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DETERMINATION OF THE THEORETICAL CHARACTERISTICS OF MIXED-FLOW PUMPS

A. Nyíri and L. Baranyi

SUMMARY

A procedure is presented to determine the theoretical characteristics of a cascade of blades on arbitrary surface of revolution with variable channel width in a pump impeller by tracing back the solution to a circular cascade.

The theoretical characteristics of a pump handling incompressible inviscid fluid can be generally determined through solving the direct problem of a cascade [1]. For the case of a radial trough-flow pump with thin blades having a certain course of channel width the task has been solved by Murata and his co-authors [2]. The results of the latter are plotted on diagrams which can be applied if the inaccuracy intrinsic to the way the data are given is admissible.

A procedure will be shown in order to extend the scope of the method worked out for radial impeller to the general case of a cascade of blades in a pump on arbitrary surface of revolution having the same width distribution as that of the radial one.

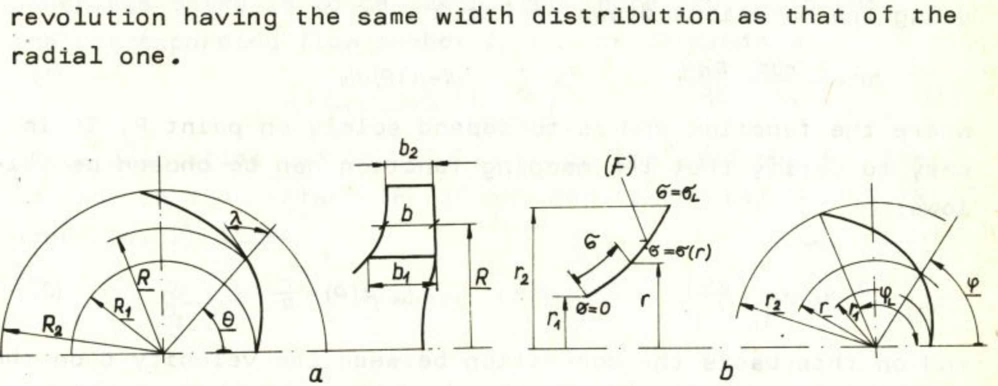

Fig.1

The logarithmic spiral shaped thin blade closing the angle λ with the radial direction is shown on Fig.1a. Generally the cascade of blades are on a surface of revolution (F) the meridional curve of which is given by the function $\sigma = \sigma(r)$ /see Fig.1b/. The channel width varies from b_1 to b_2 in both cases. The task is to find the angle and proportion preserving transformation between the two surfaces securing the same velocity potential in the same way as the conformal mapping between two complex planes.

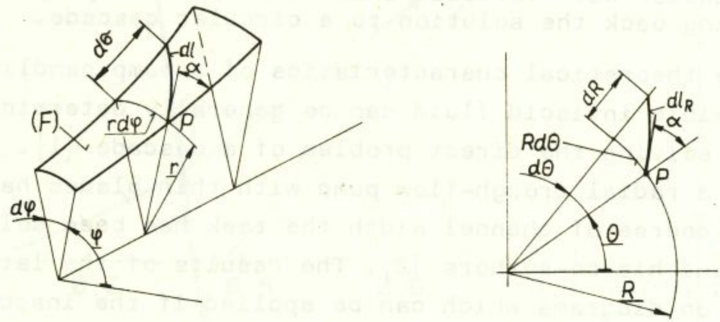

Fig.2

The above conditions are described by the following equations using the notations drawn on Fig.2

$$\tan\alpha = \frac{r\,d\varphi}{d\sigma} = \frac{R\,d\theta}{dR} \quad , \quad dl = \mu(P)\,dl_R \tag{1}$$

where the function $\mu(P)$ is to depend solely on point P. It is easy to verify that the mapping function can be chosen as follows:

$$R = R_1 \exp\left(\int_0^{\sigma} \frac{d\sigma}{r}\right) \quad ; \quad \theta = \varphi \quad ; \quad \mu(P) = \frac{r}{R} \tag{2}$$

and on this basis the connection between the velocity c on the surface (F) and the velocity c_R on the radial plane is obtained as below

$$cr = c_R R \tag{3}$$

The infinitely thin logarithmic shaped blades of angle λ and number N will correspond to the curve closing the constant angle λ with the meridional direction on the surface of revolution (F) with the same number of blades N. In order to define on both surfaces dimensionless quantities with the same values we can choose without losing the generality that the outer diameters are equal i.e. $r_2 = R_2$.

The theoretical characteristics of a circular cascade when no prewhirl exists can be determined by Murata's diagrams /to be found here also/. The equation of the theoretical characteristics is

$$\Psi_{th}(\Phi) = \Psi_o \left(1 - \frac{\tan\lambda}{k_m}\Phi\right) \tag{4}$$

where Ψ is the pressure number expressed by the specific energy Y and the peripheral velocity u_2 belonging to the radius R_2

$$\Psi = \frac{2Y}{u_2^2} \tag{5}$$

and Φ is the flow number defined by the volume rate of flow Q as follows

$$\Phi = \frac{Q}{2\pi R_2 b_2 u_2} \tag{6}$$

The values for Ψ_o and k_m in Equ. (4) are to be interpolated from the diagrams. The pressure number Ψ_s belonging to the shockless entry is to be taken from there too; with that Ψ_s the corresponding flow number Φ_s can be determined

$$\Phi_s = k_m \operatorname{cotan}\lambda \left(1 - \frac{\Psi_s}{\Psi_o}\right) \quad . \tag{7}$$

These quantities can be calculated in the same way for the cascade on the surface (F) if we compute the value of R_1/R_2 upon Equ. (2) like

$$\frac{R_1}{R_2} = e^{-a_L} \qquad \text{where} \qquad a_L = \int_0^{\sigma_L} \frac{d\sigma}{r} \quad . \tag{8}$$

In the classical stream filament theory the relative streamlines in the impeller are considered to be congruent with the blade shape. In this case when there is no prewhirl the relationship between the pressure and flow numbers is well known:

$$\Psi_\infty(\Phi) = 2(1 - \Phi\tan\lambda) \quad . \tag{9}$$

The values belonging to the shockless entry are

$$\phi_{s\infty}=\left(\frac{R_1}{R_2}\right)^2\frac{b_1}{b_2}\cot an\,\lambda \qquad ; \qquad \Psi_{s\infty}=2-2\left(\frac{R_1}{R_2}\right)^2\frac{b_1}{b_2} \quad . \tag{10}$$

Now the value of the slope factor can be calculated

$$\lambda_{p\infty}=\frac{\Psi_{th}(\phi_{s\infty})}{\Psi_{\infty}(\phi_{s\infty})}=\frac{\Psi_o\left[k_m-\left(\frac{R_1}{R_2}\right)^2\frac{b_1}{b_2}\right]}{2k_m\left[1-\left(\frac{R_1}{R_2}\right)^2\frac{b_1}{b_2}\right]} \quad . \tag{11}$$

The slope factor can be defined with the values belonging to the shockless entry of the theoretical characteristics by

$$\lambda_p=\frac{\Psi_{th}(\phi_s)}{\Psi_{\infty}(\phi_s)}=\frac{\Psi_s\,\Psi_o}{2\left[\Psi_o-k_m(\Psi_o-\Psi_s)\right]} \quad . \tag{12}$$

These last two values are close when the cascade is dense.

The previous expressions may be sufficient approximations if the blade angle is not constant on the surface (F). We can take the angle of the logarithmic spiral fitted on the leading and trailing edges of the blade by

$$\tan\lambda=\frac{\varphi_L}{a_L} \tag{13}$$

where φ_L is the span angle of a blade on its axial view /Fig.1 /.

