2D INVERSION OF BOREHOLE LOGGING DATA FOR SIMULTANEOUS DETERMINATION OF ROCK INTERFACES AND PETROPHYSICAL PARAMETERS

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In the paper a 2-D joint inversion method is presented, which is applicable for the simultaneous determination of layer thickness variation and petrophysical parameters by processing well-logging data acquired in several boreholes along the profile. The so-called interval inversion method is tested on noisy synthetic data sets generated on hydrocarbon-bearing reservoir models. Numerical experiments are performed to study the convergence and stability of the inversion procedure. Data and model misfit, function distance related to layer thickness fitting are measured as well as estimation errors and correlation coefficients are computed to check the accuracy and reliability of inversion results. It is shown that the actual inversion procedure is stable and highly accurate, which arises from the great over-determination feature of the inverse problem. Even a case study is attached to the paper in which interval inversion procedure is applied for processing of multi-borehole logging data acquired in Hungarian hydrocarbon exploratory wells in order to determine petrophysical parameters and lateral changes of layer thicknesses.

Keywords: petrophysical parameter, well-logging data, interval inversion, probe response function, layer thickness function, Legendre polynomial, global optimization, hybrid optimization.

1. Introduction

Borehole geophysical measurements play an important role both in exploration and engineering geophysics. The processing of well-logging data is applicable to determine petrophysical and geometrical properties of geological structures in the near vicinity of the borehole. Downhole logging data contain information about porosity of rocks, water and hydrocarbon saturation of the pore space, specific volume of mineral constituents including shale, hydraulic permeability and certain geometrical properties like layer thickness and local dip of the formations. Beside the quantity of information, professional practice lays ever-increasing claim also to the quality of the interpretation results. This purpose requires advanced well-log analysis methods steadily. Beside deterministic procedures geophysical inversion methods are widely used nowadays for extracting the required information from the borehole logging data set.

Well-logging inverse problems are conventionally solved by means of local inversion methods that process the data set acquired in a certain depth-point of the borehole so as to determine petrophysical model parameters only to that point. While geometrical parameters are computed out of inversion, this technique represents a narrow type of overdetermined inverse problem, because the possible number of sonde data is slightly more than that of the petrophysical unknowns in the given point. This technique leads to a set of separate inversion runs in adjacent depth points for the logging interval. Local inversion approach comprises several different mathematical methods that are quick and able to produce proper solution provided having well-chosen initial model and reliable a priori information. We can see that

industrial applications are mainly based on local inversion methods like Schlumberger Global (Mayer, 1980), Gearhart Ultra (Alberty et al., 1984) and Baker Hughes Optima (Fertl et al., 1987).

It is important fact also from the practical point of view that the marginal over-determination of the local well-logging inverse problem sets a limit to the accuracy and reliability of the parameter estimation. If we invert a data set of a greater depth-interval (like layers or zones) jointly in one inversion procedure, it can result in much more accurate and reliable estimation, because it is inevitable that the increase of overdetermination makes good improvement in quality of interpretation results. One more problem comes from the defect of local inversion, too. Methods applying gradient searching mathematics (linear optimization) during the inversion procedure tend to assign the solution to a local optimum of the objective function. This problem can effectively be reformed by using global optimization techniques like Simulated Annealing (Metropolis, 1953) that search for the absolute extremum of the objective function. Our goal was to develop a highly overdetermined global optimum seeking algorithm in order to compute higher accuracy solution for the petrophysical parameters (and other new unknowns) than can be given by local inversion.

A new well-logging inversion methodology was developed by Dobróka (1991). The so-called *interval inversion method* integrates borehole logging data acquired from different kind of borehole probes of optional depth-interval to one set, which is then inverted jointly to determine petrophysical parameters. As opposed to local inversion methods it gives not only an estimation to the point but to the entire processed depth-interval. The interval inversion method is based on the series expansion of the model parameters and straightforward modeling using depth-dependant probe response functions. Joint inversion algorithm of the integrated data set can be formulated to have order(s) of magnitude greater overdetermination ratio (data/model parameter ratio) than in case of local inversion. The development of the inversion method is the product of the Inversion and Tomography Research Team of the Department of Geophysics, University of Miskolc. The research work was also supported by the industry in the form of co-operation with the Hungarian Oil and Gas Company (2003-2008). A Ph.D. thesis on the research was written by Szabó (2004).

Interval inversion method has shown further perspectives lately in formation evaluation. Applications have been established on its property that the number of inversion unknowns can be freely increased without significant decrease of overdetermination ratio. If parameter sensitivities are high enough, the range of inversion unknowns can be extended to more well-logging interpretation parameters including non-conventional ones, too. Some types of these quantities appear in the set of probe response functions explicitly. They are called *zone parameters*, which are practically treated as constants in order to avoid ambiguity going with underdetermined local inverse problems. Zone parameters like textural, fluid and matrix properties of rocks are actually varying also within the zone and it can be shown that interpretation results are highly sensitive on choosing their right values. Having information for zone parameters only from references and laboratory data, it is advisable to compute them in situ from well logs. Interval inversion method completely supports this task. For instance, cementation exponent as a new inversion unknown was determined simultaneously with petrophysical parameters by Dobróka et al. (2007), and the three most important textural parameters (cementation, saturation exponent and tortuosity factor) were computed by Szabó (2009). Another group of inversion unknowns is the circle of *specific volumes of mineral components*. The number of matrix components is typically 1-3 (i. e. sandstone, limestone, dolomite) in case of sedimentary rocks, but in the case of metamorphic or volcanic rocks

more than 5 mineral constituents frequently occur. Being underdetermined inverse problem, local inversion methods cannot compute their values in the point. In the interval inversion procedure these new parameters can be taken into the procedure effectively. Interpretation examples of complex reservoir rocks having 5-6 mineral components including shale were presented by Dobróka et al. (2008).

Another group of non-conventional inversion unknowns is not included in probe response functions. For instance, layer boundary coordinates do not appear in the local response functions attached to the straightforward problem so they cannot be computed by any local inversion method. Log analysts handle this problem manually for the lack of any automatic procedure. However, information for the *layer thicknesses* is contained in the data set of the log, which can be extracted by the interval inversion method. Layer boundary coordinates treated as inversion unknowns were published earlier in Dobróka et al. (2002, 2003) and Szabó (2004). These papers proved that it is possible to compute the positions of interfaces of subsequent layers by inversion. As a new application we present the extension of the interval inversion method for 2-D geometries by inverting borehole logging data acquired from several boreholes jointly. The multi-well version of interval inversion procedure is useful to determine the morphology of geological structures away from the borehole while computing petrophysical parameters in the same time. This paper sums up the results of relevant research work supported by the Hungarian National Research Fund (OTKA).

2. The interval inversion method

In hydrocarbon exploration the goal of open-hole logging data processing is to separate layers according to lithology, determine their thicknesses and compute several petrophysical parameters underlying the calculation of hydrocarbon reserves. Petropysical parameters are dimensionless volumetric quantities like porosity (POR), water saturation both in the flushed (SX0) and the virgin zone (SW), shale-content (VSH) and specific volume of rock matrixes (VMA). These parameters cannot be measured directly, but are connected to well-logging data via probe response equations. As a rule, a borehole logging data set for hydrocarbon exploration consists of logs based on different physical principles and grouped by their sensitivity to lithology, porosity and saturation (Table I).

