Groundwater

Leaky Aquifer Models to Simulate the Well Flow in Fractured Aquifers with Linear Interporosity Flow

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Abstract

Following Hemker and Maas (1987) the models of two or three leaky aquifers are applied to simulate the flow to vertical wells operating in the fractured or dual porosity aquifers. The software WellTest (WT) (Székely 2015) is used for calculating the drawdown and discharge rate variation. The comparative analysis with the independent analytical solutions by Boulton and Streltsova-Adams (1978), Warren and Root (1963), Kazemi et al. (1969) concluded with acceptable agreement between the WT simulation and the alternate calculation methods. The selected field tests have been conducted in fractured limestone aquifers. The pumping test west of Copenhagen shows an example of fractured aquifer with considerable negative skin effect at the well face. The flowing well Wafra W1 in Kuwait operates in the two-zone aquifer exhibiting sufficient vertical recharge via leakage beyond a circular domain of estimated radius of 2460 m.

Introduction

The fractured or double porosity aquifers play an important role in water supply of areas characterized with the development of carbonate, sandstone, volcanic, and other hard rock formations. In such water-bearing geologic units the flow to the abstraction wells takes place predominantly along the fractures referred to as secondary porosity. These narrow flow channels convey the water stored in and drained from the extensive matrix blocks called primary porosity.

This paper evaluates and applies the models of two or three leaky aquifers with incompressible aquitards to simulate the flow in and around wells open to a single fracture of the double porosity aquifer with unconfined, semi-confined, and confined top. In this study pseudo-steady-state block-to-fissure flow (Moench 1984; Kruseman and De Ridder 1994) with incompressible matrix block is considered. This process is referred to as the linear interporosity flow prevailing between the primary and secondary porosity subdomains. The most complex case of unconfined aquifer is analyzed in comparison with the analytical solution by Boulton and Streltsova-Adams (1978) derived for drawdown around pumping well. The equivalent three-aquifer model setup was introduced and tested by Hemker and Maas (1987) for the distance of 1 m. We extended the verification for the full range of distance and time variables used in Table IV of Streltsova-Adams (1978). Parameters of this model can be changed to make it coherent with the double porosity model exhibiting confined top (Warren and Root 1963; Kazemi et al. 1969). Unlike the referenced approximate closed form solutions the equivalent models of two or three leaky aquifers are not limited in time, this feature is important in practical applications. Another modification allows for simulating the effect of semi-confined or leaky top. The options of confined and leaky top may also be modeled in the two-aquifer flow domain. Analyses of two field tests in fractured limestone aquifer document the practical application. The presented pumping test was conducted in confined aquifer whereas the evaluated constant head test was performed in a two-zone aquifer with confined and semi-confined top in the near and far zones, respectively.

Theoretical Analysis

Boulton and Streltsova-Adams (1978) developed an analytical model for the drawdown in a fractured

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unconfined aquifer caused by a well of infinitesimal radius with the effect of linear interporosity flow. Hemker and Maas (1987), Figure 4, model A) introduced a hydraulically equivalent model of three leaky aquifers where the aquifers numbered 1, 2, and 3 are used to simulate the unconfined top (water table), the fracture and the underlying matrix, respectively. The equivalent three-aquifer model involves transmissivities T_1, T_2, T_3 and storativities S_1, S_2, S_3 for aquifers as well as hydraulic resistances of aquitards c_2 and c_3 overlying the aquifers 2 and 3. In this study leakance parameters $B_2 = 1/c_2$ and $B_3 = 1/c_3$ are used to calculate the leakage through aquitards. In case of the emulated double-porosity aquifer parameters B_2 and B_3 control the delayed gravity response and the linear interporosity flow from/to the fracture, respectively. Based on the tabular dimensionless drawdown data at distance of 1 m (Streltsova-Adams 1978, Table IV) Hemker and Maas (1987) identified the parameters of the equivalent three-aquifer model as follows: $T_1 = T_3 = 0.001$ m^2/d, $T_2 = 1$ m^2/d, $B_2 = 0.01$ 1/d, $B_3 = 1$ 1/d and dimensionless storativities $S_1 = S_y = 0.1$, $S_2 = 0.0001$, $S_3 = 0.001$. The low T_1 and T_3 values approximate the zero transmissivity domains of the free surface (water table) and the matrix. The latter values result in diminishing radial flow in the relevant segments. The data obtained with the analytical double-porosity model and the replacing three-aquifer model exhibit small deviation for short and medium times, but the difference increases to 3% for the latest time (Hemker and Maas 1987, Table 3).

