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STRAIGHT CASCADE ANALYSIS FOR VARIABLE STAGGER AND PITCH

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SUMMARY

A method is shown for the solution of potential flow around the adjustable blades of a straight cascade. The basic idea of the procedure is that the governing equation is expanded into Taylor's series with respect to stagger angle. The effect of cascade solidity on the flow pattern can also be studied.

1. INTRODUCTION

Impellers of axial-flow turbomachines usually have adjustable blades in order to attain high efficiency over a wide range of operation. There are several methods available for the determination of the flow through a straight cascade of foils assuming incompressible inviscid fluid [1,3-6]. When modifying the cascade geometry by adjusting the blades, the usual procedure is to repeat most of the computational work. This paper suggests a method by which one can avoid the repeated application of the direct procedure for each cascade geometry.

2. THE GOVERNING EQUATIONS

The present method is based on the one shown in [6]. The basic notation used is shown in Fig. 1. λ is the blade stagger angle, t is the blade spacing or pitch, φ and φ' are polar angles belonging to the pivotal point P and variable point of integration P' , respectively. The inlet and outlet relative velocity vectors \vec{w}_1, \vec{w}_2 further flow angles χ_1 and χ_2 are also depicted.

The integral equation relating to the transformed contour velocity $w(\varphi)$ may be written as

$$w(\varphi) + \frac{1}{\pi} \int_0^{2\pi} K(\varphi, \varphi') w(\varphi') d\varphi' = F(\varphi) \quad (1)$$

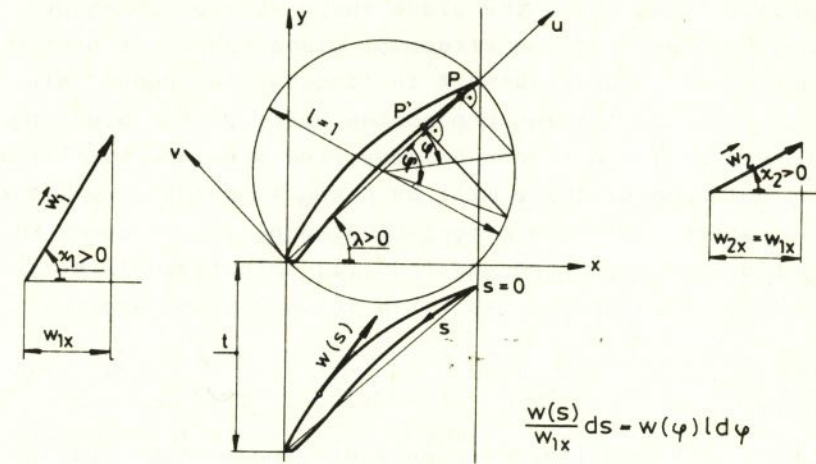


Fig. 1

where the kernel function is

$$K(\varphi, \varphi') = \frac{\pi}{t} \frac{\dot{y}(\varphi) \sinh \frac{2\pi}{t} [x(\varphi) - x(\varphi')] - \dot{x}(\varphi) \sin \frac{2\pi}{t} [y(\varphi) - y(\varphi')]}{\cosh \frac{2\pi}{t} [x(\varphi) - x(\varphi')] - \cos \frac{2\pi}{t} [y(\varphi) - y(\varphi')]} + \frac{\pi}{t} \dot{y}(\varphi) \quad (2)$$

In the limit as φ' tends to φ this becomes

$$K(\varphi, \varphi) = \lim_{\varphi' \rightarrow \varphi} K(\varphi, \varphi') = \frac{1}{2} \frac{\dot{y}(\varphi) \dot{x}(\varphi) - \dot{x}(\varphi) \dot{y}(\varphi)}{\dot{x}^2(\varphi) + \dot{y}^2(\varphi)} + \frac{\pi}{t} \dot{y}(\varphi) \quad (3)$$

The right hand side (RHS) of equation (1) can be composed of two parts

$$F(\varphi) = 2\dot{y}(\varphi) \tan \chi_1 + 2\dot{x}(\varphi) \quad (4)$$

Here $x(\varphi), y(\varphi), \dot{x}(\varphi), \dot{y}(\varphi)$ etc. are profile coordinates and their derivatives with respect to φ . By introducing numerical quadrature integral equation (1) can be approximated by a set of linear algebraic equations [3,6]. We do not

intend to deal with this problem here.

