varepsilon \,, \tag{18}$$

where ε is a uniformly distributed random number in the interval (0, 1) and $b_j^{\max}$ is an appropriately chosen limit of change in the *j*-th variable. During the random search, the energy function is calculated and compared with the previous one and the ΔE energy change is generated in every iteration steps. The acceptance probability of the new model depends on the Metropolis criteria

$$P(\Delta E, T) = \begin{Bmatrix} 1 & , if \;\Delta E \le 0 \\ \exp\left(-\dfrac{\Delta E}{T}\right) & , otherwise \end{Bmatrix}, \tag{19}$$

where the new model is always accepted when the value of the energy function is lower in the new state than in the previous one $\Delta E \le 0$. If the energy of the new model increases, there is also some probability of acceptance depending on the values of the energy function and control temperature T. The MSA method can be described by a simple and clear-cut scheme. The pseudo FORTRAN code is given by Sen and Stoffa [28]:

Start at a random location $\vec{m}_o$ with energy $E(\vec{m}_o)$ (20)

loop over temperature (T)

loop over the number of random moves/temperature

$$\Delta E = E(\vec{m}_1) - E(\vec{m}_o)$$

$$P = \exp\left(-\frac{\Delta E}{T}\right)$$

if $\Delta E \le 0$ then

$$\vec{m}_o = \vec{m}_1$$

$$E(\vec{m}_o) = E(\vec{m}_1)$$

end if

if $\Delta E \ge 0$ then

```
                        draw a random number α = U[0,1]
                        if P ≥ α then
                                m_o = m_1
                                E(m_o) = E(m_1)
                        end if
                end if
        end loop
end loop
```

This is repeated in an iterative procedure until a proper stop criterion is met. For our numerical calculations, this scheme was implemented in MATLAB code.

RESULTS AND DISCUSSION

In order to prove the validity and practical applicability of our rock physical model, the velocity and quality factor data measured by Prasad and Manghnani on a Berea sandstone rock sample were processed [29]. Since the data set contained relatively low noise we used **W** = **I** in *Equation (16)*, (where **I** is the unit matrix).

For the characterization of the accuracy, we calculated the measure of a fitting according to the data misfit formula

$$D = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{d_k^{(obs)} - d_k^{(calc)}}{d_k^{(calc)}}\right)^2} \quad , \tag{21}$$

where $d_k^{(calc)}$ and $d_k^{(obs)}$ are the calculated and observed (measured) velocity/quality factor data at the *k*-th pressure value, while N is the number of measured points. As a starting value T_0 of the control parameter, we used the D distance calculated between the measured and the calculated (on the randomly selected) start model. The control parameter was reduced by 0.1% in each iteration. The MSA inversion results can be seen in *Table 1*.

Table 1

Estimated model parameters (using the MSA method) for Berea sandstone sample

Sample	**v_0 (km/s)**	**Q_0 (–)**	**Δv (km/s)**	**ΔQ (–)**	**λ (1/MPa)**
Berea sandstone	3.684	16.4	0.925	55.0	0.0932

With these parameters, the velocity and quality factor data can be calculated at any pressure by substituting them into *Equation (6)* and *Equation (9)*. The results are shown in *Figure 1*. The continuous lines show the calculated velocity–pressure and quality factor-pressure functions while symbols represent the measured data.

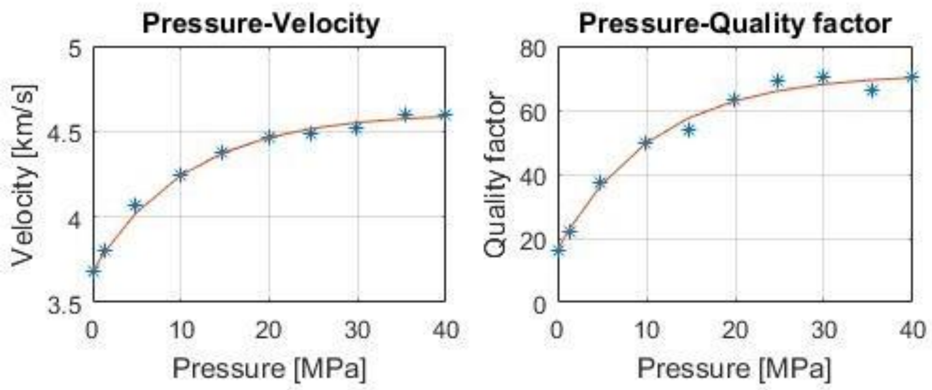

Figure 1. *The fit between the observed and calculated (with estimated model parameters) data using the global (MSA) joint inversion method*

