

DESCRIPTION OF THE PRESSURE DEPENDENT POROSITY

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ABSTRACT

The complex understanding of the subsurface structures includes the knowledge of rock physical parameters, such as porosity and permeability. They are important features of rocks because they influence the storage capacity, mobility, producibility of pore fluids (gas, oil, water) which are always in the centre of interest. In this paper, we focus only on porosity determination. It is usual in geophysics that a required parameter cannot be directly measured in situ. But some other measurable parameters are in connection with them. In our case, we use the relationship between porosity and acoustic velocity. In previous studies [1] [2] [3] it was shown how the acoustic velocities vary with changing pressure due to porosity changes. Now a rock physical model describing the pressure dependence of porosity is proposed. The applicability is proved on velocity and porosity data sets published in the literature.

INTRODUCTION

Several geophysical studies deal with the determination of porosity because it is a key parameter for hydrocarbon, water or another special (for example ore using different dissolving techniques) explorations. Rock porosity is usually measured in the laboratory, but of course, these results may differ from the in situ values. Therefore a porosity estimation technique that can be used on in situ data would be advantageous. In geophysics, different laboratory and field methods are available where the measured parameters depend on rock porosity. If the relation is well described, an in situ porosity distribution (a porosity map) can be derived for the investigated subsurface. Since in the fields scientists barely have proper control over the results, these investigations have to be started in the laboratories under well-controlled conditions.

Acoustic wave velocities are often measured characteristics [4] [5] [6] [7] [8]. Publications provide a variety of different qualitative models that concludes that the increasing pressure acting on the rock results in the closure of microcracks [9] or generally the closure of pore volume [10]. Similarly, the description of the porosity changes due to pressure increase is based on empirical relations [11] [12] [13] [14], authors list the regression parameters of the best fitting curves, but do not provide quantitative rock physical models based on physical relations between the parameters.

During the research in our institute, we introduced rock physical models describing the pressure dependence of acoustic wave velocities [1] [2]. Case studies underlined the applicability of these models. To introduce the porosity model, firstly the velocity

model based on the closing microcracks has to be understood [1]. The basic idea is formulated by Walsh and Brace [9], i.e. the main factor in the pressure dependence of velocities is the closure of microcracks. Increasing pressure closes the microcracks, so the acoustic waves can travel faster in the rock. Let us introduce the notation N for the number of microcracks in unit rock volume. In this case, the phenomenon mentioned above can be formulated with a differential equation

$$dN = -\lambda N d\sigma, \quad (1)$$

where dN is the change in the number of microcracks in the unit volume, $d\sigma$ is the change in pressure, λ rock physical parameter is the stress sensitivity of the velocities [1]. Assuming a linear relationship between the infinitesimal change in P wave velocity ($d\alpha$) and the change in the number of microcracks, a second differential equation can be formulated

$$d\alpha = -\kappa dN, \quad (2)$$

where κ is a material dependent parameter. Based on equations (1) and (2) the model equation

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - \exp(-\lambda\sigma)) \quad (3)$$

can be derived, where 3 model parameters are present: α_0 the P wave velocity at stress-free state, $\Delta\alpha_0$ the velocity drop (velocity difference between the stress-free state and the maximal pressure, where all pores are closed) and λ rock physical parameter [1]. In this paper, a porosity model is introduced as well. Its applicability was tested on laboratory data set published in the literature.

THE POROSITY MODEL

In connection with the porosity, one can distinguish different type of porosities. Based on the time when the pore volume is generated one can speak about primary porosity (ϕ_1), which is formed during the diagenesis of the rock, and the secondary porosity (ϕ_2) which is caused later by physical and/or chemical interactions in the rock. The primary porosity includes usually spherical pores, secondary porosity consists mostly of microcracks or dissolved voids. In this paper, microcracks are linked to secondary porosity. It means that the total porosity (ϕ_t) equals

$$\phi_t = \phi_1 + \phi_2. \quad (4)$$

In the velocity model, it is assumed that the microcracks are closed with increasing load. Following the analogy presented before, a differential equation describing the phenomenon – namely that the secondary porosity decreases with increasing pressure – can be written

$$d\phi_2 = -\lambda\phi_2 d\sigma, \quad (5)$$

where the λ rock physical parameter is already known from the velocity model. The solution of equation (5) is

$$\phi_2 = \phi_2^{(0)} \exp(-\lambda\sigma), \quad (6)$$

where $\phi_2^{(0)}$ is the secondary porosity in the stress-free state. Substituting equation (6) into equation (4) the

$$\phi_t = \phi_1 + \phi_2^{(0)} \exp(-\lambda\sigma) \quad (7)$$

equation can be deduced, which describes the pressure dependence of porosity. From the porosity model equation (7) it can be seen, that the total porosity tends to a limit value, namely the primary porosity.