Table I.

At first let us see the local inverse problem. All borehole logging data measured in a certain depth point are collected into the following column vector

$$\vec{d}^{(m)} = [d_1, d_2, \ldots, d_N]^T,$$

where N denotes the number of probes applied in the point (T is the symbol of conversion of the vector). Petrophysical model parameters also form a vector

$$\vec{m} = [m_1, m_2, \ldots, m_M]^T,$$

where M is the number of model parameters in the same point. In case of N>M, the inverse problem is called overdetermined. The petrophysical relation between the model and data vector is called probe response function, which can be written in a general form as

$$\vec{d} = \vec{g}(\vec{m}, \vec{c}),$$

where $\vec{c}$ includes a great number of zone parameters treated as constants during the inversion. In the straightforward modeling problem borehole logging data can be calculated by local response functions

$$d_k^{(c)} = g_k(m_1, m_2, \ldots, m_M),$$

where k denotes the k-th logging instrument (k=1,2,…,N) applied in the point. The whole formulation leads to a marginally overdetermined inverse problem, because the number of applied probes (i.e. data) is slightly more than that of the model parameters. Therefore, it is worth inverting data of a greater depth-interval jointly in one inversion procedure, which can increase the overdetermination ratio greatly. The so-called interval inversion method is based on the establishment of *depth-dependent response functions* containing vertically varying model parameters ($m_i(z)$, i=1,2,…,M). They calculate the k-th theoretical well-logging data to depth z as

$$d_k^{(c)}(z) = g_k(m_1(z), m_2(z), \ldots, m_M(z)).$$

In the above formula model parameters are continous functions that have to be discretized for the numerical computations. A discretization method based on series expansion was suggested by Dobróka (2002)

$$m_i(z) = \sum_{q=1}^{Q_i} C_q^{(i)} \Phi_q(z),$$

where C_q is the q-th expansion coefficient and Φ_q is the accordant basis function (up to Q number of additive term). Basis functions are known quantities, which may be chosen arbitrary to the actual geological situation. For instance, if we assume layer-wise homogeneous petrophysical model than unit step functions can be practically used for the series expansion

$$\Phi_q = u(z - Z_{q-1}) - u(z - Z_q),$$

where Z_q is the depth coordinate of the q-th layer. The above expression gives always zero except in the q-th layer, where C_q equals exactly m_q model parameter value. It means that each characteristic layer parameter can be described by only one series expansion coefficient. In our study let us hold down to this case, however, it must be mentioned that inhomogenious layers with more complicated or faster parameter variations can also be treated with this series expansion method (Dobróka et al., 2005). In the general case, model parameters are replaced by series expansion coefficients that have to be determined by the interval inversion procedure. Then the combined model vector as the vector of inversion unknowns can be written as

$$\vec{m} = \left[C_1^{(1)}, \ldots, C_{Q_1}^{(1)}, C_1^{(2)}, \ldots, C_{Q_2}^{(2)}, \ldots\ldots, C_1^{(M)}, \ldots, C_{Q_M}^{(M)}\right],$$

wherewith the set of probe response functions becomes to

$$d_k^{(c)}(z) = g_k\left(z, C_1^{(1)}, \ldots, C_{Q_1}^{(1)}, C_1^{(2)}, \ldots, C_{Q_2}^{(2)}, \ldots\ldots, C_1^{(M)}, \ldots, C_{Q_M}^{(M)}\right).$$

The solution of the overdetermined inverse problem is bound to the minimal misfit between the measured and calculated data. We applied the following *objective function* for the minimization

$$E = \sum_{p=1}^{P}\sum_{k=1}^{N}\left(\frac{d_{pk}^{(m)} - d_{pk}^{(c)}}{d_{pk}^{(m)}}\right)^2 \rightarrow \min,$$

where $d_{pk}^{(m)}$ and $d_{pk}^{(c)}$ denote separately the measured and predicted data for the k-th probe in the p-th depth point. The above formula shows that all data of the processed interval appear in the objective function and the overall error is minimized. There are many ways of seeking the optimum by several existing mathematical methods. Linear optimization methods are the most prevailing ones in practice, because they are very quick and effective algorithms in case of having an initial model close to the solution. However, they are not absolute minimum searching methods and generally assign the solution to a local optimum of the objective function. This problem can be reformed by the Simulated Annealing (SA) method, which performs the global optimization of the objective function by random walking in the parameter space (Metropolis et al., 1953). Only problem with global inversion is long CPU time, which can be reduced significantly by using Very Fast Simulated Annealing (see VFSA in Appendix) or hybrid (linear and global) algorithms. The hybrid optimization method using subsequent global and linear iteration steps was suggested by Dobróka et al. (2005). This combined inversion technique is not merely faster but also capable of quality checking of inversion results, which cannot be performed with pure global optimization from one inversion program run.

Method for quality checking of inversion results was adopted by us from Menke's discrete inverse theory work (1984). In case of linear methods the connection between $\underline{\underline{cov}}(\vec{d})$ data and $\underline{\underline{cov}}(\vec{m})$ model covariance matrix is

$$\underline{\underline{cov}}(\vec{m}) = \underline{\underline{A}} \cdot \underline{\underline{cov}}(\vec{d}) \cdot \underline{\underline{A}}^T,$$

where $\underline{\underline{A}}$ denotes the general inverse matrix of the actual inversion method. By this formula, it is possible to compute the error of estimated parameters if dispersion of data is known. The *estimation error* of the i-th model parameter is obtained from the main diagonal of the covariance matrix

$$\sigma_{m_i} = \sqrt{\underline{\underline{cov}}(\vec{m})_{ii}}. \qquad (1)$$

The reliability of inversion results can also be quantified by computing the *correlation coefficient*, which indicates the degree of linear dependence between the i-th and j-th model parameters

$$\underline{\underline{\mathrm{corr}}}(\vec{m})_{ij} = \frac{\underline{\underline{\mathrm{cov}}}(\vec{m})_{ij}}{\sqrt{\underline{\underline{\mathrm{cov}}}(\vec{m})_{ii}\,\underline{\underline{\mathrm{cov}}}(\vec{m})_{jj}}}\,. \tag{2}$$

If the correlation coefficient is close to ±1, there is a strong connection between the model parameters referring to unreliable solution. Reliable solution requires a model with uncorrelated parameters with correlation coefficients between 0-0.4. Sometimes it is practical to characterize the correlation matrix by one scalar, which is called the *mean spread*

$$S = \sqrt{\frac{1}{M(M\text{-}1)}\sum_{i=1}^{M}\sum_{j=1}^{M}\left(\underline{\underline{\mathrm{corr}}}(\vec{m})_{ij} - \delta_{ij}\right)^2}\,, \tag{3}$$

where δ denotes the Cronecker-delta symbol (which is 1 in case of i=j, and 0 otherwise).

