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Computation of the meridional flow pattern in hydraulic machines, based on potential theory

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COMPUTATION OF THE MERIDIONAL FLOW PATTERN IN HYDRAULIC MACHINES, BASED ON POTENTIAL THEORY

by

L. BARANYI

1. Introduction

The numerical study of fluid flow in turbomachines has been the subject of a great deal of papers because of its practical importance. Flow in a turbomachine blade row has, in general, a three-dimensional, unsteady, viscous nature, and numerical solutions for such a general type flow are very difficult even when using computers with large storage capacity. Therefore, until recently, the usual approach has been the reduction of the three-dimensional flow to the task of solving two two-dimensional problems. These are usually the meridional flow and the cascade flow. To compute one of these patterns the other one is supposed to be known. Thus the solution of the whole problem can be found only by iteration. The aim of this paper is to present a solution of the meridional flow problem.

The main numerical techniques used for predicting the meridional flow in turbomachines are as follows: the streamline curvature method [1], [2], [3], [4], the matrix method [5], [6], [7] and finite element method (as well as the ones based on the calculus of variations) [8], [9]. The potential theory can be applied succesfully to the cascade flow problem [10], [11]. We can make use of experiences obtained. Similar methods are applied to the determination of the meridional flow [12], [13].

In this paper a method is shown for the computation of the meridional flow in turbomachine handling incompressible fluid. The flow in the bladed space is supposed to be blade congruent. When deriving the equation of the problem, we use one component ot the equation of motion, the equation of continuity, a simple loss model [6] and a kinematic condi-

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tion expressing that the blade surfaces are relative stream surfaces. The method is applicable to the determination of the meridional flow in the bladed space of an impeller or that of a guide wheel with arbitrary geometry and in a duct as well.

Notation

b	factor defined by equation (3)
(B)	boundary curve of domain (D)
$\mathbf{c}$	absolute velocity vector, $\mathbf{w} + \boldsymbol{\omega} \times \mathbf{r}$
$\overline{c}$	conjugate complex velocity in the meridional plane, $c_z - ic_r$
(D)	domain
E	energy per unit mass, $U + \frac{p}{\rho} + \frac{c^2}{2}$
$\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_z$	unit vectors
$\mathbf{f}$	friction force
$\mathbf{F}$	blade force
$F(\zeta)$	complex function, $rb\overline{c}$
G	vortex-like distribution along boundary (B)
I	modified relative energy per unit mass, $E - \omega rc_\varphi$
$\mathbf{n}$	unit vector normal to the mean blade surface
p	pressure
Q	source-like distribution along boundary (B)
$\mathbf{r}$	radius vector
r, φ, z	cylindrical co-ordinates
s	arc length along boundary (B)
Y'	energy loss per unit mass
U	potential energy per unit mass
$\mathbf{w}$	relative velocity vector
α, β	blade angles defined by equation (8) (see *Fig. 1*)
ζ	complex number, $z + ir$
ρ	fluid density
χ	angle between s and z-axis (see *Fig. 2*)
ψ	stream function
ω	angular velocity vector
$\Omega^*(z, r)$	function defined by equation (11)

Subscripts

r, φ, z	radial, circumferential and axial component, respectively
0	in front of the bladed space
B	on boundary (B)

2. The derivation of the basic equation

The incompressible fluid flow is supposed to be steady and blade congruent, thus the flow will be axisymmetric. A simple loss model is assumed: the direction of the friction force **f** is opposite to the relative velocity **w** [6]. When studying the flow through turbomachines, it is convenient to use cylindrical co-ordinate system, fixed to the impeller rotating with constant angular velocity ω. In this system the equations of motion and continuity can be written as follows

$$(\text{curl } \mathbf{c}) x \mathbf{w} = \mathbf{F} - \nabla I + \mathbf{f} \quad , \tag{1}$$

$$\frac{\partial (rbc_r)}{\partial r} + \frac{\partial (rbc_z)}{\partial z} = 0 \quad . \tag{2}$$

