On the application of Combined Geoelectric Weighted Inversion in environmental exploration

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Abstract Geophysical surveying methods are of great importance in environmental exploration. Inversion-based data processing methods are applied for the determination of geometrical and physical parameters of the target model. It is presented that the use of joint inversion methods is advantageous in environmental research where highly reliable information with large spatial resolution is required. The 2D CGI (*Combined Geoelectric Inversion*) inversion method performs more accurate parameter estimation than conventional 1D single inversion methods by efficiently decreasing the number of unknowns of the inverse problem (single means that data sets of individual VES stations are inverted separately). The quality improvement in parameter space is demonstrated by comparing the traditional 1D inversion procedure with a 2D series expansion-based inversion technique. The CGI inversion method was further developed by weighting individual DC (Direct Current) geoelectric data sets automatically in order to improve inversion results. The new algorithm was named CGWI (*Combined Geoelectric Weighted Inversion*), which extracts the solution by a special weighted least squares technique. It is shown that the new inversion methodology is applicable to resolve near-surface structures such as rapidly varying layer boundaries, laterally inhomogeneous formations and pinch-outs.

Keywords Joint Inversion, Series Expansion, Combined Geoelectric Inversion, Combined Geoelectric Weighted Inversion

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1. Introduction

Applied geophysical methods are extensively used for solving geological, engineering geophysical, mining geophysical, hydrogeophysical and environmental geophysical problems. A comprehensive study of environmental geophysical surveying methods with several applications can be found in Sharma (1997). The high exploration activity of environmental geophysics is supported by an intensive surveying and interpretation method development. The development and application of inversion and tomography methods are well-documented (Dobróka et al., 1991; Ramirez et al., 1993; Hering et al., 1995; LaBrecque et al., 1996; Ramirez et al., 1996; Misiek et al., 1997; LaBrecque and Yang, 2001; LaBrecque et al., 2004; Kemna et al., 2004; LaBrecque et al., 2004; Pellerin and Wannamaker, 2005; Auken et al., 2008; Blaschek et al., 2008; Ferré et al., 2009; Szabó et al., 2012; Szabó 2012; Turai and Hursán 2012; Gyulai and Tolnai 2012).

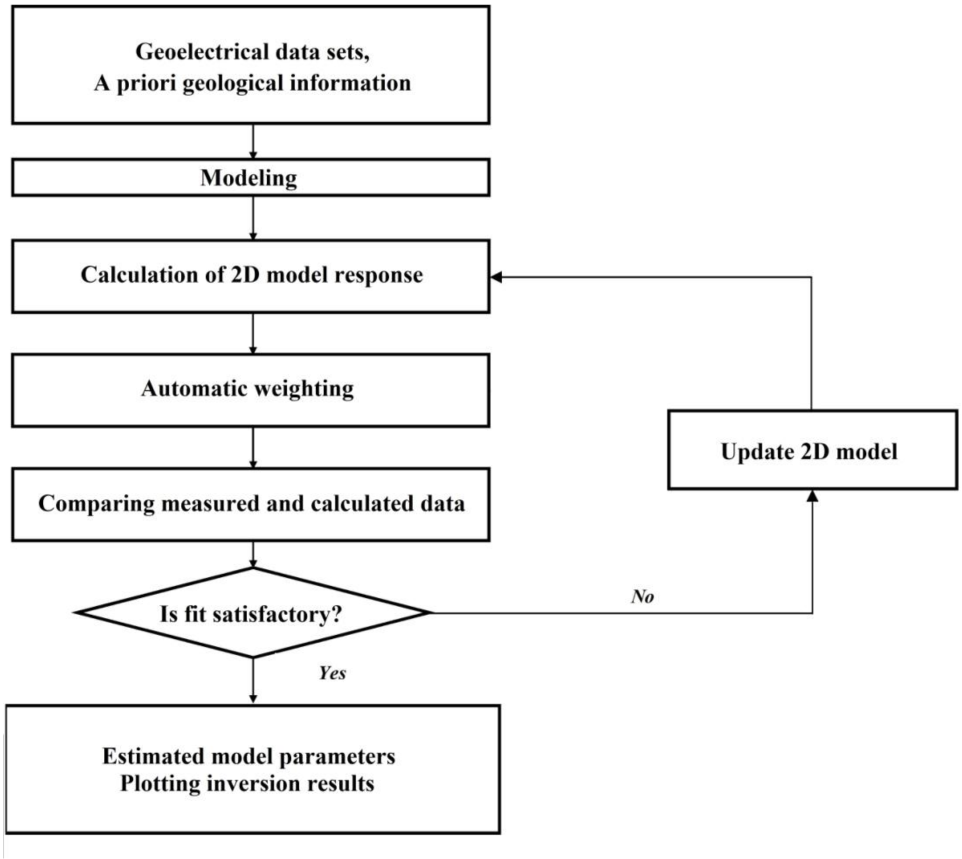
Environmental geophysical surveys are applied to sample the environment non-destructively with the aim of detecting physical contrasts and discontinuities in the rock mass. The spatial information for the structural parameters (*e.g.* layer-thickness, depth, dip, strike, azimuth, tectonics and volume) and geophysical parameters (*e.g.* mineral composition, petrophysical properties of rocks, degree of cracking and weathering, water tightness, contamination and radiological parameters) are extracted from the observations. Measurements are made continuously in space and time, but continuity is defined at a certain resolution. It is an important task to support environmental exploration with newly developed geophysical measurement and interpretation methods to achieve appropriate resolution.