3. The 2-D interval inversion method

Overdetermination can be raised, if data samples acquired in more than one borehole are integrated into the interval inversion procedure. Let us see Figure 1. Conventional local inversion method (Fig.1a) gives an estimate for the petrophysical parameter (for instance to m_2) to one point somewhere along the borehole. This is a marginally overdetermined inverse problem as having barely more data types than model parameters in the point. Obviously the position of the layer boundaries cannot be extracted from this point related information. On the contrary 1-D interval inversion method processes a data set of a greater depth interval exhibiting a greater extent of overdetermination (Fig.1b). Besides having more accurate results layer boundary coordinates (Z_1, Z_2) can be computed automatically, too. This inversion strategy can be extended to the case of 2-D models, where petrophysical parameters (m_1, m_2, m_3) and lateral changes of layer boundaries ($Z_1(x)$, $Z_2(x)$) are the unknowns of the joint inversion procedure. This is called 2-D interval inversion method (Fig.1c).

Let x be the horizontal coordinate in the Cartesian reference system along what boreholes are situated. After defining petrophysical parameters as a function of x and z coordinates, borehole logging data are computed by straightforward modeling using the following response functions

$$d_k^{(c)}(x,z) = g_k\left(m_1(x,z), m_2(x,z), \ldots, m_M(x,z)\right),$$

where $d_k^{(c)}(x,z)$ denotes the k-th calculated data in depth z of the borehole positioned at x coordinate and $m_i(x,z)$ is the value of the i-th petrophysical parameter in the same point. In order to discretize the model variables a similar series expansion technique is used than in case of the 1-D interval inversion method. The i-th model parameter is expanded into series

$$m_i(x,z) = \sum_{q=1}^{Q_i} C_q^{(i)} \Phi_q(x,z),$$

where $C_q^{(i)}$ is the q-th expansion coefficient and Φ_q is the q-th basis function of two variables given beforehand. Inversion experiments show that it is beneficial to chose orthogonal set of basis functions like Legendre polynomials so as to get the most reliable (least correlated)

model results. The system of *Legendre polynomials* is feasible and meets the following requirement

$$\int_{-1}^{1} \Phi_n(x)\Phi_m(x)dx = 0 \quad (n \neq m), \text{ where } \Phi_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2 - 1\right)^n.$$

The unknowns of the inverse problem are the C series expansion coefficients and the solution can be determined by the minimization of the following objective function

$$E = \frac{1}{FPN}\sum_{f=1}^{F}\sum_{p=1}^{P}\sum_{k=1}^{N}\left(\frac{d_{fpk}^{(m)} - d_{fpk}^{(c)}}{d_{fpk}^{(m)}}\right)^2 \rightarrow \min ,$$

where F denotes the number of boreholes, P is the number of depth points representing the interval processed and N is the number of applied probes in each well. For the minimization a hybrid optimization method was used, which applied subsequent global and linear inversion steps. Starting with VFSA (see in Appendix) the near vicinity of the optimum was approached and then we switched the algorithm to DLSQ (Damped Least Squares) in order to perform linear optimization (Marquardt, 1959). At the end of the procedure estimation errors were computed for the optimum by Eq.(1).

The accuracy of inversion results can be measured by the elements of the covariance matrix and correlation coefficients (Eqs.1-2). Besides we can define another index number, which informs about the fitting between measured and calculated data computed on the resultant model. *Relative data distance* in per cent is

$$D_d = 100\sqrt{E} \ . \tag{4}$$

For the case of synthetic tests another quantity can also be introduced, which measures the goodness of model fit. In per cent *model distance* is

$$D_m = 100\sqrt{\frac{1}{LM}\sum_{l=1}^{L}\sum_{i=1}^{M}\left(m_{li}^{(e)} - m_{li}^{(r)}\right)^2} \ , \tag{5}$$

where $m_{li}^{(e)}$ and $m_{li}^{(r)}$ denote separately the i-th estimated and exactly known (reference) model parameter in the l-th layer. It can be mentioned that normalization after summing in the formula is redundant here, because each model parameter falls in the same order of magnitude. There is an additional quantity, which is feasible. We can measure the distance between the layer boundary function determined by the 2-D interval inversion method and the reference function of the target model in case of synthetic tests. The misfit is characterized with the *relative function distance*

$$D_f = 100\sqrt{\frac{1}{(L-1)X}\sum_{l=1}^{L-1}\sum_{h=1}^{X}\left(\frac{Z_l^{(r)}(x_h) - Z_l^{(e)}(x_h)}{Z_l^{(r)}(x_h)}\right)^2} \ , \tag{6}$$

where $Z_l^{(r)}(x_h)$ and $Z_l^{(e)}(x_h)$ are separately the l-th reference and estimated layer thickness function in x_h position. The distance is obtained in per cent here, too.

4. Synthetic inversion experiments

In order to test the performance of the 2-D interval inversion algorithm numerical experiments were made on synthetic models. Inversion of noisy synthetic well-logging data sets informs us about how the procedure finds its way back to an exactly known target model. Numerical test parameters watched such as data, model, function distance, estimation error and correlation matrix are informative about the accuracy and reliability of the inversion method itself. Our inversion strategy was to determine layer-wise constant petrophysical parameters and layer thickness functions over 2-D structures. In this chapter, two cases are discussed relating to hydrocarbon-bearing reservoir models. In both cases synthetic borehole logging data were generated by means of Gearhart Ultra response equations (Alberty et al., 1984).

4.1 Inversion over 2-D shaly sand structure

A three-layered anticlinal structure (Model-A) made up of water-bearing sand, shale and hydrocarbon-bearing sand formations was chosen for first investigations (Fig. 2). In the figure, $x'=x/\Delta$, $z'=z/\Delta$ represent dimensionless lateral and depth coordinates (Δ is chosen appropriately for x and z coordinates, separately). In order to simulate real measurements, at first synthetic data were calculated in five wells (F-1–F-2) with dz=0.1m sampling interval and then 5 per cent Gaussian noise were added to N=5555 data samples. Computed well log types can be found in Table I and the logs can be seen in Fig. 3. We applied orthogonal functions for the series expansion. Layer thickness functions were generated as quadratic Legendre polynomials over the $x' \subset [-1,1]$ interval

$$Z_1'(x') = 1.2 \cdot \frac{3x'^2 - 1}{2} + 0.1x' + 2.4,$$

$$Z_2'(x) = 2.6 \cdot \frac{3x'^2 - 1}{2} + 0.1x' + 5.1,$$

The model vector contained the series expansion coefficients of lateral layer thickness functions and the petrophysical parameters assuming layer-wise homogeneous model. Table II shows the target model of the inverse problem including C series coefficients of the layer thickness functions, too. Beside M=15 inversion unknowns the overdetermination ratio was 370. The main component of rock matrix was quartz (VSD), which could be computed by the material balance equation out of inversion

$$VSD = 1 - POR - VSH.$$

Another important parameters underlying the calculation of hydrocarbon reserves could be derived from the ones given in Table II. The following formulas were used to compute movable (SCHM) and irreducible hydrocarbon saturation (SCHIR) as well as absolute permeability (PERM)

$$\mathrm{SCHM} = \mathrm{SX0} - \mathrm{SW},$$
$$\mathrm{SCHIR} = 1 - \mathrm{SX0},$$
$$\mathrm{PERM} = 0.136 \cdot \frac{(100 \cdot \mathrm{POR})^{4.4}}{(100 \cdot \mathrm{SWR})^{2}},$$

where SWB=0.2 denotes the bound water saturation. Water saturation in the undisturbed zone was computed by $SW=SX0^{0.2}$, which is a frequently used formula in well log analysis. The initial model comprised first-guess values of petrophysical parameters and $Z_1(x)=2$, $Z_2(x)=5$ planar layer boundaries (Fig. 2).