In this last equation the blade thickness is taken into account by the factor b:

$$b = 1 - \frac{s_\varphi}{T} \quad , \tag{3}$$

where s_φ is the angular blade thickness, T is the cascade blade pitch. In equation (1) I is the modified relative energy, **F** is a nonconservative blade force vector normal to the mean blade surface, which is in turn a relative streamsurface, consequently

$$\mathbf{F} \cdot \mathbf{w} = 0 \ . \tag{4}$$

This force **F** originally introduced by *Lorenz* [14] provides the energy transfer between the blades of the impeller and the fluid. It follows from equations (1) and (4) that only the friction force **f** has influence on the change of the modified relative energy I along the relative flow path. Integrating the equation of motion (1) along a relative streamline we find for steady flow

$$I = I_0 - Y' \quad , \tag{5}$$

where I_0 is the reference value of I, which remains constant along any streamline for inviscid flow, and Y' is the energy loss. Using test results Y' can be considered as a known distribution along the trailing edge of the blade, and it can be approximated by using some sort of interpolation method along a streamline. By knowing this distribution and the relative velocity **w**, the streamwise equation of motion can be used to determine the friction force:

$$\mathbf{f} = -\frac{\mathbf{w}}{w^2} \frac{dY'}{dt} \quad . \tag{6}$$

To reshape the equation of motion (1), let us multiply it by $\mathbf{n} \times \mathbf{w}/w^2$, so we obtain

$$\mathbf{n} \cdot \text{curl } \mathbf{c} = -\frac{1}{w^2} \nabla I \cdot (\mathbf{n} \times \mathbf{w}) \ , \tag{7}$$

where **n** is the unit vector normal to the mean blade surface. This equation can be studied in detail by introducing two angles α and β defined by the local geometry of the mean blade surface, (*Fig. 1*).