The determination of the theoretical characteristics outlined above may be a good assistance in designing impeller blading in the simple classical way.

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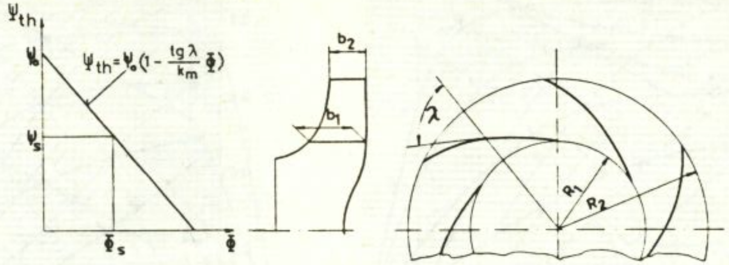
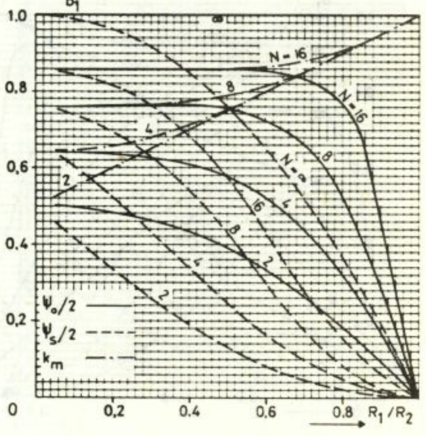


$\frac{b_2}{b_1} = 1{,}0$; $\lambda = 10°$



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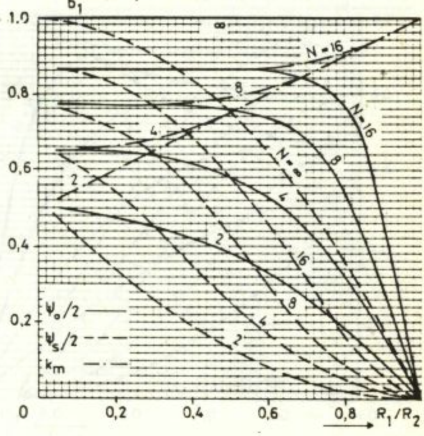

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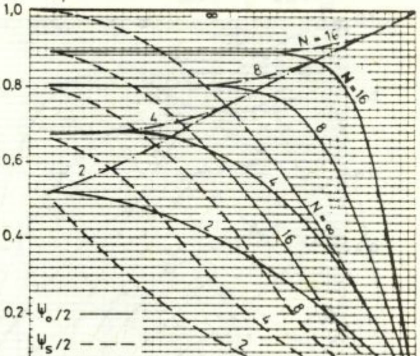

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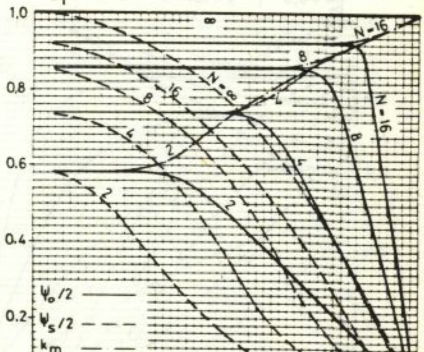

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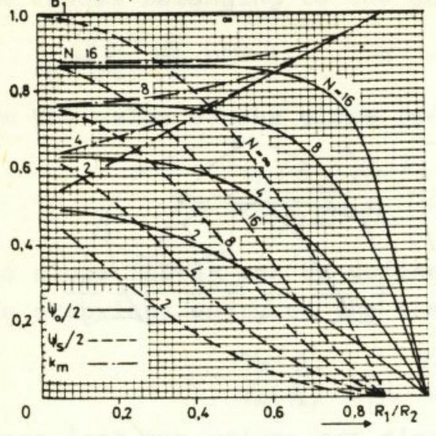