In environmental exploration the use of data processing and interpretation techniques provided with quality check tools are of great importance. They are applicable for calculate the estimation errors of structural and physical parameters. Beside exploration purposes, a special emphasis is laid on the accuracy and reliability of data supporting the protection of the health of people and mineral resources that may be damaged by mining or engineering activity. Because of these reasons, it has always been an important task to develop new geophysical surveying and interpretation methods with improved resolution and reliability. Our goal is to introduce a new series expansion-based inversion methodology into the environmental geophysical practice, which provides high accuracy and reliable estimation results. The power of the inversion method is based on the formulation of a highly overdetermined inverse problem resulting in a stable, accurate and less noise-sensitive inversion procedure that maximizes the amount of environmental information extracted from the data. In the paper, the advantages of the new inversion method are highlighted, and its application is demonstrated using DC geoelectric data.

2. Geophysical inversion and related problems

Structural and petrophysical information about geological structures can be extracted by indirect analysis of geophysical surveying data. This procedure is called geophysical inversion, which is the most accepted and widely used data processing technique nowadays. In the paper we propose a new weighted 2D inversion method (Section 2.2). The workflow of the inversion procedure can be seen in **Fig. 1** The 2D inverse problem is solved in stages.

Fig. 1 The flowchart of the CGWI geophysical inversion procedure.



The first stage of inverse modeling is data acquisition, which is followed by several data processing steps in the interest of parameter estimation. During the iterative procedure the model describing the geological structure is gradually refined in order to achieve a good fit between the measured and calculated data. Mathematically, it is an optimization procedure resulting in an optimal set of model parameters, which are plotted as sections for the environmental interpretation. The theoretical background of inversion methods is detailed in Menke (1984) and Tarantola (2005). Near-surface geoelectric applications and several references can be found in Pellerin and Wannamaker (2005).

Along with several advantages geophysical inversion methods have some limitations, too. Since data are always contaminated with some amount of noise, the model parameters estimated by inversion are also erroneous. In addition to noise caused by instrumental errors and environmental effects, the error of model construction (i.e. uncertainty from the difference between the inversion model applied to sounding response functions and the real geological structure) is also expected, which is not possible to be quantified. On the other hand, the number of model parameters should be specified properly to describe the geological structure. Either too large or too small number of parameters can cause less accurate inversion results, *i.e.* many parameters may cause instable inversion procedure or large estimation errors and small number of parameters does not represent the structure well.. Both marginally over determined or underdetermined inverse problems related to complicated models in most of the cases result in ambiguous solution. Treating this problem it is possible to collect large number of data, but the amount of inherent information in data about the structure is sometimes not enough for a unique solution. The problem of ambiguity can be caused by low parameter sensitivities, which originates from the small variability of data influenced by structural and geophysical parameters. Consequently, less sensitive parameters cannot be determined with the required accuracy and resolution. Parameter sensitivity functions can be calculated in order to check whether a parameter is applicable as an inversion unknown. They were introduced into the near-surface geophysical practice for studying the absorption and dispersion behavior of guided waves by Dobróka (1988) and were extensively used in geoelectric modeling by Gyulai (1989). Inverse problems require the highest possible amount of a priori information about the environmental geophysical model. Small amount of a priori information, low physical contrasts, small variations in geometry, non-relevant boundaries for different measured quantities and oversimplification of the model can be the source of the failure of the inversion procedure. The above detailed problems may cause non-reliable structural and physical parameters and incorrect interpretation results.

There are some options for the improvement of inversion results. Efforts can generally be made such as reduction of data noise, searching for high parameter sensitivities, proper simplification of the model and searching for high physical contrasts etc. A more advanced way of achieving higher resolution and reliability of the estimated parameters is the application of joint inversion methods, in which several data sets based on different physical principles measured over the same structure are integrated into one inversion procedure and processed simultaneously to determine an extended geophysical model. The principles of the joint inversion technique were introduced by Vozoff and Jupp (1975). The concept of joint inversion has also been used for a wide range of environmental geophysical problems. In novel applications different geophysical and hydrological data sets are coupled in one inversion procedure (Yeh and Simunek 2002; Finsterle and Kowalsky 2008; Hinnel et al. 2010; Dafflon et al., 2011).

According to our interpretation, the term “different physical principle” not only means that different geophysical parameters (*e.g.* resistivity, velocity, density) but also different types of measurement methods of the same physical principle containing distinct information about the geological structure are involved in the inversion procedure. The information content of the individual data sets can be quantified equivocally by the Fisher Information Matrix (Salát et al., 1982). The inversion processing of data sets (measured by the same principle along a line - surface or borehole - or over an area) representing different information from the same structure is reasonably called structurally linked joint inversion (Gyulai and Ormos 1999; Li and Oldenburg 2000). The inverse of the information matrix is the covariance matrix, which contains important information about the quality of the estimated model parameters. Quality checking of inversion is of great importance in accepting the geophysical inversion results (Menke 1984; Salát and Drahos 2005).