Table II.

Inversion was performed by applying a hybrid VFSA+DLSQ optimization algorithm. The development of convergence during the inversion procedure can be seen in Fig. 4. Series expansion coefficients of layer thickness functions were looked for between iteration steps 1-300 series and found at 350. Then, under the 20 per cent model distance level, petrophysical parameters were refined slowly at regular rate of convergence. The optimum was found at D_d=5.00 per cent data and D_m=1.32 per cent model distance (Eq. 4-5). It is observable that layer thickness functions were reconstructed very accurately (Fig. 2). The relative function distance based on Eq.(6) was D_f=0.67 per cent (when having 5 per cent Gaussian noise in the data space). Estimates for petrophysical parameters were also given close to desired values (Table III) with small estimation errors (Eq. 1). Realistic and accurate values were obtained for non-inversion petrophysical parameters derived from interval inversion results (Table IV). It can be seen that such delicate parameter like absolute permeability can be estimated accurately by the method. Mean spread computed by Eq.(3) was S=0.20, which confirmed that the solution is not just accurate but reliable.

Table III.
Table IV.

4.2 Inversion over 2-D calcareous sandstone structure

In the next stage we made the variation of layer thickness functions complicated and increased the number of petrophysical unknowns of the 2-D interval inversion problem. Theoretically fast changes of boundaries can be treated well by the interval inversion method, but it is important to watch that the more complicated the variation is, the more series coefficients we have to apply for describing them. This can cause the decrease of the overdetermination ratio, which may be affecting the quality of inversion results badly. Moreover, having high number of inversion unknowns can make the inversion procedure numerically unstable. According to our preliminary experience it is advisable to settle for a tenth-degree polynomial approximation for the 2-D case. However, the number of series expansion coefficients required depends on the number of other petrophysical unknowns, too. Our new objective model (Model-B) was chosen for a 3-layered hydrocarbon structure made up of shale, shaly-calcareous sand and limestone formations (Fig. 5). Model-B is a bit more complicated than Model-A, because we have got a new matrix component (i.e. limestone) as an additional inversion unknown and two layer boundary functions changing by 4th-degree Legendre polynomials

$$Z_1'(x') = -0.15 \cdot \frac{35x'^4 - 30x'^2 + 3}{8} + 0.25 \cdot \frac{5x'^3 - 3x'}{2} - 0.15 \cdot \frac{3x'^2 - 1}{2} + 0.4x' + 3.32,$$

$$Z_2'(x') = -0.61 \cdot \frac{35x'^4 - 30x'^2 + 3}{8} + 0.97 \cdot \frac{5x'^3 - 3x'}{2} - 0.59 \cdot \frac{3x'^2 - 1}{2} + 2.65x' + 7.20.$$

Borehole logging data charged with 5 per cent Gaussian noise were generated in well W-1–W-6 resulting in N=6666 data samples (Fig.6). Having M=22 inversion unknowns (Table V.) the overdetermination ratio was 303. In order to release one inversion unknown the specific volume of limestone was computed by the material balance equation during the inversion procedure

$$VLM = 1 - POR - VSH - VSD.$$

SW, SCHM, SCHIR, PERM quantities as non-inversion parameters were derived by means of deterministic formulas out of inversion.

Table V.

The interval inversion procedure proved to be stable and convergent. In Fig. 7 the development of convergence during inversion can be seen. Escaping from several local minima during the global optimization phase can be seen in Fig. 7a. The position of layer boundaries was caught by the 400th iteration step. Petrophysical parameters were refined then gradually until the end of the procedure. Table VI shows good estimates for the petrophysical parameters. The solution was given at D_d=5.42 per cent data and D_m=0.70 per cent model distance. The latter quantity even changed for the better compared with the inversion results of Model-A. The mean spread S=0.17 also represented a reliable solution. Determined layer thickness functions can be seen in Fig. 5. The relative function distance was D_f=0.01 per cent. Having a noisy data set, it is still a very accurate approximation of the position of rock interfaces. Moreover, an interesting geometrical feature was also recognized by the procedure. The lack of the second layer in well W-1 was automatically identified and then the appearance of the same layer was detected as deepening continuously downwards along well W-2–W-6. Non-inversion parameters derived from interval inversion results were also obtained accurately except the permeability of the second layer (Table VII). Nevertheless, it must be mentioned that this parameter is enough to be known as a value of the right order of magnitude, because there is no method to determine it better then qualitatively. From this point of view the permeability result is acceptable, which shows the significant presence of a permeable formation (as a reservoir) between two impermeable layers.

Table VI.
Table VII.

4. Case history

After synthetic tests done, in-situ well-logging data acquired in four boreholes side by side situated in a Hungarian hydrocarbon field were processed by means of the 2-D interval inversion method. Well D3, D1, D5 and D6 fell upon a profile 1.4km in length. Data samples corresponding to given GR, K, TH, U, ZDEN (lithodensity), AT, RS, RT (true resistivity) logs (Table I) were measured with sampling interval dz=0.1m in each well.

Interval inversion procedure was performed by using VFSA method. The maximum number of iterations was 5000. Our initial model respect to layer thickness (in order to be precise zone thickness) included $Z_1(x)$=50m, $Z_2(x)$=200m, $Z_{12}(x)$=85m values, where $Z_1(x)$ and $Z_2(x)$ were the depth of upper and lower boundary of the gas reservoir and $Z_{12}(x)$ was the depth of gas-water contact at position x. Layer thickness functions were assumed to be varying by 4-th degree power functions (where C4 corresponded to the coefficient term of the zeroth power of depth). The solution was obtained at D_d=10.61% relative data distance. The resultant 2-D section can be seen in Fig. 8 (where depth is transformed not real one) and parameters are in Table VIII.

Table VIII.

