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[Manuscript received November, 20 1970]

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Skid-free rolling of the steered front wheels of motor cars in curves may be realized with a reasonable approximation with the aid of the familiar trapezoidal steering gear, by conveniently selecting the lengths and angles of the lock of each rod of the steering linkage. However, the divergence between the cornering angles of the wheels theoretically needed for pure rolling and those practically realized by the trapezoidal steering linkage in using this system may only be neglected with regard to skidding in the case if the wheels are, in a geometrical concept, flat circular discs situated perpendicularly to the plane of the road surface and the steering knuckle pins around which the wheels may be turned, are also normal to the road surface. Owing to the actual position of the medium plane of the front wheels and to the finite extent of the rubber tyres, noticeable differences may occur between the theoretical and effective radii of cornering. This paper shows how it is possible to design a trapezoidal steering linkage which, by assuming the tyres to be circular rings and taking into account the effective steering angle of the wheels, makes the rolling of the front wheels in cornering — at least in curves of a frequently applied average radius — theoretically perfectly, and in practice to be accurate with a fair approximation. The analysis concerns a car having a rigid front axle.

1. The wheels are flat planes perpendicular to the road surface; the axis of the steering knuckle pin is also normal to the road surface

1.1. *Condition of pure rolling*

If the wheels of a car rolling on horizontal ground are replaced by circular planes perpendicular to the ground surface and the front wheels, as is normal, may be turned around vertical pivots, then the condition of pure rolling in cornering is that the extension of the axles of all of the wheels should intersect at one single point [1]. Then, using the symbols of Fig. 1 (in a nearside cornering)

$$\left.\begin{aligned} \tan\varphi_n &= \frac{a}{\varrho-b} \\ \tan\varphi_0 &= \frac{a}{\varrho+b} \end{aligned}\right\} \quad (|\varrho| \neq b).$$

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Here

a = wheel base,

2b = wheel track,

φ_n, φ_0 = angles at which the nearside and offside front wheels, respectively, are inclined to the longitudinal axis of the car,

ϱ = distance of the centre of curve C to the longitudinal axis of the car.

The condition of pure rolling is expressed by the relations

$$\cot\varphi_0 - \cot\varphi_n = \frac{2b}{a}, \quad \text{(in nearside cornering)}$$
$$\cot\varphi_n - \cot\varphi_0 = \frac{2b}{a}, \quad \text{(in offside cornering).} \qquad (1)$$

derived from the above equations.

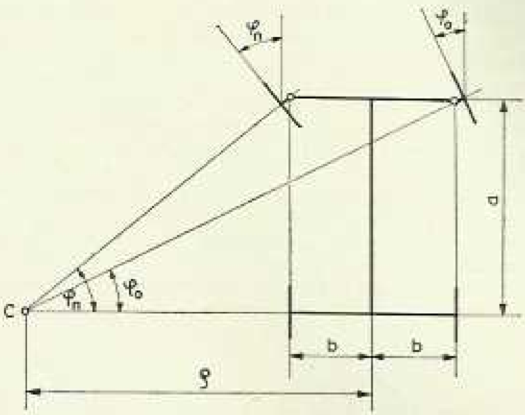

Fig. 1

The appropriate turning of the wheels is realized with a reasonable approximation by the usual steering trapezoid (Fig. 2). This mechanism assures with a very good approximation the realization of the conditions defined by relations (1) in the cornering of a large radius; the approximation is, in the case of cornerings of smaller radii more unfavourable, namely, the front wheels roll and skid and at the same time quicken the wear of the tyres, although this is not so bad because, in general, drivers travel more slowly in bends having small radii of curvature.

According to the arrangement represented in Fig. 2, the lengthening lines of the arms of length r of the steering linkage intersect at the midpoint of the rear axle in the case where the planes of wheels are parallel with the medium axis of the wheel track (i.e., when the car rolls along a tangent line), inclining to the longitudinal axis with an angle α, thus:

$$\tan\alpha = \frac{b}{a}; \quad \cos\alpha = \frac{a}{\sqrt{a^2+b^2}}, \quad \sin\alpha = \frac{b}{\sqrt{a^2+b^2}}. \qquad (2)$$

Between the geometrical parameters of the hinged four-member linkage consisting of a member of a length $2b$, the arms of the trapezoid and the spur rod of length l, the following relation exists:

$$r \sin \alpha = \frac{2b - l}{2};$$

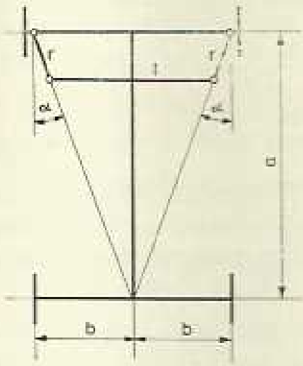

Fig. 2

By making use of the term in (2)

$$\frac{rb}{\sqrt{a^2 + b^2}} = \frac{2b - l}{2}. \tag{3}$$

It may be proved [2] that in the case of the steering trapezoid shown in Fig. 2, the relation between φ_0 and φ_n with the symbols

$$f = \frac{l^2 - 4b^2}{2lr}, \qquad g = \frac{2b}{l} \tag{4}$$

is as follows

$$\varphi_0 = \arctan \frac{l\sqrt{1 - [f + g \sin(\varphi_n + \alpha)]^2}}{r - l[f + g \sin(\varphi_n + \alpha)]} + \arctan \frac{r \cos(\varphi_n + \alpha)}{2b - r \sin(\varphi_n + \alpha)} + \alpha - 90^\circ. \tag{5}$$