In case of the usual direct problem the stagger angle λ and blade spacing t are fixed. We shall consider these quantities as independent variables, and study their effects upon the cascade flow, while the blade shape is kept unchanged. We note that knowledge of the effect of blade spacing t on the flow may be of good assistance in finding the proper value of blade load. Since the two procedures built up for studying the effects of λ and t upon cascade flow are very similar in nature, only one of these will be shown in detail. Let it be the one referring to the effect of stagger λ . It seems straightforward to expand equation (1) into Taylor's series with respect to λ around its fixed value λ_0 . We have

$$\sum_{i=0}^{\infty} \frac{\partial^i w}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} + \frac{1}{\pi} \int_0^{2\pi} \sum_{i=0}^{\infty} \frac{\partial^i K}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} \sum_{i=0}^{\infty} \frac{\partial^i w}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} d\varphi = \sum_{i=0}^{\infty} \frac{\partial^i F}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} \quad (5)$$

By separating the different powers of $(\lambda - \lambda_0)$ this equation can be resolved into an infinite set of integral equations for the derivatives of the velocity with respect to λ . The first few equations of this set are as follow

$$\begin{aligned} w(\varphi; \lambda_0) + \frac{1}{\pi} \int_0^{2\pi} K(\varphi, \varphi'; \lambda_0) w(\varphi'; \lambda_0) d\varphi' &= F(\varphi; \lambda_0) \\ \frac{\partial w(\varphi; \lambda)}{\partial \lambda} \Big|_{\lambda_0} + \frac{1}{\pi} \int_0^{2\pi} K(\varphi, \varphi'; \lambda_0) \frac{\partial w(\varphi'; \lambda)}{\partial \lambda} \Big|_{\lambda_0} d\varphi' &= \frac{\partial F(\varphi; \lambda)}{\partial \lambda} \Big|_{\lambda_0} - \frac{1}{\pi} \int_0^{2\pi} \frac{\partial K(\varphi, \varphi'; \lambda)}{\partial \lambda} \Big|_{\lambda_0} w(\varphi'; \lambda_0) d\varphi' \\ \frac{\partial^2 w(\varphi; \lambda)}{\partial \lambda^2} \Big|_{\lambda_0} + \frac{1}{\pi} \int_0^{2\pi} K(\varphi, \varphi'; \lambda_0) \frac{\partial^2 w(\varphi'; \lambda)}{\partial \lambda^2} \Big|_{\lambda_0} d\varphi' &= \frac{\partial^2 F(\varphi; \lambda)}{\partial \lambda^2} \Big|_{\lambda_0} - \frac{1}{\pi} \int_0^{2\pi} \frac{\partial^2 K(\varphi, \varphi'; \lambda)}{\partial \lambda^2} \Big|_{\lambda_0} w(\varphi'; \lambda_0) d\varphi' - \\ &\quad - \frac{2}{\pi} \int_0^{2\pi} \frac{\partial K(\varphi, \varphi'; \lambda)}{\partial \lambda} \Big|_{\lambda_0} \frac{\partial w(\varphi'; \lambda)}{\partial \lambda} \Big|_{\lambda_0} d\varphi' \end{aligned} \quad (6)$$

All of these equations have the same kernel and their right hand sides contain solutions of the preceding equations and derivatives of the kernel function.

Using solutions

$$w(\varphi; \lambda_0) ; \frac{\partial w(\varphi; \lambda)}{\partial \lambda} \Big|_{\lambda_0} ; \frac{\partial^2 w(\varphi; \lambda)}{\partial \lambda^2} \Big|_{\lambda_0} ; \dots$$

occurring in equations (6), the transformed contour velocity belonging to stagger angle λ can be obtained as

$$w(\varphi; \lambda) = \sum_{i=0}^{\infty} \frac{\partial^i w(\varphi; \lambda)}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} \quad (7)$$

The solution of the first few equations of set (6) requires the evaluation of the derivatives of the kernel function $K(\varphi, \varphi'; \lambda)$ (3) and that of the function $F(\varphi)$ (4). These can be obtained through differentiation of quantities $x(\varphi; \lambda)$, $y(\varphi; \lambda)$, $\dot{x}(\varphi; \lambda)$, $\dot{y}(\varphi; \lambda)$. Looking at Fig. 1, relations between coordinates x, y and u, v can be derived easily in the form

$$\begin{aligned} x(\varphi; \lambda) &= u(\varphi) \cos \lambda - v(\varphi) \sin \lambda \\ y(\varphi; \lambda) &= u(\varphi) \sin \lambda + v(\varphi) \cos \lambda \end{aligned} \quad (8)$$