2.1. Series expansion-based inversion methodology

The series expansion based inversion method can be used effectively for the interpretation of different types of geophysical surveying data, of which applicability has been proven using gravity (Dobróka and Völgyesi 2010), geoelectric (Gyulai and Ormos 1999; Turai et al. 2010; Gyulai et al. 2010a; Gyulai et al. 2010b), seismic (Dobróka 1994; Ormos and Daragó 2005; Paripás and Ormos 2011), well-logging data sets (Szabó 2004; Dobróka et al. 2009; Dobróka and Szabó 2010) and in general data processing (Vass 2009). The series expansion-based inversion method can be considered as a joint inversion method, where each datum measured along a profile (or over an area) assists in the determination of the series expansion coefficients describing the geological structure. The inversion technique allows to further coupling data measured by different physical principles in one inversion procedure, which can improve the reliability of the interpretation results (Dobróka and Szabó 2005; Drahos 2005).

The principle of the series expansion-based inversion method is that variations of layer boundaries and physical parameters along the profile are described by continuous functions. The discretization of model parameters is based on series expansion (Dobróka 1993)

$$p_k(x) = \sum_{q=1}^{Q_k} C_q^{(k)} \Phi_q(x), \tag{1}$$

where p_k denotes the k-th physical or structural parameter $(k=1, 2, \ldots, K)$, C_q is the q-th expansion coefficient and Φ_q is the q-th basis function (up to Q

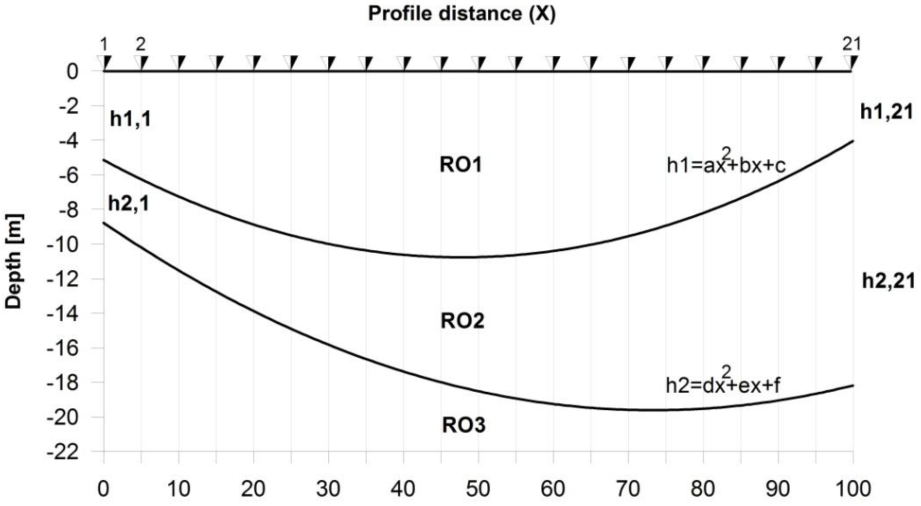
number of additive terms), which is the function of the independent variable *x*. Basis functions are known quantities, which can be chosen arbitrarily for the environmental geological setting. In earlier studies, it was demonstrated that geoelectric structures can be described properly by periodic functions (Gyulai and Ormos 1999; Gyulai et al. 2010a). In a simpler way a set of power basis functions can be used in **Equation 1**

$$\Phi_q(x) = x^{q-1}. \tag{2}$$

According to our experiences orthogonal functions (e.i., trigonometric functions) can be used effectively in the discretization process. For instance, an application can be found in Dobróka et al. (2009) for the use of Legendre polynomials. By the above formulation all data measured along the profile are inverted simultaneously to determine the series expansion coefficients. The advantage of this technique is that the variation of structural and physical parameters can be described with significantly less unknowns (*i.e.* series expansion coefficients) than data and a highly overdetermined inverse problem is formulated, which is more favorable to be solved than a marginally overdetermined or an underdetermined inverse problem. The strategy of choosing the number of expansion coefficients is detailed in Gyulai et al. (2010a).

In order to demonstrate the increase of the overdetermination (data-to-unknowns) ratio, conventionally used 1D single inversion results and a 2D series expansion-based inversion method using VES (*Vertical Electric Sounding*) data are compared. In **Fig. 2** the inversion scheme of VES data measured along a profile is shown.

Fig. 2 1D single and 2D series expansion based inversion of VES data sets measured along a profile (h1 and h2 denote the layer-thicknesses; RO1, RO2, RO3 are the resistivities of the three-layered geoelectric model).