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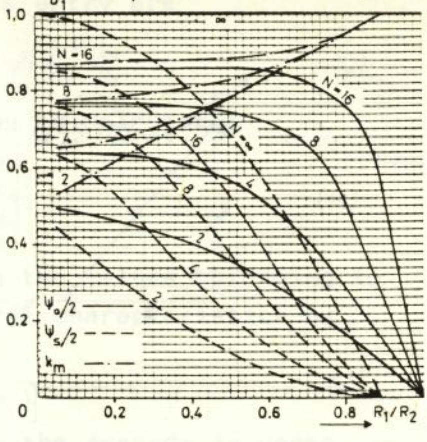

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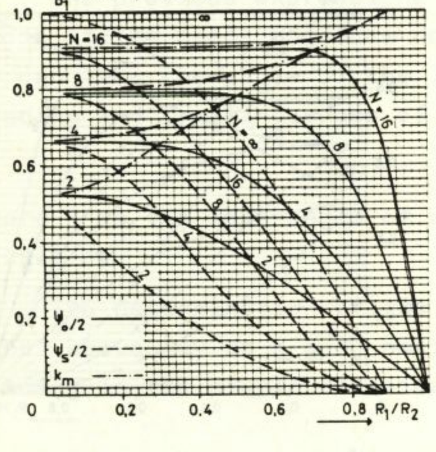

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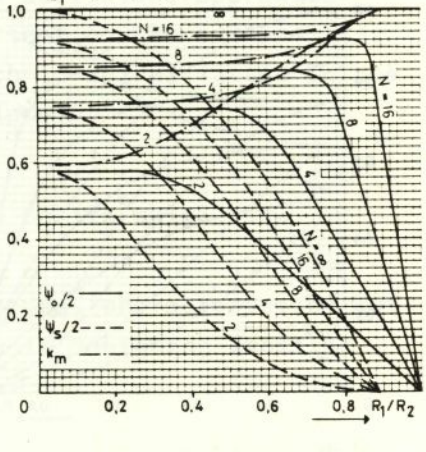

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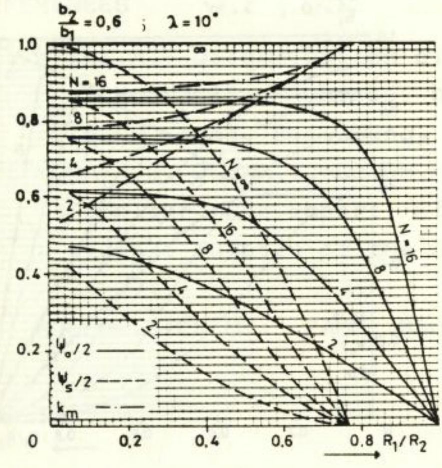

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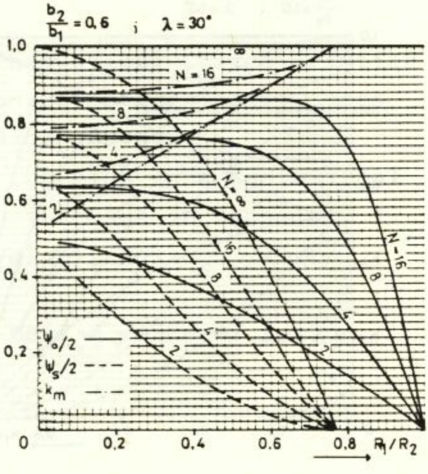

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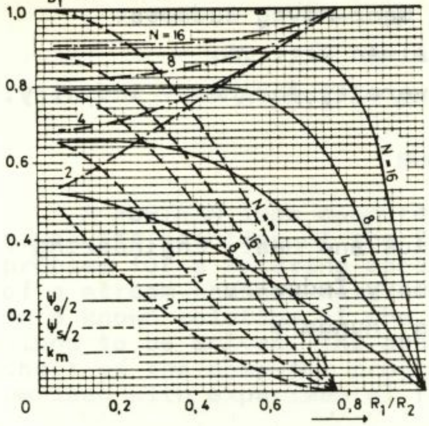