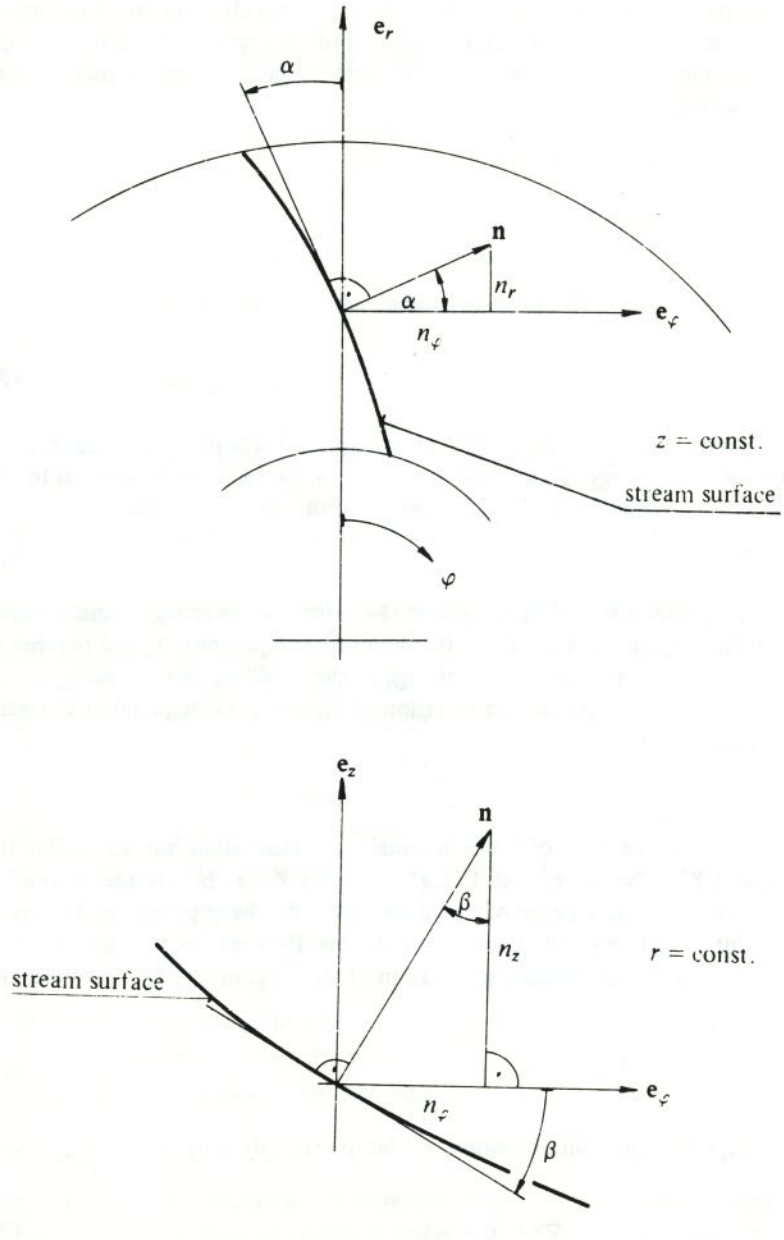

Fig. 1 **Definition of blade angles α and β**

$$\operatorname{tg}\alpha = \frac{n_r}{n_\varphi} \quad ; \qquad \operatorname{ctg}\beta = \frac{n_z}{n_\varphi} \quad . \tag{8}$$

A stream function ψ may be defined from the equation of continuity (2) where

$$\frac{\partial \psi}{\partial r} = r\,b\,c_z\,; \qquad \frac{\partial \psi}{\partial z} = -\,r\,b\,c_r \quad . \tag{9}$$

By using equations (7), (8), (9) the following

$$\Delta\psi = \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} = \Omega^* \tag{10}$$

Poisson-like partial differential equation is obtained for ψ, where

$$\Omega^* = c_z\left(b + r\,\frac{\partial b}{\partial r}\right) - rc_r\,\frac{\partial b}{\partial z} + b\,\frac{\partial (rc_\varphi)}{\partial r}\operatorname{ctg}\beta - b\,\frac{\partial (rc_\varphi)}{\partial z}\operatorname{tg}\alpha +$$

$$+\,\frac{rb}{c^2 - 2\omega r c_\varphi + \omega^2 r^2}\left\{\frac{\partial I}{\partial r}[\,c_z - (c_\varphi - \omega r)\operatorname{ctg}\beta] +\right.$$

$$\left. +\,\frac{\partial I}{\partial z}[\,(c_\varphi - \omega r)\operatorname{tg}\alpha - c_r]\right\} \quad . \tag{11}$$

This last function is determined by the blade geometry, the modified relative energy I, the moment of momentum rc_φ and the meridional velocity. Since velocity components c_z and c_r occur in the term of Ω^* (11), the problem can be solved by using iterative method only.

Let us see now what the corresponding equation for the stream function in an axisymmetric steady duct flow is. Equations (1)–(6) are valid now also if the angular velocity ω and the blade force **F** are put equal to zero and if $b = 1$. By using the circumferential component of the equation of motion (1) and equation (6) we have the following relation

$$\frac{d(rc_\varphi)}{dt} = -\,\frac{rc_\varphi}{c^2}\,\frac{dY'}{dt} \quad . \tag{12}$$

This expresses the conservation of moment of momentum for an inviscid flow. When there are losses, however, equation (12) shows that the moment of momentum decreases in the direction of flow. Our basic equation for the duct flow can be obtained from the equation of motion for the direction $\mathbf{e}_\varphi \times \mathbf{c} = c_z\mathbf{e}_r - c_r\mathbf{e}_z$ which leads to a relation with no component of the friction force. By inserting ψ (9) into it we have

$$\Delta\psi = \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} = c_z +$$

$$+\,\frac{r}{c_z^2 + c_r^2}\left[c_z\,\frac{\partial E}{\partial r} - c_r\,\frac{\partial E}{\partial z} + \frac{c_r c_\varphi}{r}\,\frac{\partial (rc_\varphi)}{\partial z} - \frac{c_z c_\varphi}{r}\,\frac{\partial (rc_\varphi)}{\partial r}\right], \tag{13}$$

where E is the energy. It is easily verified, that replacing $\mathrm{tg}\alpha$ and $\mathrm{ctg}\beta$ in equation (11) formally by

$$\mathrm{tg}\alpha = -\frac{c_r c_\varphi}{c_z^2 + c_r^2} \qquad \text{and} \qquad \mathrm{ctg}\beta = -\frac{c_z c_\varphi}{c_z^2 + c_r^2} \tag{14}$$

(and of course $\omega = 0$, $b = 1$) equations (10) and (11) reduce to equation (13).