INVERSION

Velocity and porosity data are processed together in quality-checked joint inversion, where 5 model parameters ($\alpha_0, \Delta\alpha_0, \lambda, \phi_1, \phi_2^{(0)}$) are determined. Joint inversion can be applied when at least one common parameter links the data sets. In the present case, the rock physical parameter λ provides the connection which is part of both the equations (3) and (7). Data are almost noise-free and the problem is overdetermined (number of data is greater than the number of model parameters), therefore the Gaussian Least Squared Method was used. The workflow of the inversion is summarized in Figure 1. First, the start model is defined based on the measured values. In the first iteration, the forward modelling is done. It means that the theoretical velocity and porosity data are calculated by substituting them into the response equations describing the physical relationship between the parameters of the problem. The results are compared to the measured ones. At this point the relative data distance (D) between the k -th measured ($d_k^{(m)}$) and calculated ($d_k^{(c)}$) data is determined (N_d is the number of data) as

$$D = \sqrt{\frac{1}{N_d} \sum_{k=1}^{N_d} \left(\frac{d_k^{(m)} - d_k^{(c)}}{d_k^{(m)}} \right)^2} \cdot 100 [\%]. \quad (8)$$

This parameter characterizes the accuracy of the inversion. If it is not small enough, the model parameters are changed and theoretical data are recalculated. After reaching the stop criteria defined by data distance or the number of iteration steps, the model parameters used in the last iteration step are considered the solution of the problem. According to the method proposed by Menke [15], the estimation errors of the model parameters (σ_{m_i}) can be determined as well. They build up the diagonal of the covariance matrix of the model parameters ($cov(m)$)

$$\sigma_{m_i} = \sqrt{\text{cov}(m)_{MM}}, \quad (9)$$

where $M=1,2,3,4,5$.

The reliability of the model can be characterized by the mean spread (S). It is calculated according to the following equation [15]

$$S = \sqrt{\frac{1}{M(M-1)} \sum_{i=1}^M \sum_{j=1}^M (\text{corr}(m)_{ij} - \delta_{ij})^2}, \quad (10)$$

where δ is the Kronecker-delta symbol (it equals 1 if $i=j$, else 0), M is the number of model parameters and $\text{corr}(m)$ is the correlation matrix.

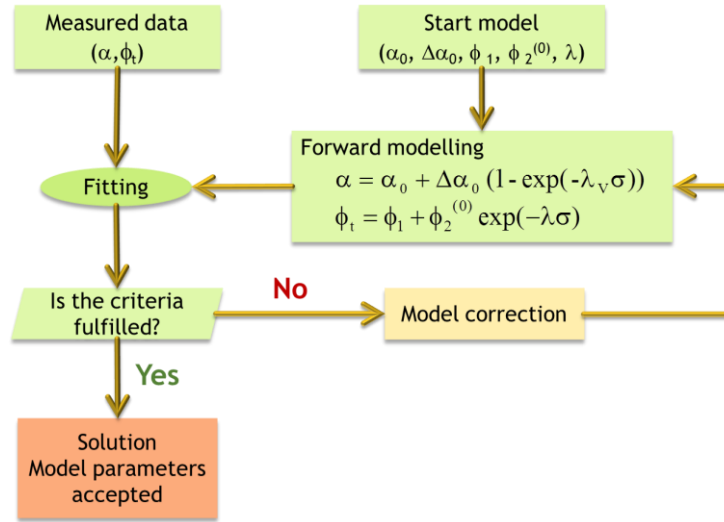


Fig. 1
Workflow of the joint inversion

CASE STUDY

In the rock physical laboratory of the Department of Geophysics P and S wave velocities can be measured under varying loading very accurately, but currently, the determination of pore volume during the measurements is not available. Therefore the applicability of the proposed porosity model is tested on data sets published in literature. Unfortunately, it is still not an easy task, because only a few publications are available, where both the velocity and the porosity were measured under varying loading.