The present analysis extends the verification procedure to the all seven distances $r = 0.1$, 0.2, 0.5, 1, 2, 3, and 5 m in the Table IV (Streltsova-Adams 1978). The WT (WellTest) software (Székely 2015) includes the option of simulating the drawdown in the leaky three-aquifer systems by using Stehfest's (1970) numerical Laplace inversion (NLI) method (Hemker and Maas 1987) employing data based on the previous hydraulic parameters. Dimensionless drawdown and dimensional time are used for plotting. With seven exceptions the tabular dimensionless analytical drawdown $W_1(r,\theta)$ data by Streltsova-Adams exhibit low (< 0.2) deviation from the WT-NLI simulation (Figure 1), circles mark these data. Black squares visualize the seven deviating data. At distance $r = 0.1$ m the four deviating data points may be fitted to the curve by using the Warren and Root (1963) method for calculating the drawdown. Parameters T_2, S_2, B_3, and S_3 are used in calculations. The early and late time intervals with reduced slopes are the results of delay effects caused by the induced linear inflow from the matrix and the delayed gravity response (delayed water table depletion), respectively. The above numerical testing has also been performed by using the axi-symmetrical FD (Finite Difference) technique available with the WT software. The comparative analysis concluded with similar deviation between the analytical and WT-FD simulated drawdown data.

In this study two modifications of the above base model are used for practical analysis. One may notice

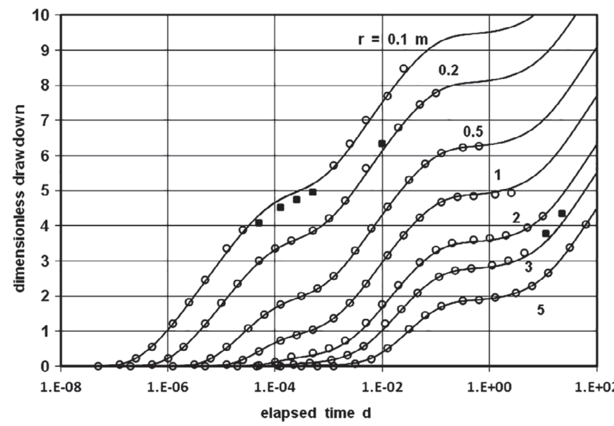

Figure 1. WT-NLI simulation of dimensionless drawdown data in Table IV by Streltsova-Adams (1978).

that by setting $S_1 = S_y = 0$ or $B_2 = 0$ the water table gets cleared or disconnected, respectively. In both cases the system turns into a confined fractured aquifer analyzed by Warren and Root (1963) and Kazemi et al. (1969). This option is referred to as model I. By letting $S_1 = S_y \to \infty$ the water table depletion diminishes which is equivalent to the fix head (zero drawdown) top providing vertical recharge to the fracture by means of leakage. This option is called as model II. At long elapsed time the model II reaches the steady-state flow conditions, at this stage the drawdown follows the De Glee solution (De Glee 1930; Kruseman and De Ridder 1994) where $L = (T_2 / B_2)^{1/2}$.

Figure 2 compares the drawdown in the fracture at distance of 1 m for the options of unconfined ($S_1 = 0.1$ model A by Hemker and Maas 1987), confined ($S_1 = 0$ model I) and semi-confined ($S_1 = 10^8$ model II) fractured aquifer. Solid lines display the result of the three-aquifer WT-NLI simulation. Circles along the upper line follow the well known closed form solution for confined double-porosity aquifers (Warren and Root 1963 Equation 15, Kazemi et al. 1969 Equation 9, Kruseman and De Ridder 1994 Equations 17.1 and 17.9) with the option of strata-type fractured aquifer. These theories require the dimensionless interporosity flow coefficient λ which for the equivalent three-aquifer flow domain can be written as $\lambda = r^2 B_3/T_2$ where r is used for the drilling radius r_d or for the distance to the observation point r_{ob}. According to the time limitation of Equations 9 (Kazemi et al. 1969) and 17.9 (Kruseman and De Ridder 1994) for the current parametric values the closed-form approximation is valid for the time interval $t > 0.005$ d. Circles for the model A correspond to the data in the last column of Table 3 (Hemker and Maas 1987). Finally circles at the end of the lower curve show the steady-state drawdown according to De Glee (1930).