The differentiation of these quantities with respect to λ yield

$$\frac{\partial x(\varphi; \lambda)}{\partial \lambda} = -y(\varphi; \lambda) ; \quad \frac{\partial y(\varphi; \lambda)}{\partial \lambda} = x(\varphi; \lambda) \quad (9)$$

Since equation (1) is a linear one, further taking into account equation (4), $w(\varphi; \lambda)$ can be synthesized from a combination of two basic solutions

$$w(\varphi; \lambda) = w_A(\varphi; \lambda) \tan \lambda + w_B(\varphi; \lambda) \quad (10)$$

Bearing in mind equation (10), it is obvious that the right hand sides of equations (6) can also be resolved into two parts. Therefore basic solutions $w_A(\varphi; \lambda)$, $w_B(\varphi; \lambda)$ may be obtained separately as

$$w_{A,B}(\varphi; \lambda) = \sum_{i=0}^{\infty} \frac{\partial^i w_{A,B}(\varphi; \lambda)}{\partial \lambda^i} \Big|_{\lambda_0} \frac{(\lambda - \lambda_0)^i}{i!} \quad (11)$$

Now we can derive the dimensionless circulations A and B as

$$A, B = \frac{1}{t} \int_0^{2\pi} w_{A,B}(\varphi; \lambda) d\varphi \quad (12)$$

further the outlet flow angle χ_2 [6] is determined by

$$\tan \chi_2 = (1-A)\tan \chi_1 - B \quad (13)$$

It was assumed here that the rear stagnation point is fixed regardless of stagger and incidence, further that the flow is attached to the blade surfaces.

We note that the effect of changing the λ is more pronounced when cascade solidity (l/t) is high. When applying this method to a single foil as a limiting case of a cascade, we arrive at the following well-known conclusion: potential flow around a single aerofoil can be described by using one angle (incidence) only, while the description of the cascade flow requires two angles (incidence and stagger).

Our experiences show that even the solution of the first three equations of system (6) enables us to determine the cascade flow over a relatively wide λ or t domain.

3. NUMERICAL ANALYSIS AND SAMPLE CALCULATIONS

A FORTRAN computer program was developed for the solution of the problem based on the one shown in [6]. By introducing numerical quadrature and satisfying integral equations (6) at M different points on the surface, we arrive at sets of linear algebraic equations. The first of these is identical with the one relating to flow around cascade with fixed geometry [6]

$$\sum_{j=1}^M K_{ij} w_j = 2y_i \tan \chi_1 + 2\dot{x}_i \quad (i=1,2,\dots,M) \quad (14a)$$

It is known that special care has to be paid to the steepness of the kernel function when the pivotal point and the variable point of integration are opposite profile points. This problem is overcome by applying interpolation and numerical quadrature in the vicinity of these points [6]. The numerical smoothing of input profile data is also very important.

The second and third equations of set (6) become

$$\sum_{j=1}^M K_{ij} w_j = 2y_i' \tan \chi_1 + 2\dot{x}_i' - \sum_{j=1}^M K_{ij}' w_j \quad (i=1,2,\dots,M) \quad (14b)$$

$$\sum_{j=1}^M K_{ij} w_j'' = 2y_i'' \tan \chi_1 + 2\dot{x}_i'' - \sum_{j=1}^M K_{ij}'' w_j - 2 \sum_{j=1}^M K_{ij}' w_j' \quad (i=1,2,\dots,M) \quad (14c)$$

The superscripts ' and '' refer to first and second derivatives with respect to λ or t . It means a substantial advantage that the coefficient matrices of the linear systems (14a,b,c) are the same, i.e. K_{ij} , further that both the matrix and its inverse are already available. Only the RHS-s of systems (14b,c) have to be evaluated. A further advantage is that the simple trapezoidal rule has proved to be sufficient for numerical treatment here. The sum of each column of K_{ij}' and K_{ij}'' was checked and gave the result

$$\sum_{i=1}^M K_{ij}' \cong 0 \quad ; \quad \sum_{i=1}^M K_{ij}'' \cong 0 \quad (j=1,2,\dots,M) \quad (15)$$

as it was expected upon Martensen's work [4].