The figure shows that the calculated functions are in good accordance with the measured data, which proves that the petrophysical models suggested in *Equations (6)* and *(9)* apply well in practice. This is proved also by the data misfit value, which is D = 0.025 in our case.

CONCLUSIONS

We presented new petrophysical models for describing the connection between the propagation velocity of the acoustic wave and rock pressure as well as the quality factor and pressure. The models are based on the idea that microcracks are opened and closed as pressure changes. Within the confines of the models, differential equations were set up and solved describing the phenomenon of pressure dependence. It was found that the quality factor and the seismic velocity increases with increasing pressure.

The models were applied to acoustic velocity and quality factor data measured on a core sample. By means of global inversion-based processing (Metropolis Simulated Annealing), the model parameters were determined from measurement data. The described inverse problem was significantly overdetermined. Therefore, the inversion procedure was stable and could be handled properly by the global MSA inversion technique. The calculated data matched accurately with measured data, proving that the petrophysical model applies well in practice. Inversion results confirmed the accuracy and feasibility of our petrophysical models.

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LIST OF SYMBOLS

Symbol	Description	Unit
$A_\perp$	total effective area of the open microcracks projected to a plane perpendicular to the direction of the uniaxial load	m^2
$A_\perp^{(0)}$	total effective area of the open microcracks in a stress-free state	m^2
b_j^{max}	limit of change in the *j*-th variable	–
D	data misfit	%
$dA_\perp$	the change in the effective area of the open microcracks	m^2
d^{calc}	calculated data vector	–
$d_k^{(calc)}$	calculated velocity/quality factor at the *k*-th pressure value	m/s
d^{obs}	observed (measured) data vector	–
$d_k^{(obs)}$	observed (measured) velocity/quality factor at the *k*-th pressure value	m/s
dQ	the change in the quality factor of the longitudinal wave	–
dv	change of propagation wave velocity	m/s
dσ	stress increase	MPa
e	deviation vector	–
E	weighted norm	–
g	response function	–
I	unit matrix	–
K	integration constant	–
m	model parameter vector	–
m_0	start model parameter vector	–
m_j^{new}	*j*-th new model parameter vector	–
m_j^{old}	*j*-th old model parameter vector	–
m_Q	model parameter vector of quality factor model equation	–
m_v	model parameter vector of velocity model equation	–
N	number of measured points	–
N_Q	number of the measured quality factor data	–
N_v	number of the measured velocity data	–

Symbol	**Description**	**Unit**
Q	quality factor of the longitudinal wave	–
Q_0	quality factor of the longitudinal wave at stress-free state	–
Q^{calc}	calculated quality factor data of the longitudinal wave	–
Q^{obs}	observed (measured) quality factor data of the longitudinal wave	–
T	temperature	°C
T_0	start temperature	°C
v	longitudinal wave velocity	m/s
v_0	longitudinal wave velocity at stress-free state	m/s
v^{calc}	calculated longitudinal wave velocity data	m/s
v_{max}	longitudinal wave velocity at maximum pressure	m/s
v^{obs}	observed (measured) velocity data	m/s
W	weight	–
α	proportionality factor	1/(m s)
β	proportionality factor	1/ m^2
ΔE	energy change	°C
ΔQ	quality factor range of the longitudinal wave	–
Δv	velocity range	m/s
ε	random number in the interval (0,1)	–
λ	material dependent rock physical parameter	1/MPa
σ	stress	MPa
σ*	maximum stress	MPa

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