The three-aquifer models I and II with confined and semi-confined (leaky) top are considered as the proper base approximation of the fissured aquifers in the forthcoming two case studies. One may notice that in these three-aquifer models the top segment 1 can be ignored or may be replaced with a constant head source

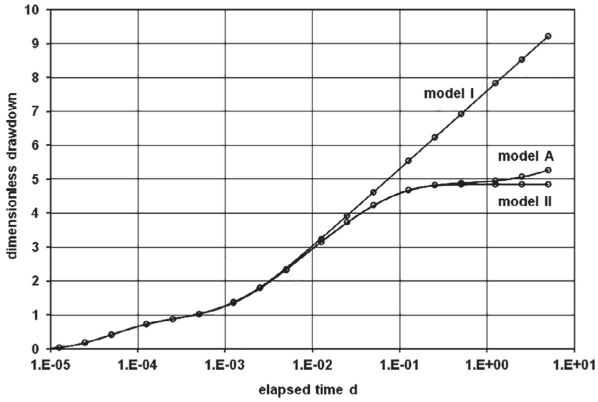

Figure 2. WT-NLI simulation of dimensionless drawdown data for fractured models A, I, and II.

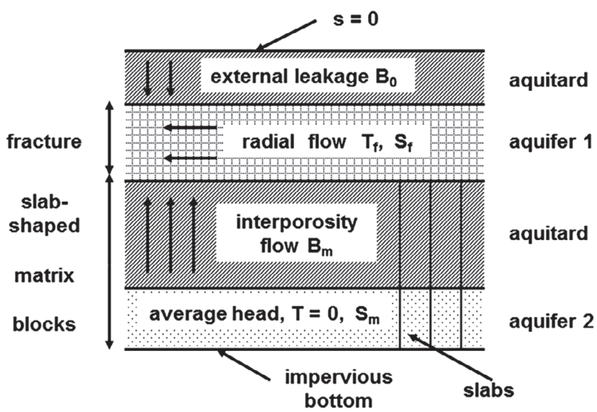

Figure 3. Leaky aquifer approximation of the formation with single fracture.

bed, respectively. These circumstances allow for removing the top segment and applying reduced but hydraulically equivalent two-aquifer models.

In the case of emulating the confined model I the fracture is to be overlain by an impermeable aquiclude, this reduced version is referred to as model I*. In the case of approximating the semi-confined model II the two-aquifer equivalent has an incompressible aquitard above the fracture with zero drawdown $s = 0$ condition at the top. This version is called model II*. These reduced, two-aquifer models have been successfully verified against the previous three-aquifer tests with options of $S_1 = 0$ and $S_1 = 10^8$.

The geometry of the two-aquifer model is shown in Figure 3. The aquifer 1 corresponds to the fracture or secondary porosity and provides the radial flow toward the well. The aquifer 2 of zero transmissivity stores the water of the matrix blocks or primary porosity. The interbedded aquitard controls the linear interporosity flow between the matrix blocks and the fracture. The top aquitard provides the optional external recharge for semi-confined formations. In confined flow systems this unit is to be replaced with an aquiclude.

Based on the agreement of the results obtained with the three and two aquifer models the latter are applied in the field test analyses. According to Figure 3 in this reduced model option symbols T_f and S_f are used for the transmissivity and storativity of the fracture, S_m denotes the storativity of the matrix, B_m is used for the leakance controlling the linear interporosity flow and B_0 denotes the leakance of the optional top aquitard controlling the leakage from the constant head source bed to the fracture(s). In this notation the dimensionless interporosity flow coefficient is defined as $\lambda = r^2 B_m / T_f$. In both case studies the skin loss is in effect, the option of thin resistant skin layer on the drilling face (Kruseman and De Ridder 1994) is considered.