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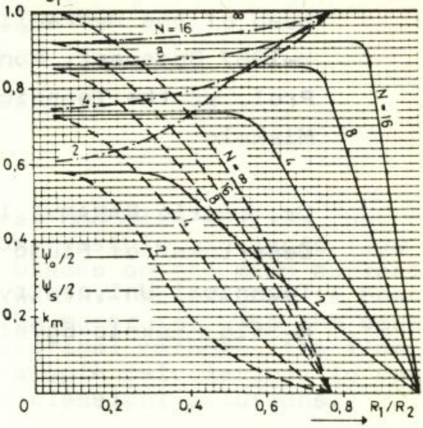

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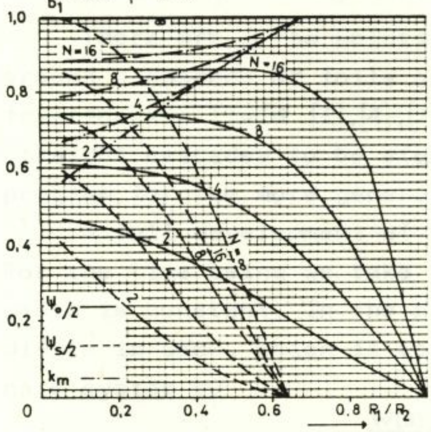

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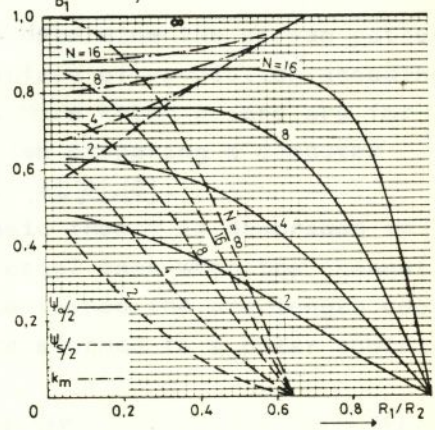

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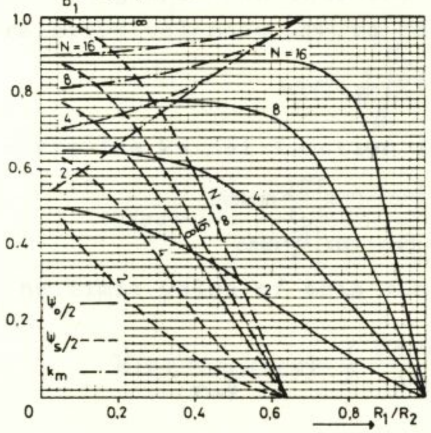

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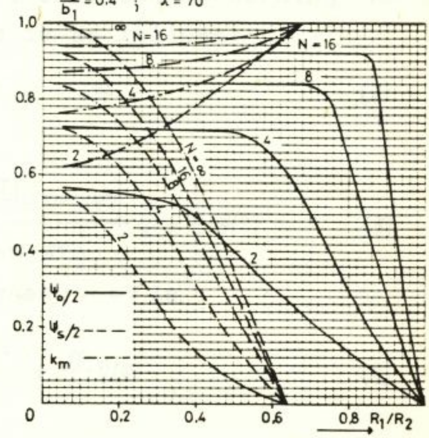

Dr. András NYIRI
Head of the Hydraulic Dep. Ganz-Mávag Budapest
H-1967 Budapest, Könyves Kálmán krt. 76.
Prof. at the Technical University for Heavy Industry, Miskolc

Dr. László BARANYI lecturer
Department of Fluid Mechanics and Heat Engineering
Technical University for Heavy Industry
H-3515 Miskolc-Egyetemváros, Hungary