This analysis, leading to equations (10), (11) is applicable to the flow either in the impeller, the guide wheel ($\omega = 0$) or the duct region of a mixed flow turbomachine [$\omega = 0$, $b = 1$, (14)]. For a pump impeller $\omega > 0$, and $\omega < 0$ for a turbine.

If the flow is a potential one in front of the stationary or rotating blade row, and the loss on the cascade can be neglected, the equation of motion (7) reduces to

$$\mathbf{n} \cdot \mathrm{curl}\, \mathbf{c} = 0 \ ,$$

since ∇I vanishes identically. This equation shows that the curl vector is contained in the tangent plane of the mean blade surface.

3. The solution of the basic equation of flow

The *Poisson*-like differential equation (10) will be solved by Green's integral theorem. Let $P(z, r)$ be a given point and (D) a single connected, bounded domain the closed boundary of which is (B) (*Fig. 2*), then

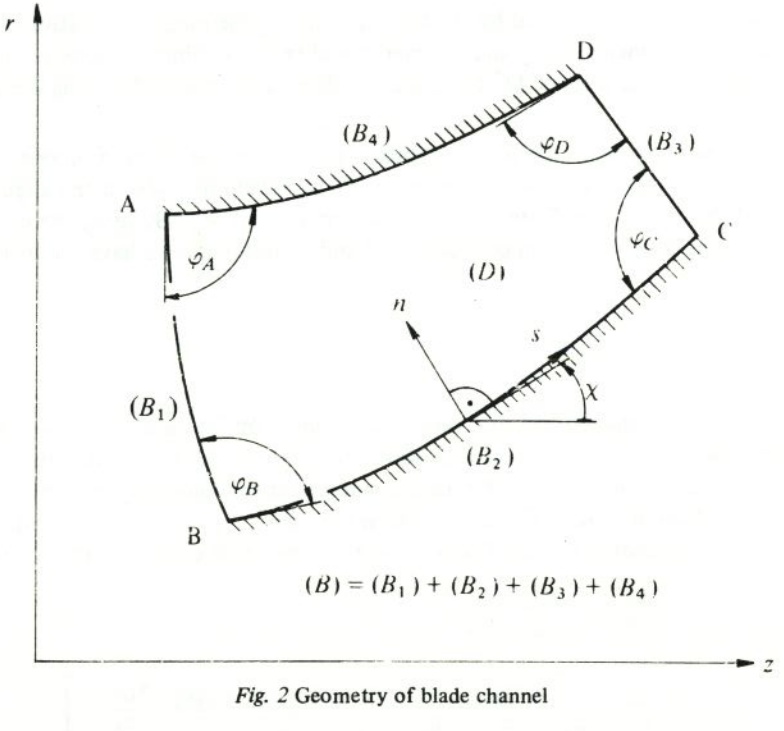

Fig. 2 Geometry of blade channel

$$\left.\begin{array}{ll} P\in(D), & \psi(P) \\ P\notin(D), & 0 \end{array}\right\} = \frac{1}{2\pi}\int_{(D)}\Delta\psi \ln d\, dA + \frac{1}{2\pi}\oint_{(B)}\frac{\partial\psi}{\partial n'}\ln d\, ds' - \frac{1}{2\pi}\oint_{(B)}\psi\frac{\partial}{\partial n'}(\ln d)\, ds'. \quad (15)$$

Here d is the distance between the point under consideration $P(z, r)$ and the variable point of the integration $P'(z', r')$, dA is the element of surface, ds is the element of arc length along boundary (B) and $\partial/\partial n$ denotes differentiation in the direction of the inward normal to the curve (B).

It is convenient to use the complex analysis to the solution of this problem. Let us introduce the complex variable $\zeta = z + ir$ relating to the point under consideration $P(z, r)$. ζ' and ζ'_B will denote the complex vectors of the variable points in domain (D) and on boundary (B), respectively. Applying the $(\partial/\partial r + i\partial/\partial z)$ complex differential operator to equation (15) we obtain

$$\left.\begin{array}{ll} \zeta\in(D), & \dfrac{\partial\psi}{\partial r}+i\dfrac{\partial\psi}{\partial z} \\ \zeta\notin(D), & 0 \end{array}\right\} = \frac{i}{2\pi}\int_{(D)}\frac{\Delta\psi dA}{\zeta-\zeta'} + \frac{i}{2\pi}\oint_{(B)}\frac{\partial\psi}{\partial n'}\frac{ds'}{\zeta-\zeta'_B} -$$

$$-\frac{i}{2\pi}\oint_{(B)}\psi\frac{\partial}{\partial n'}\left(\frac{1}{\zeta-\zeta'_B}\right)ds' .$$