There are 21 VES stations, each of them is provided with 20 data (totally 420 data). The total number of local layer-thickness ($h_{1,t}$ and $h_{2,t}$, where t=1,2,…,21) and resistivity values (*RO1,RO2,RO3*) of the three-layered structure is 105 (5 model parameters and 21 VES stations), which have to be determined by a set of 1D single inversion procedures, respectively. However, layer-thicknesses and resistivities can be expanded into series by using power functions. For demonstrating the problem, the layer thicknesses are assumed to be described by quadratic functions using **Equation 1**. and **2**. If resistivities were unvarying, only 9 unknowns (*a,b,c,d,e,f,RO1,RO2,RO3*) would be estimated for the same number of data. In the latter case the overdetermination ratio is seven times higher, which reduces significantly the uncertainty of parameter estimation. After an estimate is given for the series expansion coefficients by the inversion procedure, the structural and physical parameters can be derived by **Equation 1**. If resistivities showed relatively smooth lateral variations, the overdetermination ratio does not decrease significantly with respect to the case of constant layer resistivities.

2.2 The 2D inversion algorithm

The theory of 1D inversion of geoelectric sounding data is detailed in Koefoed (1979). For Schlumberger-array measurements the apparent resistivity ρ_a at profile distance *s* can be written as

$$\rho_a(s) = \rho_a(\boldsymbol{p}(s), AB/2), \tag{3}$$

where the model vector of the inverse problem is

$$\boldsymbol{p}(s)=\begin{bmatrix} h_1(s),\ldots,h_t(s),\ldots,h_{N-1}(s), \\ \rho_1(s),\ldots,\rho_t(s),\ldots,\rho_N(s) \end{bmatrix}^T , \qquad (4)$$

where $h_t(s)$ and $\rho_t(s)$ denote the local thickness and resistivity of the t-th layer at profile distance s, and AB/2 is the power electrode spacing (N is the number of layers and T is the symbol of transpose).

For 2D inversion a FD (*Finite Difference*) forward modeling algorithm developed by Spitzer (1995) is used. The essential part of the inversion procedure is the parameterization of a 2D geoelectric model in terms of the series expansion of layer-thicknesses and resistivities based on **Equation 1**

$$h_t(x)=\sum_{q=1}^{Q_t} B_q^{(t)} \Phi(x), \qquad (5)$$

$$\rho_t(x)=\sum_{w=1}^{W_t} C_w^{(t)} \Phi(x), \qquad (6)$$

where B_q and C_w expansion coefficients are the unknowns of the 2D inverse problem. Thus the model vector of **Equation 4** becomes

$$\boldsymbol{p}=\begin{bmatrix} B_1^{(1)},\ldots,B_{Q_1}^{(1)},\ldots,B_1^{(N-1)},\ldots,B_{Q_{N-1}}^{(N-1)}, \\ C_1^{(1)},\ldots,C_{W_1}^{(1)},\ldots,C_1^{(N)},\ldots,C_{W_N}^{(N)} \end{bmatrix}^T . \qquad (7)$$

By this formulation, all data measured along the profile are integrated into the observed data vector

$$\boldsymbol{\rho_a^{(o)}}=\begin{bmatrix} \rho_{a,1}^{(o)}(s_1),\ldots,\rho_{a,N}^{(o)}(s_1),\ldots, \\ \rho_{a,1}^{(o)}(s_V),\ldots,\rho_{a,N}^{(o)}(s_V) \end{bmatrix}^T , \qquad (8)$$

where V is the total number of VES stations along the profile. The connection between data and model defined in **Equation 3** is extended to the entire profile, thus the calculated resistivity data vector is

$$\boldsymbol{\rho_a^{(c)}}=\boldsymbol{\rho_a}(\boldsymbol{p}) . \qquad (9)$$

The formulation of the series expansion-based inverse problem assures a high overdetermination ratio, which results in a more stable and robust inversion procedure than a marginally overdetermined or an underdetermined one. The inversion method was named CGI (*Combined Geoelectric Inversion*) by Gyulai et al. (2010a).

The solution of the overdetermined CGI inverse problem can be given at the minimal distance between the measured and calculated data. A proper objective function suggested by Dobróka and Szabó (2012) was used

$$E=\sum_{l=1}^{N\cdot V}\left(\frac{\rho_{a,l}^{(o)}-\rho_{a,l}^{(c)}}{\rho_{a,l}^{(c)}}\right)^2=min, \qquad (10)$$

where $\rho_l^{(o)}$ and $\rho_l^{(c)}$ denote the l-th observed and calculated data, respectively (V is the number of VES stations along the profile, N is the number of resistivity data in one station). The above function allows that the contribution of data having different magnitudes to the solution would be the same. Depending on the error statistics of measured data similar objective functions to **Equation 10** can be implemented, for instance, having outliers in the data set an L_1 norm based error function is preferred. There are several inversion techniques for seeking the optimum of **Equation 10**. Linearized optimization methods are the most prevailing inversion techniques, because they are quick and effective in case of having an initial model close to the solution (Marquardt 1959; Menke 1984). However, they are not absolute minimum searching methods and can assign the solution to a local optimum of the objective function. In that case global optimization methods are recommended to be used such as Simulated Annealing (Metropolis et al. 1953) or Genetic Algorithms (Holland 1975). The subsequent combination of linear and global inversion techniques forms a fast algorithm resulting in the most reliable estimation (Dobróka and Szabó 2005).