Discussion and conclusions

In the paper we demonstrated a new perspective of the interval inversion of borehole logging data relating to multi-well data processing application. Lateral variation of layer boundaries along the profile of boreholes can be determined together with petrophysical parameters by means of the 2-D interval inversion method. Interval inversion procedure is an appropriate tool to make automatic layer boundary determination for geological correlation purposes in case of not too complicated geological structures. It was shown that hydrocarbon zones can also be detected well by the method. For the case of more refined image it is possible to apply a model made up of more number of thin layers within the processed interval. Higher vertical resolution can be achieved by choosing arbitrary depth interval for the investigations. Only criterion is the maintenance of high overdetermination ratio, which essentially makes the method powerful and effective. It is a problem of model construction how detailed picture can be produced by means of the least number of unknowns. Too many inversion unknowns can lead us to less stable solution numerically. For longer processing intervals we suggest the application of interval inversion method in a loop in order to avoid high number of unknowns and numerical problems. Since there is no restriction to the number of data applied, the future perspective of the development of interval inversion method may be going forward to the extension to 3-D features by using proper series expansion technique.

Appendix

Pseudo Fortran code of VFSA algorithm by Sen et al. (1997)

Start at a random location $\vec{m}_0$ with energy $E(\vec{m}_0)$

loop over temperature (T)

  loop over number of random moves/temperature

    loop over model parameters i=1,N

      number generation

      $u_i = U[0,1]$

      $y_i = \text{sign}(u - .5)\, T_i \left[(1 + \frac{1}{T_i})^{|2u_i - 1|} - 1 \right]$

      updating model

      $m_i^{new} = m_i^{old} + y_i (m_i^{max} - m_i^{min})$

    end loop

    now we have a new model m^{new}

    $\Delta E = E(\vec{m}^{new}) - E(\vec{m}_0)$

    $P = \exp(-\frac{\Delta E}{T})$

    if $\Delta E \le 0$ then

      $\vec{m}_0 = \vec{m}^{new}$

      $E(\vec{m}_0) = E(\vec{m}^{new})$

    end if

    if $\Delta E \ge 0$ then

      draw a random number $\alpha = U[0,1]$

      if $P \ge \alpha$ then

        $\vec{m}_0 = \vec{m}^{new}$

        $E(\vec{m}_0) = E(\vec{m}^{new})$

      end if

    end if

  end loop

end loop

Acknowledgements

The research work was supported by the Hungarian National Research Fund (OTKA project number: T049852). Special thanks to Hungarian Gas and Oil Company for providing field data for the research. Thanks are also due to the Erasmus project supporting the co-operation between the University of Rome "La Sapienza" and the University of Miskolc. As a member of the MTA-ME Research Group for Geophysical Inversion and Tomography the authors express thanks to the Hungarian Academy of Sciences.

References

Alberty M, Hasmhmy K, 1984: Application of ULTRA to log analysis. SPWLA 25th Annual Logging Symposium, 1984-Z.

Dobróka M, Szabó N P, 2002: The MSA inversion of openhole well log data. Intellectual Service for Oil & Gas Industry. Ufa State Petroleum Technologocal University & University of Miskolc. Analysis, Solutions, Perspectives Vol 2. pp. 27-38.

Dobróka M, Szabó N, Hursán L, 2003: The interpretation of well log data by means of Float Encoded Genetic Algorithm. 65th EAGE Conference & Exhibition (Stavanger), P261.

Dobróka M, Szabó P N, 2005: Combined global/linear inversion of well-logging data in layer-wise homogeneous and inhomogeneous media. Acta Geodaetica et Geophysica Hungarica, Vol. 40(2), pp. 203-214.

Dobróka M, Kiss B, Szabó N, Toth J, Ormos T, 2007: Determination of cementation exponent using an interval inversion method. 69th EAGE Conference & Exhibition (London), P092.

Dobróka M, Szabó N, Ormos T, Kiss B, Toth J, Szabó I, 2008: Interval inversion of borehole geophysical data for surveying multimineral rocks. 14th European Meeting of Environmental and Engineering Geophysics (Cracow), P11.

Fertl W, Ball M, Chace D, 1987: The Well Data System (WDS): „An Advanced Formation Evaluation Concept in a Microcomputer Environment", SPE 17034.

Marquardt D W, 1959: Solution of nonlinear chemical engineering models. Chem. Eng. Prog., 55/1959 No. 6.

Mayer C, 1980: GLOBAL, a new approach to computer-processed log interpretation. SPE 9341.

Menke W, 1984: Geophysical data analysis – Discrete inverse theory. Academic Press, Inc. London Ltd.

Metropolis N, Rosenbluth A, Rosenbluth M, Teller A, Teller E, 1953: Equation of state calculations by fast computing machines. J. Chem. Phys. 21, pp. 1087-1092.

Sen M, Stoffa P L, 1997: Global optimization methods in Geophysical inversion (In Seismic Exploration, edited by K. Helbig and S. Treitel, Elsevier Science)

Szabó N, 2004: Modern inversion methods for the interpretation of well-logging data. Ph.D. thesis, University of Miskolc.

Szabó N P, 2004: Global inversion of well log data. Geophysical Transactions, Loránd Eötvös Geophysical Institute of Hungary, Vol. 44. Nos. 3-4., pp. 313-329.

Szabó N, Dobróka M, Vass P, 2009: The determination of textural parameters using interval inversion of borehole geophysical data. 15th European Meeting of Environmental and Engineering Geophysics (Dublin), P60.

Legends

Tables

Table I.

Log type	Log name	Sensitive to	Unit
SP	spontaneous potencial	lithology	mV
GR	natural gamma ray		API
K	spectral (potassium) gamma ray		per cent
U	spectral (uranium) gamma ray		ppm
TH	spectral (thorium) gamma ray		ppm
PE	photoelectric absorption index		barn/e
CN	compensated neutron	porosity	p.u.
CD	compensated density		g/cm^3
AT	acoustic travel-time		μs/m
RS	shallow resistivity	saturation	ohmm
RD	deep resistivity		ohmm

Table II.

Layer	POR	SX0	VSH	Boundary	C0	C1	C2
1 (water sand)	0.25	1.0	0.15	$Z_1(x)$	2.4	0.1	1.2
2 (shale)	0.10	1.0	0.80	$Z_2(x)$	5.1	0.1	2.6
3 (gas sand)	0.30	0.80	0.10				

Table III.

Layer	Parameter	Target value	Estim. value	Estim. error
1	POR	0.2500	0.2509	±0.0008
	SX0	1.0000	1.0000	±0.0007
	VSH	0.1500	0.1490	±0.0006
2	POR	0.1000	0.1029	±0.0015
	SX0	1.0000	0.9983	±0.0007
	VSH	0.8000	0.7968	±0.0016
3	POR	0.3000	0.3002	±0.0006
	SX0	0.8000	0.8001	±0.0004
	VSH	0.1000	0.0998	±0.0003

Table IV.

Layer	Parameter	Target value	Estim. value
1	SW	1.0000	1.0000
	VSD	0.6000	0.6001
	SCHM	0	0
	SCHIR	0	0
	PERM	481 mD	489 mD
2	SW	1.0000	0.9915
	VSD	0.1000	0.1003
	SCHM	0	0.0068
	SCHIR	0	0.0017
	PERM	8.54 mD	9.68 mD
3	SW	0.3277	0.3279
	VSD	0.6000	0.6000
	SCHM	0.4723	0.4722
	SCHIR	0.2000	0.1999
	PERM	1073 mD	1076 mD

Table V.