The present paper is based on the data sets measured and published by Yang et al. [16]. They determined the P wave velocity with 1 MHz frequency using the impulse transmission technique [17]. The breccias and gouges are originated from the rupture zone of the Wenchuan earthquake happened in 2008. In this paper data obtained on samples „D3s-36” and „D3s-52” are processed. A short sample description can be found in Table 1.

Table 1
Sample description

Sample	Lithology	Mineral composition (weight%)	Density (g/cm ³)
D3s-36	black gouge	quartz39, albite5, microcline3, calcite21, dolomite2, total clay29	2,571
D3s-52	green-grey gouge	quartz8, albite49, calcite7, total clay36	2,766

Velocity and porosity data were processed according to the method presented in the section „Inversion”. The estimated model parameters with their estimation errors are summarized in Table 2. These parameters provide the possibility of the determination of velocities and porosities for any pressure by substituting them into the model equations (3) and (7).

Table 2

Model parameters (and their estimation errors) determined by linearized inversion

Sample	α_0 (km/s)	$\Delta\alpha_0$ (km/s)	λ (1/MPa)	ϕ_1 (-)	$\phi_2^{(0)}$ (-)
D3s-36	2,03(±0,05)	1,48(±0,03)	0,0079(±0,0005)	0,02(±0,01)	0,24(±0,01)
D3s-52	1,97(±0,11)	2,27(±0,10)	0,0081(±0,0005)	0,05(±0,01)	0,22(±0,01)

The results are plotted in Figure 2 and 3. Red dots indicate the measured data [16], the blue lines are the values calculated based on the rock physical models. It can be seen, that the distances between measured and calculated data are small, it is supported by the relative data distance values in Table 3 as well. For the sample „D3s-36” it is 3,6%, in case of the sample „D3s-52” it is a little bit higher, 8,4%, but it is still a perfectly acceptable result in engineering practice. The mean spread can be also seen in Table 3, it indicates that the model parameters are in moderate correlation.