A new inversion methodology was suggested by Drahos (2008), which gave an estimate for the geoelectric model by using automatic weighting to apparent resistivity data. Consider two different geophysical surveying methods, where the data and the corresponding response functions are denoted by vectors $\boldsymbol{d_1},\boldsymbol{G_1}(\boldsymbol{m})$ and $\boldsymbol{d_2},\boldsymbol{G_2}(\boldsymbol{m})$, respectively. The unknown model parameters are the components of vector $\mathbf{m}$. The relationship between the measured and the calculated data are

$$\boldsymbol{d_1}=\boldsymbol{G_1}(\boldsymbol{m})+\boldsymbol{e_1} \qquad (11)$$

and

$$\boldsymbol{d_2}=\boldsymbol{G_2}(\boldsymbol{m})+\boldsymbol{e_2}, \qquad (12)$$

where $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ represent the random noise. The standard deviations of these vectors are σ_1 and σ_2 that are also unknowns. The joint objective function based on the L_2 norm is

$$\lambda = w_1 \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m})\right)^2 + w_2 \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m})\right)^2 , \qquad (13)$$

where w_1 and w_2 are unknown weight factors, n_1 and n_2 are the numbers of data, respectively. Drahos (2008) applied the maximum likelihood method for solving the optimization problem. It was also concluded that the values of the weights are:

$$w_1 = \frac{1}{2\sigma_1^2} \text{ and } w_2 = \frac{1}{2\sigma_2^2} . \qquad (14)$$

If the standard deviations are unknown, the minimization must also be done with the respect to σ_1 and σ_2 , too. If conditions

$$\frac{\partial \lambda}{\partial \sigma_1} = 0 \text{ and } \frac{\partial \lambda}{\partial \sigma_2} = 0 \qquad (15)$$

are fulfilled, data variances are derived as

$$\sigma_1^2 = \frac{1}{n_1} \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m})\right)^2 \qquad (16)$$

and

$$\sigma_2^2 = \frac{1}{n_2} \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m})\right)^2 . \qquad (17)$$

Combining **Equation 16** and **17** with **Equation 13**, objective function λ will not contain σ_1 and σ_2 explicitly

$$\lambda = \frac{n_1}{2} \ln\left(\frac{1}{n_1} \sum_{i=1}^{n_1} \left(d_{1,i} - G_{1,i}(\mathbf{m})\right)^2 \right) + \frac{n_2}{2} \ln\left(\frac{1}{n_2} \sum_{j=1}^{n_2} \left(d_{2,j} - G_{2,j}(\mathbf{m})\right)^2 \right) . \qquad (18)$$

After finding the minimum of **Equation 18**, and the optimal model is estimated, the estimates of σ_1 and σ_2 can be calculated from **Equation 16** and **17**. They can be directly used in calculating the standard deviations $\sigma(m_i)$, which measure the uncertainty of the model parameter estimates Menke (1984). The first 2D geoelectric application of the above inversion method can be found in Drahos et al. (2011).

The CGI method was further developed by using the above optimization strategy. The inversion algorithm was renamed CGWI (*Combined Geoelectric Weighted Inversion*). Weighting prevents the simultaneous inversion procedure from giving less accurate results (performance) caused by very noisy data sets. The CGWI method applies an objective function, which is related analytically to the standard deviations of data. The minimization of the objective function with respect to the model parameters results in an automatic optimization according to the standard deviations of data. The method can be regarded as the generalization of the CGI inverse problem.

2.3. *The quality check of the inversion results*

Linearized inversion methods give an opportunity for checking the quality of the inversion results. It is known that given quantitative information about the uncertainty of data, it is possible to derive the estimation errors of the model parameters. Menke (1984) suggested a relationship between the data and model covariance matrices

$$cov\boldsymbol{p} = \boldsymbol{A} cov\boldsymbol{\rho}_{\boldsymbol{a}}^{(\boldsymbol{o})} \boldsymbol{A}^{\boldsymbol{T}} , \qquad (19)$$

where $\boldsymbol{A}$ denotes the general inverse of the actual inversion method. Data covariance matrix ($cov\boldsymbol{\rho}_{\boldsymbol{a}}^{(\boldsymbol{o})}$) contains data variances in its main diagonal. The estimation error of the *k*-th model parameter (σ_k) is derived as the square root of the *k*-th element of the main diagonal of the model covariance matrix ($cov\boldsymbol{p}$).

The elements of the model covariance matrix are defined for the case of series expansion at a given VES station as

$$\sigma_k(x_m) = \frac{\sqrt{\sum_{i=1}^{J(k)} \sum_{j=1}^{J(k)} \left(\Phi_{ki}(x_m) \Phi_{kj}(x_m) cov_{ij} \right)}}{p_k(x_m)} \qquad (20)$$

where $\sigma_k(x_m)$ denotes the estimation error of the *k*-th model parameter (resistivity or layer-thickness) at the *m*-th VES station along direction *x*. The $\Phi_{ki}(x)$ and $\Phi_{kj}(x)$ are the *i*-th and *j*-th basis functions (*J(k)*) is the number of basis function elements in the series expansion describing the *k*-th model parameter) and cov_{ij} represents the covariance matrix element of the estimated series expansion coefficients Gyulai et al. (2010a). In order to give an overall characteristic of the parameter estimation for the 2D model the mean estimation error is introduced as

$$F=\sqrt{\frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}\sigma_k^2(x_m)}\cdot 100\%. \tag{21}$$