Layer	POR	SX0	VSH	VSD	Boundary	C0	C1	C2	C3	C4
1 (shale)	0.05	1.00	0.80	0.10	$Z_1(x)$	3.32	0.40	-0.15	0.25	-0.15
2 (gas sand)	0.30	0.80	0.10	0.60	$Z_2(x)$	7.20	2.65	-0.59	0.97	-0.61
3 (limestone)	0.05	1.00	0.10	0.05						

Table VI.

Layer	Parameter	Target value	Estim. value	Estim. error
1	POR	0.0500	0.0487	± 0.0012
	SX0	1.0000	1.0000	± 0.0007
	VSH	0.8000	0.8177	± 0.0016
	VSD	0.1000	0.0903	± 0.0011
2	POR	0.3000	0.2817	± 0.0006
	SX0	0.8000	0.8097	± 0.0004
	VSH	0.1000	0.1074	± 0.0003
	VSD	0.6000	0.6113	± 0.0006
3	POR	0.0500	0.0503	± 0.0004
	SX0	1.0000	0.9990	± 0.0008
	VSH	0.1000	0.0997	± 0.0006
	VSD	0.0500	0.0503	± 0.0017

Table VII.

Layer	Parameter	Target value	Estim. value
1	SW	1.0000	1.0000
	VLM	0.0500	0.0503
	SCHM	0	0
	SCHIR	0	0
	PERM	0.40 mD	0.36 mD
2	SW	0.3277	0.3480
	VLM	0	0.0001
	SCHM	0.4723	0.4617
	SCHIR	0.2000	0.1903
	PERM	1073 mD	814 mD
3	SW	1.0000	0.9950
	VLM	0.8000	0.7997
	SCHM	0	0.0040
	SCHIR	0	0.0010
	PERM	0.40 mD	0.41 mD

Table VIII.

Zone	POR	VSH	SW	Boundary	C0	C1	C2	C3	C4
1 (shale)	0.08	0.65	1.00	surface					
2 (gas sand)	0.25	0.10	0.26	$Z_1(x)$	9.862	10^{-4}	-13.96	-29.08	52.20
3 (water sand)	0.25	0.10	1.00	$Z_{12}(x)$	27.487	10^{-4}	-163.8	201.04	89.90
4 (shaly sand)	0.12	0.45	1.00	$Z_2(x)$	-18.57	$2 \cdot 10^{-4}$	87.78	-89.16	205.10

Figures

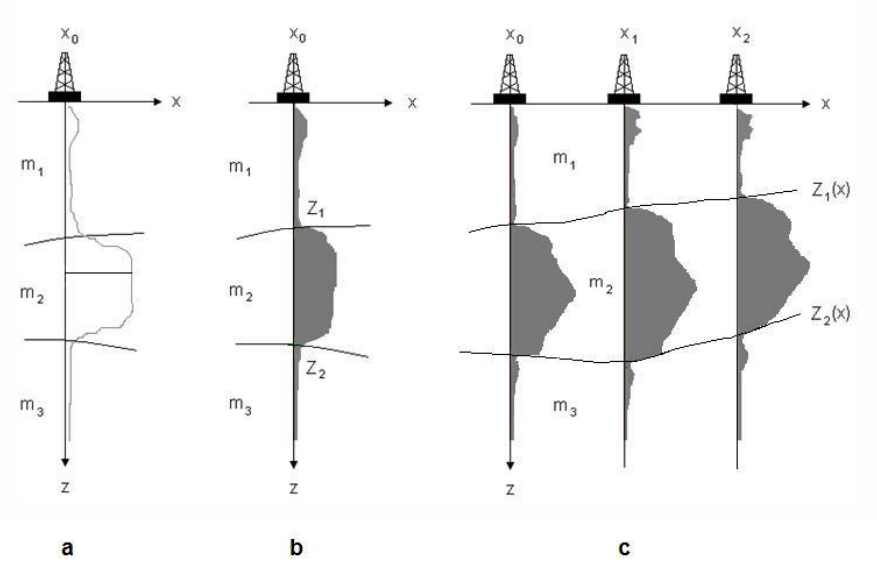


Figure 1

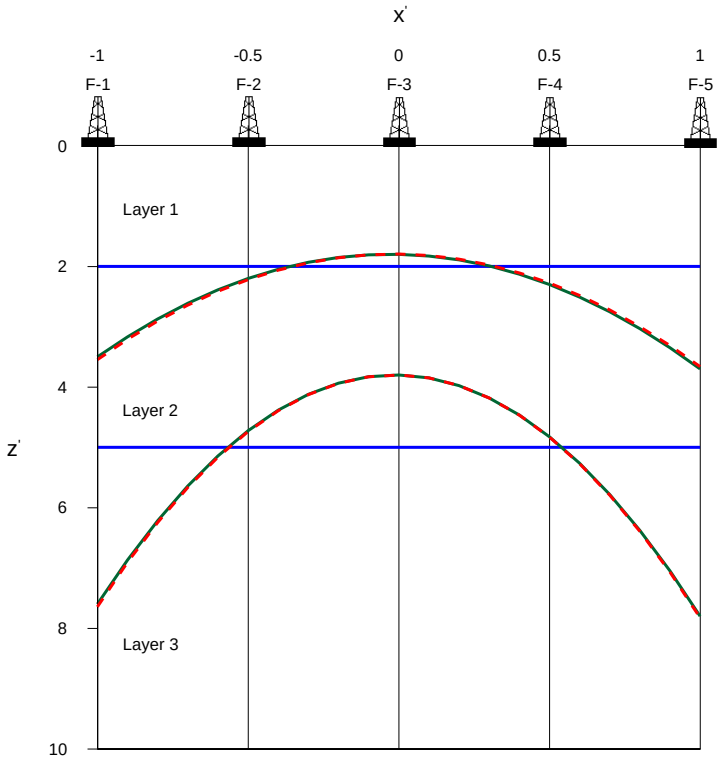
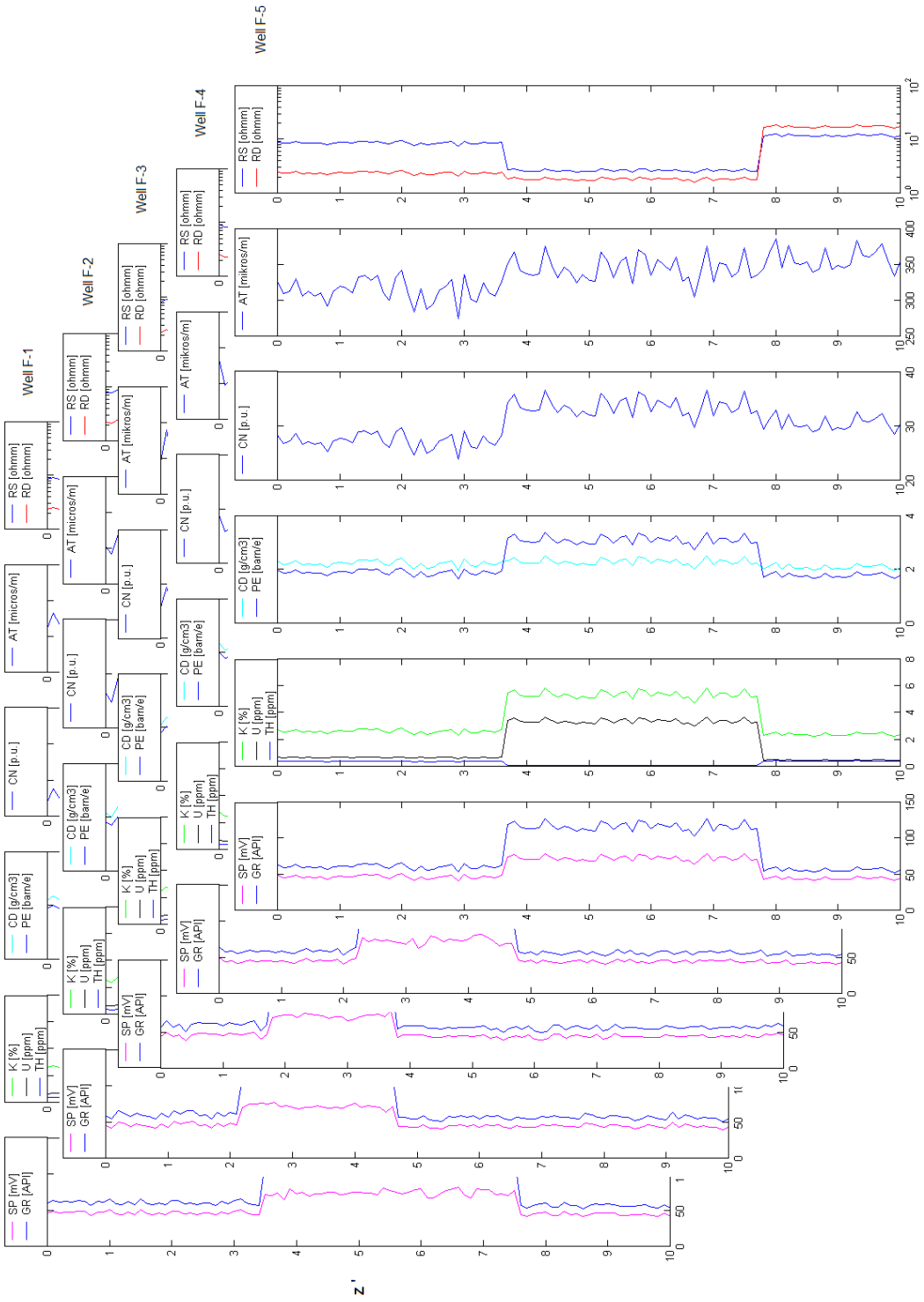