As $1/(\zeta - \zeta'_B)$ is an analitic function along boundary (B), the last term of the previous equation can be evaluated as

$$-\frac{i}{2\pi}\oint_{(B)}\psi\frac{\partial}{\partial n'}\left(\frac{1}{\zeta-\zeta'_B}\right)ds' = \frac{1}{2\pi}\oint_{(B)}\psi\frac{\partial}{\partial s'}\left(\frac{1}{\zeta-\zeta'_B}\right)ds' =$$

$$= \frac{1}{2\pi}\left[\frac{\psi}{\zeta-\zeta'_B}\right]_{s'=0}^{L} - \frac{1}{2\pi}\oint_{(B)}\frac{\partial\psi}{\partial s'}\frac{ds'}{\zeta-\zeta'_B} ,$$

where L is the total length of curve (B). Since ψ is a singlevalued function, the first term on the right hand side of this relation is zero, thus

$$\left.\begin{array}{ll} \zeta\in(D), & \dfrac{\partial\psi}{\partial r}+i\dfrac{\partial\psi}{\partial z} \\ \zeta\notin(D), & 0 \end{array}\right\} = \frac{i}{2\pi}\int_{(D)}\frac{\Delta\psi dA}{\zeta-\zeta'} + \frac{i}{2\pi}\oint_{(B)}\left(\frac{\partial\psi}{\partial n'}+i\frac{\partial\psi}{\partial s'}\right)\frac{ds'}{\zeta-\zeta'_B} . \quad (16)$$

Let us denote the complex function occuring on the left hand side of equation (16) by $F(\zeta)$, and utilizing the definition of ψ (9) we have

$$F(\zeta) = \frac{\partial\psi}{\partial r} + i\,\frac{\partial\psi}{\partial z} = rb(c_z - ic_r) = r\,b\,\overline{c} \quad , \tag{17}$$

where $\overline{c}$ is the conjugate complex velocity. The value of the function $F(\zeta)$ jumps crossing the boundary (B); two limit values are defined as follows

$$\lim_{\substack{\zeta \to \zeta_B \\ \zeta \in (D)}} F(\zeta) = F^+(\zeta_B)\,, \qquad \lim_{\substack{\zeta \to \zeta_B \\ \zeta \notin (D)}} F(\zeta) = F^-(\zeta_B)\ . \tag{18}$$

According to equations (16) and (17) $F^-(\zeta_B)$ is zero identically. The difference between the limit values of $F(\zeta)$ on the two sides of the contour can be written in the following form

$$F^+(\zeta_B) - F^-(\zeta_B) = F^+(\zeta_B) = [G(s) - iQ(s)]e^{-i\chi}\ . \tag{19}$$

Here χ is the angle between s and z-axis (*Fig. 2*). The tangential and normal component of function $F^+(\zeta_B)$, viz. the G and Q can be represented as vortex- and source-like distribution along the boundary, respectively. The walls of the blade channel (B_2) and (B_4) being streamlines there Q distribution is zero identically. Generally, however, both G and Q differ from zero at the inlet and outlet sections of the channel (viz. along the curves (B_1) and (B_3) on *Fig. 2*, respectively).

Let us return now to equation (16). Tankig into account equations (10), (17) and the relation

$$\frac{\partial\psi}{\partial n'} + i\,\frac{\partial\psi}{\partial s'} = \left(\frac{\partial\psi}{\partial r'} + i\,\frac{\partial\psi}{\partial z'}\right)e^{i\chi'} = F^+(\zeta_B)\,e^{i\chi'} = G(s') - iQ(s')$$

equation (16) will have the form

$$\left.\begin{matrix} \zeta \in (D)\,, & F(\zeta) \\ \zeta \notin (D)\,, & 0 \end{matrix}\right\} = \frac{i}{2\pi}\int_{(D)} \Omega^*(z',r')\,\frac{dA}{\zeta - \zeta'} + \frac{i}{2\pi}\oint_{(B)} [G(s') - iQ(s')]\,\frac{ds'}{\zeta - \zeta'_B}\ . \tag{20}$$