For characterizing the fit between the measured ($\boldsymbol{\rho}^{(o)}$) and calculated data ($\boldsymbol{\rho}^{(c)}$) the relative data distance is defined as

$$d=\sqrt{\frac{1}{N}\sum_{l=1}^{N}\left(\frac{\rho_l^{(o)}-\rho_l^{(c)}}{\rho_l^{(c)}}\right)^2}\cdot 100\%. \tag{22}$$

The reliability of the inversion results can be quantified by the correlation coefficients indicating the degree of linear dependence between the *i*-th and *j*-th model parameters

$$(corr\mathbf{p})_{ij}=\frac{(\mathrm{cov}\mathbf{p})_{ij}}{\sigma_i\sigma_j}. \tag{23}$$

If the absolute value of the correlation coefficient is close to zero, there is no connection between the model parameters, which refers to a reliable solution. Only uncorrelated or poorly correlated model parameters can be resolved individually by inversion. CGWI method results in relatively low correlation between the inversion parameters reducing the amount of ambiguity. This is especially true if orthogonal basis functions are used in serious function, therefore periodic functions were applied in this study. In case of having many inversion unknowns, a scalar can be derived from the elements of **Equation 23**. For measuring the average correlation among the model parameters, the mean correlation *S* is easier to be used

$$S=\sqrt{\frac{1}{K(K-1)}\sum_{i=1}^{K}\sum_{j=1}^{K}\left(corr\mathbf{p}_{ij}-\delta_{ij}\right)^2}, \tag{24}$$

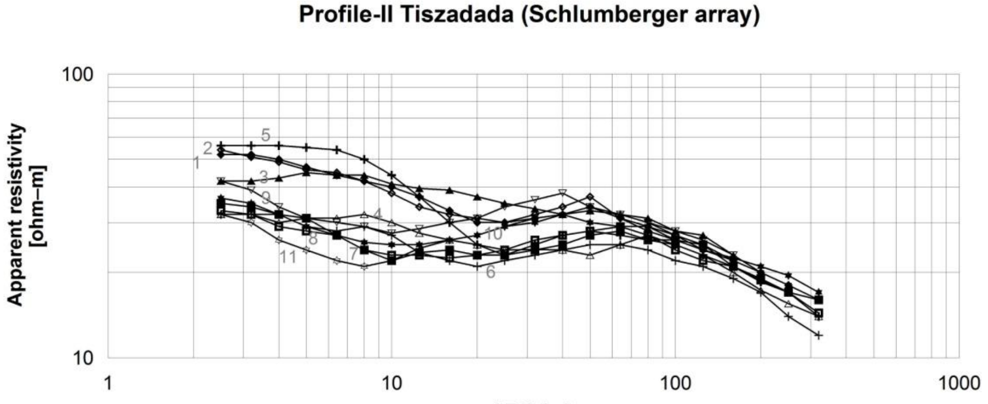
where correlation coefficients of the main diagonal are not included in the calculation of the average correlation (where $\delta_{ij}=1$ when $i=j$, otherwise $\delta_{ij}=0$).

3. Field study

An environmental study is presented for comparing the 1D single inversion method with the series expansion-based 2D CGI and 2D CGWI inversion procedure, respectively. The field example shows the processing of VES data by both inversion methods. The aim of the environmental exploration was to map an aquifer situated in Tiszadada at the bank of River Tisza (North-East Hungary).

The geological target was an inhomogeneous gravel sequence with a clayey basement. An earlier paper dealt with the interpretation of another profile (Profile-I) from the same area (Gyulai et al. 2010a). In this research a new 2000m profile (Profile-II) was investigated, along which apparent resistivity data were collected at 11 VES stations in Schlumberger array (see **Fig. 3**). Based on the preliminary interpretation of Profile-I, the geological structure was approximated by a four-layered model, where both layer-thicknesses and resistivities were allowed to vary laterally.

Fig. 3 Measured resistivity sounding curves at eleven parallel VES stations along Profile-II in Tiszadada (North-East Hungary).



At first a set of 1D single inversion procedures were performed for the determination of local layer-thicknesses and resistivities defined in **Equation 4**. In **Table 1**, the estimated model parameters and their estimation errors are represented. The parameters and their measurement units are: profile distance *x* (meter), resistivity ρ_i (ohm-m), layer-thickness h_i (meter). The measure of estimation error σ indicated in round brackets after each parameter is percent.