Figure 2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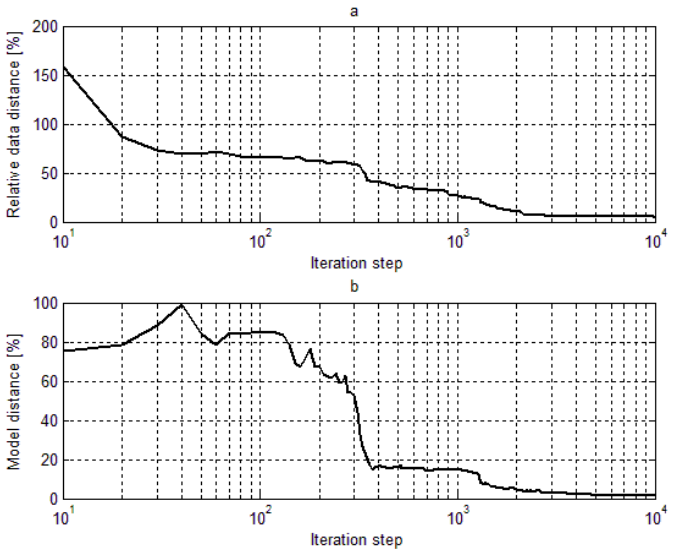
Well F-2
Well F-3
Well F-4
Well F-5
SP [mV]
GR [API]
K [%]
U [ppm]
TH [ppm]
CD [g/cm3]
PE [barn/e]
CN [p.u.]
AT [micros/m]
RS [ohmm]
RD [ohmm]
z

Figure 3



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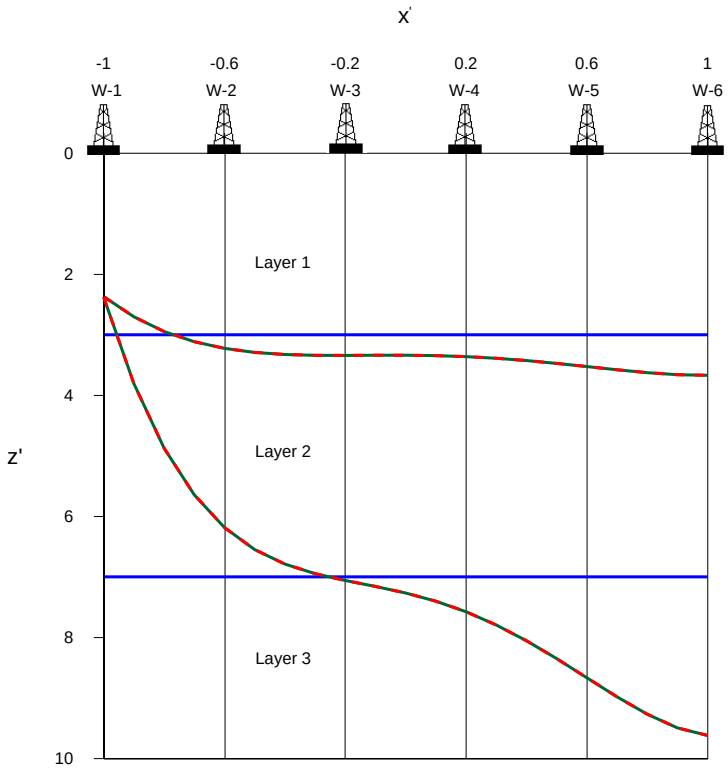