In the WT modeling environment (Székely 2015) the double-porosity models represented by leaky aquifers have the ability of (1) simulating pumping, constant head, and slug tests, (2) modeling multiple fractures, (3) considering two-zone flow domains, and (4) applying time-variant skin loss parameters. With two exceptions (slug test and multiple fractures) these options are used in the forthcoming case studies.

Pumping Test West of Copenhagen

Nielsen (2007) described a pumping test performed west of Copenhagen in the fractured Hoeje Taastrup limestone. The 0.25 d-long test involved a pumping ($r_d = 0.1$ m) and an observation well ($r_{ob} = 18$ m). Constant pumping rate of $Q_w = 362.4$ m^3/d was applied during testing. The author applied the linear interporosity flow model. In parameter estimation the analytical method by Warren and Root (1963) and the type curve fitting method by Streltsova (1976) are used for the pumping and observation wells, respectively. The first analysis resulted in negative dimensionless skin factor of $\sigma = -4.14$, the two independent analyses yielded two different sets of T_f, S_f, λ and S_m parameters. The formation parameters T_f, S_f, and S_m of the first and second calibrations show moderate (less than a magnitude) difference: $T_f = 1002$ and 1443 m^2/d, $S_f = 0.000712$ and 0.00223, $S_m = 0.0203$ and 0.0207, respectively.

In this alternate analysis the confined fractured aquifer model I* is used for simulation. The WT-NLI modeling involved (1) very low (10^{-10} m^2/d) transmissivity for the matrix, (2) the option of finite well radius (Hemker 1999), and (3) coupled analysis by simulating the drawdown evolution in both wells. In the automatic parameter estimation the available σ parameter and the arithmetic average values of the formation parameters by Nielsen (2007) are assumed as initial guess in calibration.

The HC software (Székely 2015) is used for the calibration, this program employs the nonlinear multiregression code by Garbow et al. (1996). The parameter optimization (1) ignored the very early three outlier head measurements in the pumping well at values of less than 5 cm and (2) considered an undivided aquifer test producing 132 drawdown measurements in the two wells. The model calibration concluded with the following result: $\sigma = -3.418$ for the pumping well and $T_f = 1152$ m^2/d,

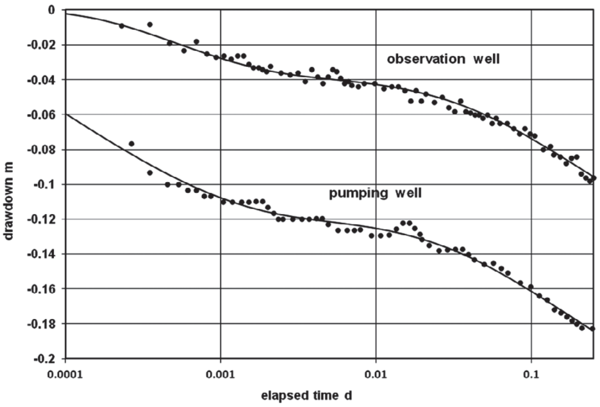

Figure 4. The simulated (solid lines) and measured (dots) drawdown in the pumping and observation wells west of Copenhagen.

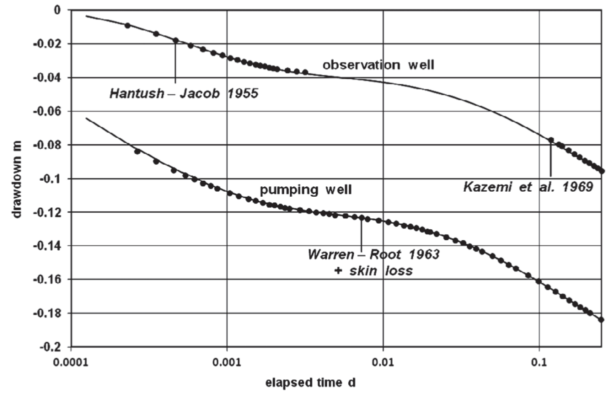

Figure 5. The WT-NLI simulated (solid lines) and the independent analytical (dots) drawdown in the pumping and observation wells west of Copenhagen.