Table 3

Relative data distances and mean spread in the last iteration step

	D3s-36	D3s-52
D (%)	3,6	8,4
S	0,57	0,58

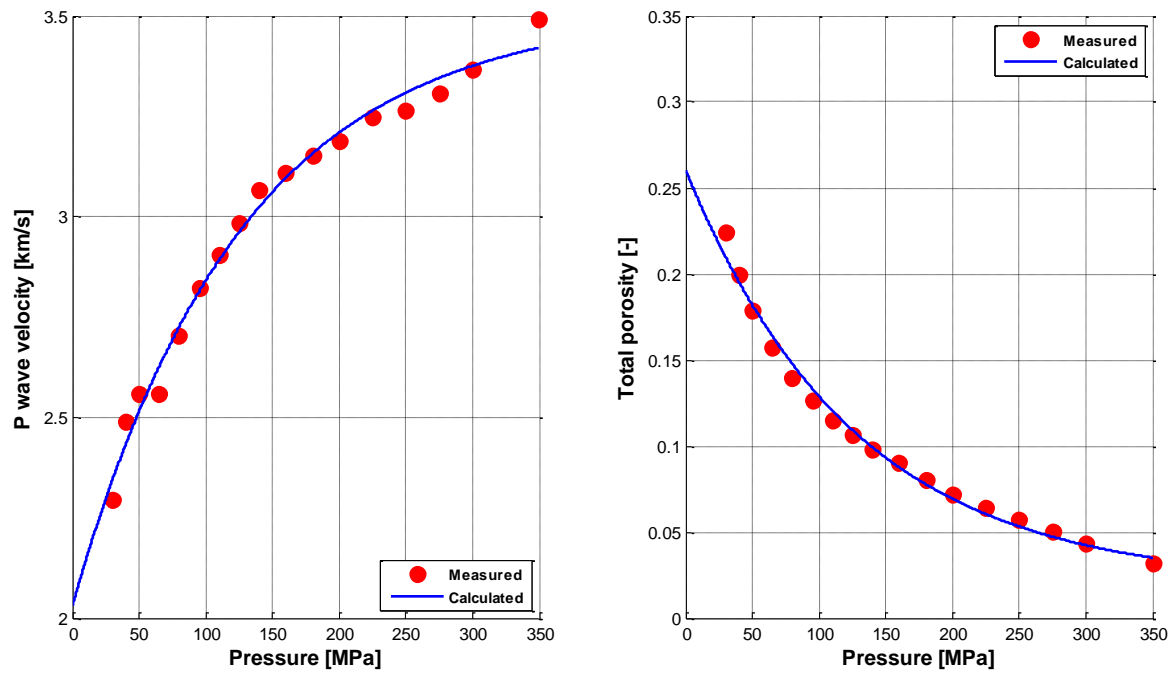


Fig. 2

Measured and calculated velocity and porosity data vs. pressure for sample D3s-36

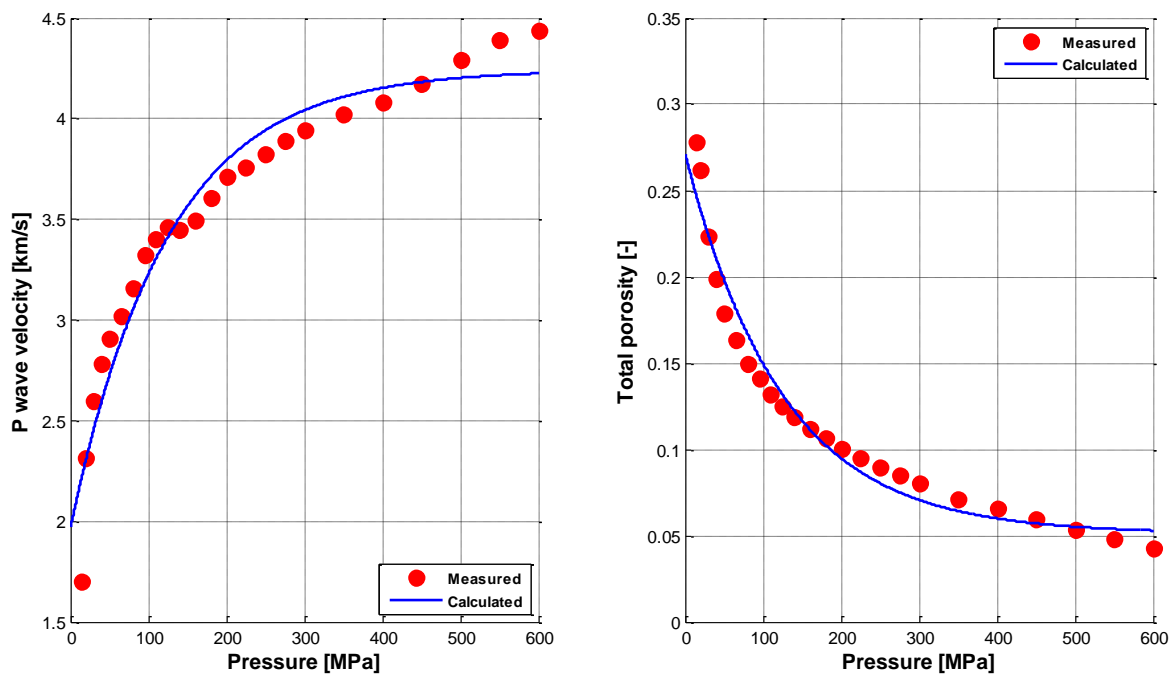


Fig. 3

Measured and calculated velocity and porosity data vs. pressure for sample D3s-52

CONCLUSIONS

In this paper, a rock physical model describing the pressure dependence of porosity was presented. The basic idea follows the analogy of previously introduced and successfully applied velocity model, namely that the increasing pressure closes the microcracks forming the secondary porosity of the rock. The model parameters in the

model equations were determined in quality-checked joint inversion because the λ rock physical parameter provided the required link between the velocity and porosity data sets. The models were applied with success on laboratory data published in the literature. Although the results are precise enough for the engineering problems, the model can be further refined by introducing new parameters.

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