In order to determine $F(\zeta)$ on the boundary, the limiting case of equation (20) is to be considered, when $\zeta \to \zeta_B$. Applying the Plemelj's formula [15] we arrive to the following equation:

$$F(\zeta_B) = \left(\frac{1}{2} - \frac{\varphi}{2\pi} \pm \frac{1}{2}\right)[G(s) - iQ(s)]\,e^{-i\chi} +$$

$$+ \frac{i}{2\pi}\int_{(D)} \Omega^*(z',r')\,\frac{dA}{\zeta_B - \zeta'} + \frac{i}{2\pi}\oint_{(B)} [G(s') - iQ(s')]\,\frac{ds'}{\zeta_B - \zeta'_B}\ , \tag{21}$$

where

$$\varphi = \begin{cases} \text{one of the corner angles } \varphi_A, \varphi_B, \varphi_C, \varphi_D \text{ (Fig. 2),} \\ \text{if } \zeta_B \text{ is a corner point,} \\ \\ \pi, \text{ otherwise} \end{cases}$$

In equation (21) the upper sign holds for the inner side of boundary (B), and the lower sign relates ot the outer side. Here the surface integral is continuous over the whole plane, and the line integral means a Cauchy principal value which always exists if $F^*(\zeta_B) = [G(s) - iQ(s)]e^{-i\chi}$ is a continuous function of arc length s.

Let us see now what sort of conditions must be fulfilled for G and Q. For this purpose the line integral of $F^*(\zeta_B)$ along (B) is carried out. Since the angle $\varphi = \pi$ almost everywhere in equation (21), the line integral of $F^*(\zeta_B)$ along (B) remains unchanged when φ is put equal to π. We can write

$$\oint_{(B)} F^*(\zeta_B) d\zeta_B = \frac{1}{2} \oint_{(B)} [G(s) - iQ(s)] e^{-i\chi} d\zeta_B +$$

$$+ \frac{i}{2\pi} \oint_{(B)} \int_{(D)} \Omega^*(z', r') \frac{dA d\zeta_B}{\zeta_B - \zeta'} + \frac{i}{2\pi} \oint_{(B)} \oint_{(B)} [G(s') -$$

$$- iQ(s')] \frac{ds' d\zeta_B}{\zeta_B - \zeta'_B} .$$

By using the Cauchy integral and separating the real and imaginary parts of this integral we have

$$\oint_{(B)} G(s) ds = - \int_{(D)} \Omega^*(z', r') dA \quad , \tag{22}$$

$$\oint_{(B)} Q(s) ds = 0 \ . \tag{23}$$

The first of these equations shows the well-known relation between the circulation and the vortex distribution. The second one is the equation of continuity.

4. The computation of the velocity distribution

Our task is to determine the meridional flow in a given meridional section of a runner (or guide wheel) with prescribed energy and loss distribution over that and the inlet velocity distribution is also supposed to be known.

Considering equations (11) and (20) it is obvious that the distribution of $F^*(\zeta_B)$ along the walls and at the exit of the channel, furthermore the velocity field **c** over the domain (D) must also be known. Therefore our task can be solved by iteration only. The values of the function $F^*(\zeta_B)$ can be assumed upon test results at the inlet and outlet of the channel, but these distributions are to fulfil the equation of continuity (23). When curves (B_1) and (B_3) are not close to the bladed space (see *Fig. 2*), then G and Q can be assumed unchanged on them. So the function Q has a fixed distribution along curve (B) as it has been mentioned $Q \equiv 0$ on the walls.