$S_f = 0.002209$, $S_m = 0.04206$, $B_m = 1.334$ 1/d for both wells. The transmissivity and the storativity of the fracture are within the range of the Nielsen's values, the storativity of the matrix exceeds data obtained with that dual calibration.

Figure 4 displays the simulated (solid lines) and the measured (circles) drawdown data in both wells. As a result of the model calibration the starting averaged absolute model discrepancy of 0.00962 m with reference to the measured data reduced to 0.00223 m.

The effect of nonzero transmissivity of the matrix domain is numerically evaluated. WT-FD simulations have been conducted with transmissivity values of 0 and 10^{-10} m^2/d. No deviation between the two results is detected.

In order to verify the above parameters numerical groundwater flow modeling with the FLOW (Székely 1998) and the MODFLOW based GMS packages (AQUAVEO 2013) was also conducted. The FLOW software is based on the point centered scheme with nodes at the corners of the blocks. The simulation concluded with low mean absolute model discrepancy of 0.00237 m between the simulated and the measured drawdown data. The block centered GMS package showed close result of 0.00270 m.

The WT-NLI simulated data of the presented field test analysis are in coherence with the relevant analytical solutions. Figure 5 documents this verification test. The solid lines show the result of the WT-NLI simulation whereas dots mark the alternate, independent analytical drawdown data referring to the observation time applied during the testing. In the case of the pumping well the analytical, closed form solution by Warren and Root (1963) is valid over the whole testing time. After correction for the skin loss the averaged absolute model discrepancy between the analytical and the WT-NLI simulated drawdown data gets as low as 0.000187 m. For the observation well two analytical solutions may be used for approximation. At late time the closed form solution by Kazemi et al. (1969) may be applied. At short time the drawdown in the matrix may be negligible small and the equation by Hantush and Jacob (1955) for the single leaky aquifer can be appropriate for the head depletion in both wells. In the observation well the mean absolute model discrepancy between the analytical and the WT-NLI simulated drawdown data is 0.000382 m.

Free Flow Test in South Kuwait

The aquifer systems in Kuwait comprise mainly the clastic Neogene-Quaternary Kuwait Group and the underlying carbonate Eocene Dammam Formation. The latter is underlain with the practically impervious Rus formation (Mukhopadhyay et al. 1994). In the Wafra farm area the Dammam Formation aquifer is composed of fissured, karstified chalky limestone and dolomite overlain by a low-permeability chert.

The 97-m deep, high-capacity free-flowing well WW1 of a 310-mm internal diameter has been drilled in the Al-Wafra Farm Area, located in the southern part of the State of Kuwait. The upper section of the well is cased in the Kuwait Group aquifer, whereas the lower section, at depth between 65 and 97 m, is open in the carbonate Dammam Formation. This aquifer has an estimated thickness of 150 m. At the well site the chert along with the dense layers of siliceous gypsum and anhydrite at the top of the Dammam formation create a confining bed above the carbonate aquifer. During the well construction a sharp increase of static level and a gushing, free flow of brackish water at an exceptionally high rate were detected at the bottom of the hole, and the drilling was stopped.

The overflow experiment was conducted by the Ministry of Electricity and Water in 1974, between May 15 and 21. The discharge rate through an orifice pipe with a 0.229-m diameter was measured in the range of 30,920 and 25,460 m^3/d. Circles in Figure 6 exhibit the measured discharge rate data. The original 22.25-m static piezometric head above the closed wellhead recovered within 2 d after shutdown.