Figure 5

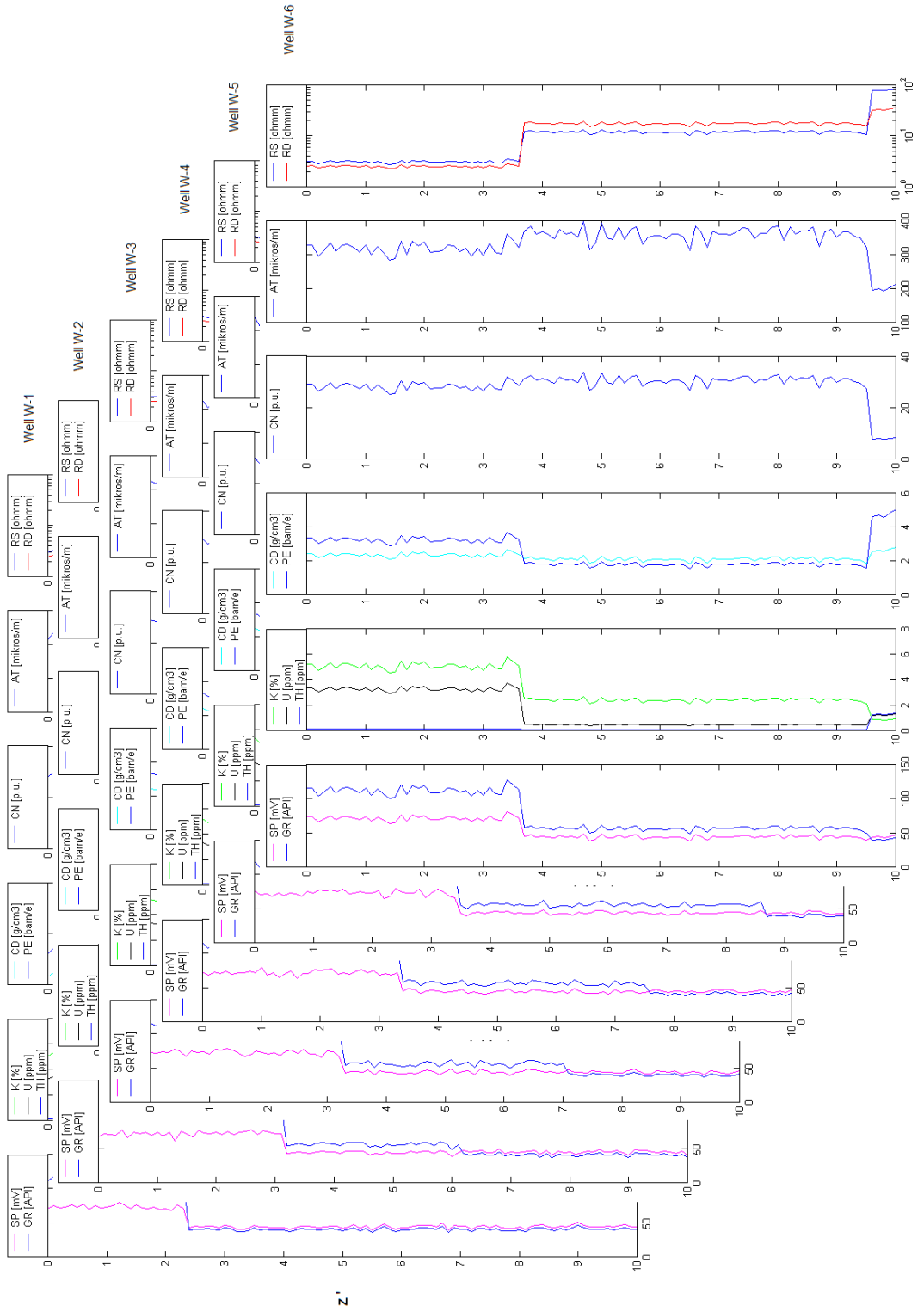
Well W-1
Well W-2
Well W-3
Well W-4
Well W-5
Well W-6
SP [mV]
GR [API]
K [%]
U [ppm]
TH [ppm]
CD [g/cm3]
PE [barn/e]
CN [p.u.]
AT [mikros/m]
RS [ohmm]
RD [ohmm]
z

Figure 6

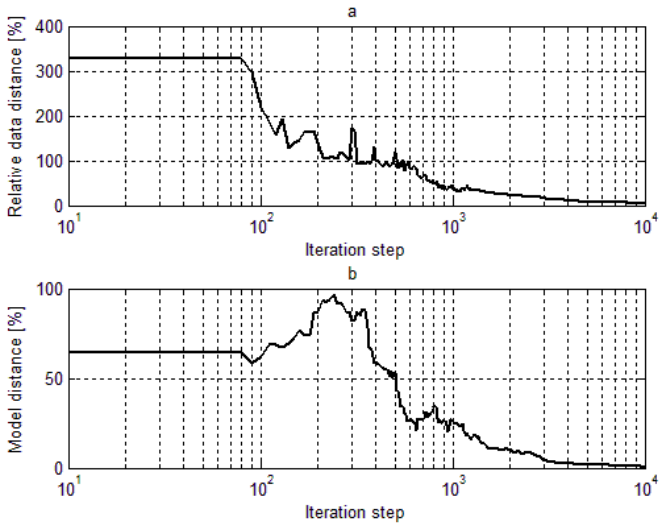

Figure 7

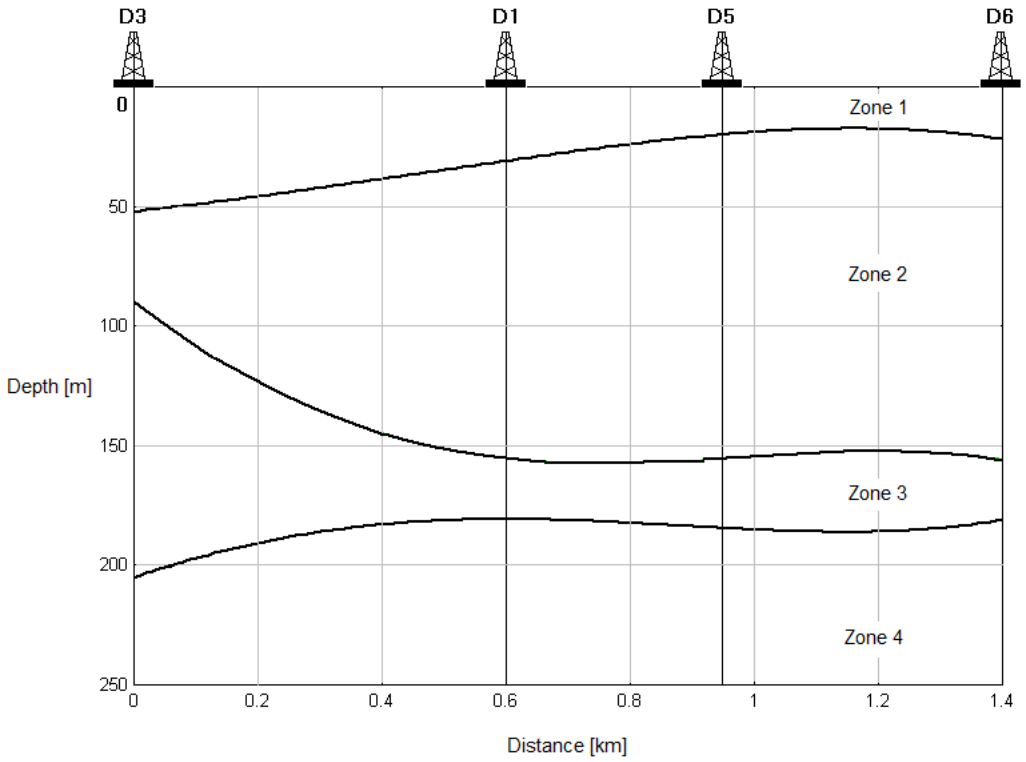

Figure 8