Table 1 The results of 1D single inversion of VES data measured along Profile-II in Tiszadada (North-East Hungary).

x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
ρ_1	51.7 (2)	52.5 (3)	43.6 (1)	31.8 (1)	59.8 (3)	32.8 (3)	38.6 (4)	34.0 (4)	51.5 (8)	40.2 (4)	37.4 (5)
ρ_2	21.0 (72)	19.0 (105)	28.4 (19)	17.6 (32)	21.1 (11)	20.0 (6)	21.1 (4)	18.4 (15)	25.6 (3)	20.0 (10)	14.3 (21)
ρ_3	37.3 (14)	40.7 (15)	41.3 (80)	54.0 (200)	42.3 (121)	38.5 (73)	43.4 (54)	31.5 (6)	55.0 (39)	35.2 (4)	38.0 (3)
ρ_4	15.5 (3)	15.5 (3)	13.8 (4)	11.8 (5)	12.0 (9)	9.3 (6)	14.0 (4)	12.6 (4)	20.8 (6)	17.5 (2)	15.7 (2)
h_1	5.1 (29)	4.7 (35)	8.8 (25)	8.2 (26)	5.0 (10)	3.9 (17)	2.1 (12)	2.7 (21)	1.4 (12)	2.1 (14)	1.4 (16)
h_2	6.3 (170)	5.3 (193)	20.9 (193)	17.3 (127)	22.1 (105)	21.3 (66)	19.3 (39)	7.3 (46)	14.8 (29)	5.5 (31)	3.6 (35)
h_3	39.4 (39)	35.7 (35)	34.0 (185)	24.0 (253)	33.7 (190)	35.1 (110)	29.1 (82)	56.8 (15)	26.0 (69)	38.5 (13)	42.1 (8)

The mean estimation error defined in **Equation 21** was 74%. This value represented the average case in geoelectric inversion practice. The largest estimation errors were related to the second and third layer-thicknesses. Relatively higher accuracy were obtained at VES stations Nos. 7-11 (see estimation errors at s=1200-2000m in **Table 1**). The average value of local data distances based on **Equation 22** was 2.7%. The above found results showed a good fitting in data space by relatively low accuracy of the estimations. The reliability of the estimated geoelectric model was relatively poor, because the correlation matrix indicated highly correlated parameters. The average value of mean spreads defined in **Equation 24** was 0.68.

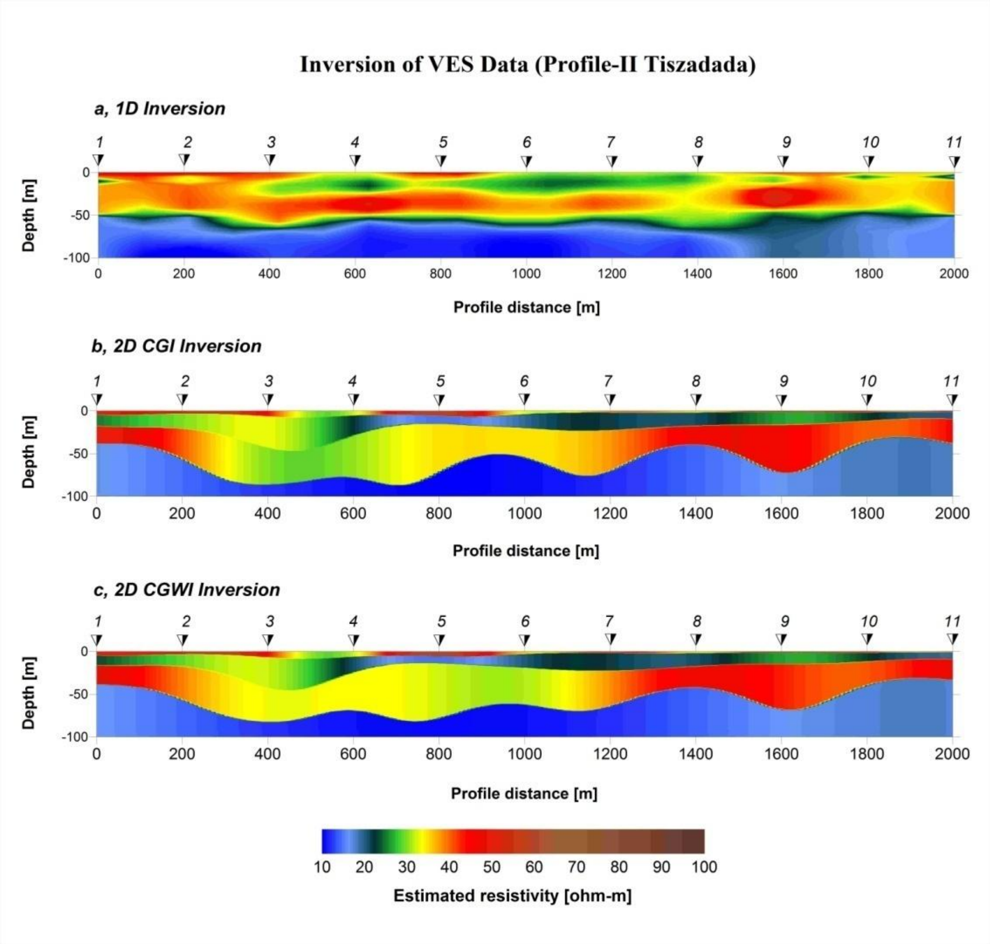
The CGI and CGWI inversion of the same data set applied a 2D forward modeling procedure for a more accurate computation of apparent resistivities than 1D forward modeling. The optimal numbers of series expansion coefficients of the periodic (sine) basis functions were (11, 11, 11) for the layer-thicknesses and (11, 9, 7, 3) for the resistivities. The optimal number of coefficients was selected by the strategy detailed in Gyulai et al. (2010a). The quality results of the 1D and CGWI inversion procedures can be compared in **Table 2**. Relative data distances d_1 (measured in percent), mean estimation errors F_1 (measured in percent) and mean spreads S_1 are referring to local 1D inversion results. Relative data distances d_2 (measured in percent), mean estimation errors F_2 (measured in percent), and mean spread S_2 are referring to 2D CGWI inversion results.