The earlier computer-aided model calibration (Székely 2015) assumed the Dammam aquifer as a two-aquifer system exhibiting high and low transmissivities

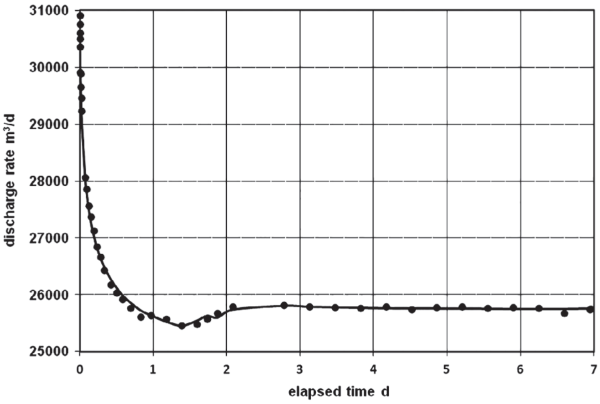

Figure 6. The simulated (solid lines) and measured (dots) discharge rates in the flowing well Wafra W1 in Kuwait.

in the upper and lower sections, respectively. The analysis pointed out the two-zone nature of the tapped formation. The near zone around the well was assumed as confined domain with dense siliceous gypsum and anhydrite sections at the top covered by the very low permeability chert at the bottom of the clastic Kuwait Group aquifer. This lithological feature sustains the high head difference between the Dammam and Kuwait Group aquifers. By contrast the far zone beyond the estimated radius $R_{far} = 2460$ m was considered as the area receiving induced vertical recharge or leakage from the Kuwait Group aquifer during the overflow test. This leakage takes place through some huge deep faults known in the nearby area. They may break up the low permeability top of the Dammam Formation and at some locations can provide the vertical hydraulic contact necessary for recharge from the layered Kuwait Group aquifer. This recharge caused stabilization of the discharge rate of the well after the elapsed time of 2.8 d.

The mathematical model for hydraulic analysis (Székely 2015 Equations III.7–8) includes three important well bore effects controlling the free flow rate variation during the test: (1) the kinetic energy carried out by the water jet leaving the well bore, (2) the time variant turbulent skin loss, and (3) the inertia effect controlling the acceleration of the water column in the well bore. The first and second energy terms reduce the available drawdown at the drilling face. The last process controls the early time rise of the discharge rate from zero to the maximum value.

The presented new well test data analysis involves fractured aquifer models I^* and II^* in the near and far zones, respectively. The parameters of the previous analysis by Székely (2015) are used as initial guess in model calibration and the WT-FD technique is applied in the WT simulation. This method allows for $T = 0$ m^2/d in the matrix domain. The optimization with the HC software yielded the following parameters: $T_f = 2175$ m^2/d, $S_f = 3.856 \times 10^{-6}$, $S_m = 2.095 \times 10^{-4}$, $B_m = 0.01541$ 1/d in both zones, whereas $B_0 = 0$ and 2.697×10^{-4} 1/d in the near and far zones, respectively. The quadratic skin loss due to the turbulent flow is defined according to equation 14.2 in Kruseman and De Ridder (1994). The temporal increase of the discharge rate prior to the flow rate stabilization indicated reduction of the skin loss parameter with time. The quadratic well-loss coefficient C decreased from 8.0×10^{-10} to 5.7×10^{-12} d^2/m^5 during the test. This change is probably associated with the sufficient cleanup of the karstic flow channels owing to the high flow velocity around the well. The averaged absolute model discrepancy between the measured and simulated discharge rate data is 51.28 m^3/d. Figure 6 documents the good fit between the measured (circles) and simulated (solid line) overflow rate data.

Summary

The theoretical analysis along with numerical testing proved the similarity of flow to pumping wells operating in fractured and leaky aquifers. As shown by Hemker and Maas (1987) the model of three leaky aquifers is suitable for approximating the drawdown in unconfined dual-porosity aquifer analytically evaluated by Streltsova-Adams (1978). The well flow in fractured aquifers with confined or semi-confined top may be approximated with analytical models developed for two leaky aquifers. The computer software WT (Székely 2015) is used for numerical comparison. The pumping test west of Copenhagen is evaluated to define parameters of the fractured aquifer and the skin factor of the pumping well. The drawdown evolution in the pumping and observation wells is analyzed. The concept of dual-porosity aquifer is also applied to simulate the flow rate variation in a high capacity flowing well discharging a two-zone fractured aquifer in Kuwait. The effects of kinetic energy variation and turbulent skin loss with time variant skin parameters are considered.

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