The following integral equation is obtained from equation (21):

$$F^*(\zeta_B) = [G(s) - iQ(s)]\, e^{-i\chi} = \frac{i}{\varphi} \int_{(D)} \Omega^*(z', r') \frac{dA}{\zeta_B - \zeta'} +$$

$$+ \frac{i}{\varphi} \oint_{(B)} [G(s') - iQ(s')] \frac{ds'}{\zeta_B - \zeta'_B} \quad . \tag{24}$$

The solution of the problem can be reached by the repeated application of equations (20) and (24). In the konwledge of the $(j-1)$ -th approximation the j – th can be determined as follows. By inserting the $(j-1)$-th values of Ω^* and G into equation (24) the j – th value of G will be obtained

$$G^{(j)}(s) = Re\, [F^{*(j)}(\zeta_B) e^{i\chi}] \ , \tag{25}$$

where Re [ ] denotes the real part of the function in the bracket.

The recurrence formula is introduced

$$G^{(j)*} = G^{(j)} + k[G^{(j)} - G^{(j-1)*}] \tag{26}$$

with taking $G^{(0)*} = G^{(0)}$. Putting the values of $G^{(j)*}$ and $\Omega^{*(j-1)}$ into equation (20) the j – th approximation of the meridional velocity components c_z and c_r will be obtained

$$c_z^{(j)} = \frac{1}{rb} \mathrm{Re}\, [F^{(j)}(\zeta)] \ ; \qquad c_r^{(j)} = -\frac{1}{rb} Im\, [F^{(j)}(\zeta)]$$

where Im [] denotes the imaginary part of the function in the bracket.

Upon these velocity distributions the j – th approximation of Ω^* can be evaluated, by which all the j – th values are known.

By selecting the value of k in equation (26) properly the convergence of the procedure will be faster than with the value $k = 0$, viz. in case of $G^{(j)*} = G^{(j)}$. The iteration is to be continued until two successive flow patterns are sufficiently close to each other.

5. Summary

In this paper a method is shown for the computation of the meridional flow in a turbomachine handling incompressible fluid. The flow in the bladed space is supposed to be blade congruent. The basic equation of the problem is derived by means of the equations of motion, continuity, a simple loss model and a geometric condition relating to the three components of velocity. This is a Poisson-like partial differential equation which is solved by Green's integral theorem using iteration. The method is rather general in nature. It is applicable to the determination of the meridional flow in the bladed space of an impeller or that of a quide wheel with arbitrary geometry and in a duct as well.

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POTENTIALTHEORETISCHE BERECHNUNG DER MERIDIANSTRÖMUNG IN STRÖMUNGSMASCHINEN

von
L. BARANYI

Zusammenfassung

In dieser Arbeit wird eine Methode zur Berechnung der in mit einem inkompressiblen Medium arbeitenden strömungstechnischen Maschinen entstehenden Meridianströmung vorgestellt. Die Strömung im Schaufelraum wird als schaufelkongruent angesehen. Die Grundgleichung für die Aufgabe ergibt sich unter Benutzung der Bewegungs- und Kontinuitätsgleichungen, eines einfachen Verlustmodells sowie der kinematischen Bedingung für die drei Geschwindigkeitskomponenten. Die Grundgleichung ist eine Poissonsche partielle Differentialgleichung und wird unter Anwendung der Greenschen Integralformel, durch Iteration gelöst. Die Methode ist sehr allgemein: sie kann zur Ermittlung der Meridianströmung im Schaufelraum eines Lauf- oder Leitrades beliebiger Geometrie sowie in einem radfreien drehsymmetrischen Kanal verwendet werden.

РАСЧЕТ МЕРИДИОНАЛЬНОГО ПОТОКА, ФОРМИРУЮЩЕГОСЯ В ГИДРАВЛИЧЕСКИХ МАШИНАХ С ПОМОЩЬЮ ТЕОРИИ ПОТЕНЦИАЛА

Л. БАРАНИ

Резюме

В настоящей работе показывается один из методов для расчета меридионального потока, формирующегося в аэрогидродинамических машинах, работающих с несжимаемой средой. Предположим, что в области лопастей число лопастей бесконечно. Основое уравнение задачи получается с помощью уравнения движения, неразрывности, одной простой модели потери и кинематического условия, относящегося к трем компонентам скорости. Это уравнение – которое решается применением интеральной формули Грина, с помощью итерации – похоже на дифференциальное уравнение Пуассона в частных производных. Этот метод общеприменим, им пользоваться для определения меридионального потока, формирующегося в области лопастей рабочего колеса или направляющего аппарата имеющего произвольную геометрию так же, как в канале свободном от лопастей.