Table 2 The quality results of 1D and CGWI inversion of VES data measured along Profile-II in Tiszadada (North-East Hungary).

	x	0	200	400	600	800	1000	1200	1400	1600	1800	2000
1D	d_1	3.0	3.5	2.1	2.5	3.2	3.4	2.9	2.9	2.4	1.8	1.8
	F_1	72.4	85.3	106	132	106	56.2	41	21.9	33.2	14.5	18.0
	S_1	0.70	0.70	0.73	0.72	0.73	0.70	0.63	0.65	0.64	0.66	0.67
CGWI	d_2	2.8	4.0	2.4	4.0	3.5	3.2	3.1	2.6	4.4	1.5	2.2
	F_2	18.6	15.5	17.4	13.3	12.4	18.2	16.8	18.9	30.7	15.7	20.3
	S_2	0.25										

The introduction of individual weights in the CGWI procedure was justified by having different data distances (d_1) for VES stations in case of 1D inversion (average of 2,7%). The average of data misfits (d_2) of the CGWI procedure was 3.1%. It was concluded that approximately the same data misfit was achieved (the difference of misfit arises from numerical calculations). However, the quality of CGWI inversion results has been highly increased. The average of mean estimation errors (F_2) for structural parameters and resistivities has been decreased from 74% to 18%, which represented four times higher accuracy in the parameter space. The major part of estimation error originated from estimates of the first thin layer, which covered the errors of other estimations at the same stations (*e.g.* see F_2 at s=1600m in **Table 2**). If the first layer had been discarded, better overall estimation would have been given. The correlation coefficients of the 1D models showing strong dependence between the model parameters were displaced by smaller correlation coefficients. The mean spread (S_2) decreased from 0.68 to 0.25, which referred to much more reliable inversion results (the mean spread in this particular case is computed for a correlation matrix containing all unknowns of the 2D structure). The inversion results showed the univocal advantage of using series expansion technique which can be used for avoiding the ambiguity problem. The lateral changing of resistivities and layer-thicknesses can be seen in **Fig. 4**, where the results of 1D inversion, 2D (unweighted) CGI and 2D (weighted) CGWI inversion procedures are plotted. It can be seen that the geological structure obtained by CGWI procedure had been slightly modified compared to the result of unweighted CGI procedure. Comparing 1D to CGWI results, it can be seen that on the last third part of Profile-II (at VES stations Nos. 7-11) the layers are relatively homogeneous. In that case, the 1D approximation was good enough and there were not large differences between the 1D and 2D inversion results. The locally computed estimation errors of both inversion methods were smaller here than on the other parts of the profile. It can be seen that the CGI (or CGWI) technique visualized the pinch-out of layers with nearly the same value of resistivities in adjacent layers (see the second and third layers at s=300m). It can be mentioned that a similar phenomenon was previously detected in boreholes by using a 2D series expansion-based inversion method Dobróka et al. (2009). Comparing the given values of quality check parameters it is concluded that the model estimated by CGWI is more acceptable than that of given by 1D inversion. The resolution of the model can be further improved by using different types of basis functions.

Fig. 4 Inversion of apparent resistivity data collected at eleven parallel VES stations along Profile-II in Tiszadada (North-East Hungary). The inversion results are: interpolated 1D single inversion (case a), 2D unweighted CGI inversion (b) and 2D weighted CGWI inversion (c).



4. Discussions and conclusions

It was shown that several geophysical surveying methods can be used for the observation of the environment. The basis of the determination of geometrical parameters of the structure is the difference that exists between the physical parameters on different sides of a layer boundary.

However, the information inherent in measured data is sometimes not enough to resolve the environmental structures and physical parameters, because of the data noise and low parameter sensitivities (problem of ambiguity). Using oversimplified models for decreasing the number of unknowns of the geophysical inverse problem is not a practical alternative, because reducing the number of unknowns causes poor accuracy and reliability of the inversion results. On the other hand, applying too complicated models leads to inevitably poor reliability of the inversion estimations. The suggested series expansion-based inversion method can give an accurate and reliable estimate for complex structures including inhomogeneous layers with laterally varying boundaries in a stable inversion procedure, which is formulated as a highly overdetermined inverse problem. The inversion method performs simultaneous processing of different kinds of geoelectrical surveying data sets. Compared to traditional 1D inversion, the 2D CGWI procedure may produce at least one order of magnitude improvement in the parameter space. In an earlier study Gyulai et al. (2010a) compared CGI algorithm with a classical smoothness constrained 2D inversion program (RES2DINV by Geotomo Softwares), where the former produced sharper boundaries and more accurate and reliable parameters of 2D models. In this study, the significant improvement of accuracy and reliability of estimations was demonstrated by a field case representing an environmental problem. As a consequence, the application of the 2D interpretation method using a new model parameterization technique enables to study the environmental structures and phenomena in a more precise way.

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