As known, systems (14a,b,c) are indeterminate. Applying Wilkinson-type Kutta-Joukowski condition

$$w_M = -\alpha w_1 \quad (16)$$

-where $\alpha \neq 1$ - the systems become determinate. Taking into account equations (10) and (11), each of the systems of (14a,b,c) is solved for two RHS-s, by simply multiplying the inverted matrix by the separate RHS vectors. Matrix K_{ij} is a very well conditioned one, thus its inverse can be determined easily.

The computer program has been tested against several sample calculations. Only two of these will be shown here. Fig.2 represents the effect of blade stagger upon the flow through a cascade built up from primary series of turbine blades [2]. Full curves demonstrate the solutions of the direct problem for both λ and λ_0 [6], x-s and circles refer to linear (14a,b) and quadratic (14a,b,c) approximations of the present method, respectively. The experimental, theoretical as well as the linear and quadratic approximative values of outlet flow angle χ_2 are also depicted. Fig.3 demonstrates the effect of cascade solidity on the flow through a cascade built up by using C4 profiles. The change in solidity is large, nevertheless the solution seems to be quite acceptable.

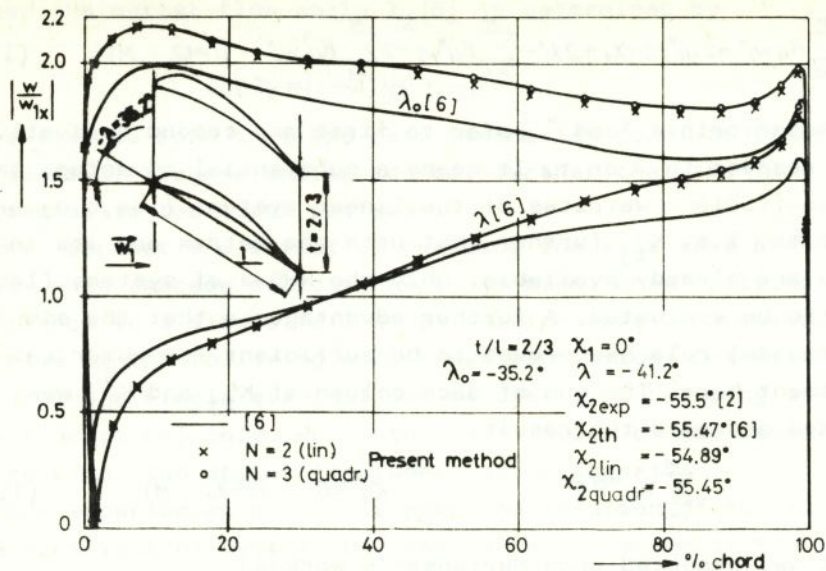


Fig. 2

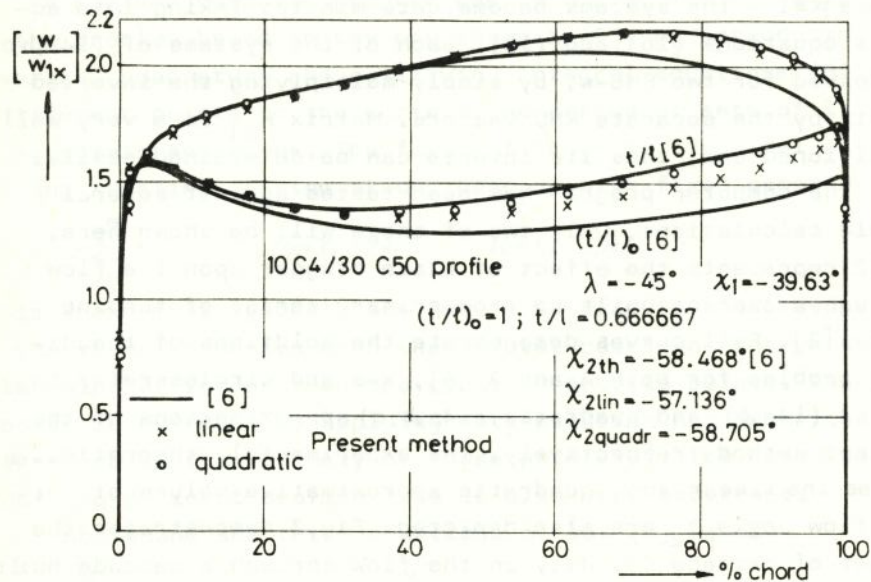


Fig. 3

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