By using formulas (1) and (5), the deviations in the locking angles of the wheels may be found if the geometrical parameters are given.

Consider the example of Fig. 3. Here

$$a = 2 \text{ m},$$
$$b = 0{,}5 \text{ m},$$
$$l = 0{,}9 \text{ m}.$$

Then, according to (3)

$$r = \frac{\sqrt{a^2 + b^2}}{b} \cdot \frac{2b - l}{2} = 0{,}206 \text{ m}$$

$$\alpha = \text{arc tan} \frac{b}{a} = 14°02'10'' ,$$

$$f = \frac{l^2 - 4b^2}{2\, lr} = -0{,}512 ,$$

$$g = \frac{2b}{l} = 1{,}111 ,$$

$$\alpha - 90° = -75°57'50'' .$$

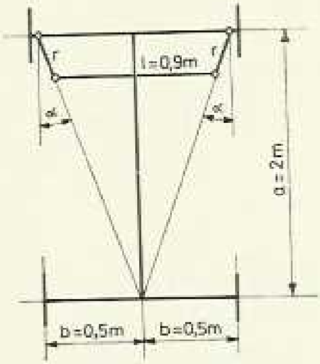

Fig. 3

Substituting these numerical data into Eq. (5), one obtains

$$\varphi_0 = \text{arc tan} \frac{0{,}9 \sqrt{1 - [1{,}111 \sin(\varphi_n + 14°02'10'') - 0{,}512]^2}}{0{,}206 - 0{,}9\,[1{,}111 \sin(\varphi_n + 14°02'10'') - 0{,}512]} +$$
$$+ \text{arc tan} \frac{0{,}206 \cos(\varphi_n + 14°02'10'')}{1 - 0{,}206 \sin(\varphi_n + 14°02'10'')} - 75°57'50'' .$$

The condition of the exact rolling with the given data will be

$$\varphi_0 = \text{arccot}\,(0{,}5 + \cot \varphi_n) .$$

On this basis, the correct value of φ_0 and the approximate value of φ_{0a} realized by the mechanism should be calculated for some values of the angle φ_n. The values of the radius of curvature ϱ associated with φ_n are also given:

$$\varrho = \frac{a}{\tan \varphi_n} + b ,$$

with the given numerical values

$$\varrho = \frac{2}{\tan \varphi_n} + 0{,}5 .$$

The results are listed in Table I.

Table I

φ_n	φ_0	φ_{id}	$\varphi_{id}-\varphi_0$	ϱ
0°	0°	0°	0°	∞
10°	9° 12′56″	9° 33′02″	0°20′06″	11,84 m
20°	17° 06′55″	17° 57′33″	0° 50′38″	5,99 m
30°	24° 08′21″	25° 40′17″	1° 31′56″	3,96 m
40°	30° 35′51″	31° 43′31″	1° 07′40″	2,88 m

1.2. *Geometrical arrangement of the steering gear assuring pure rolling*

In taking into account the conditions of pure rolling, relations should be established between the geometrical parameters defining the positions of the planes of the front wheels which would permit the construction of an ideal steering gear.

a) It seems to be practicable to follow the intersection line of the planes of the front wheels in cornering or — which from the viewpoint of the analysis is equivalent to this — the piercing point M on the ground plane of this intersection line.

In Fig. 4, the system of co-ordinates xy, fixed to the car, lies in the ground plane. The origin of the system of co-ordinates is the midpoint of the front axle of the car, the x axis being the centre line of the car directed to the driving direction.

Instead of the intersection point of the traces of the planes of wheels, the point of intersection M of the lines (e_n) and (e_0) intersecting the points $L(O; b)$ and $R(O; -b)$ will be considered at different wheel positions.

In nearside cornering the equation of the line (e_n) is:

$$y - b = x \tan \varphi_n ,$$

and the equation of the line (e_0):

$$y + b = x \tan \varphi_0 .$$

In the case of pure rolling:

$$\tan \varphi_n = \frac{a}{\varrho - b} , \quad \tan \varphi_0 = \frac{a}{\varrho + b} , \quad (|\varrho| \neq b) .$$

Substituting these terms into the equations of the lines, yields

$$(e_n): \quad y - b = \frac{a}{\varrho - b} x ,$$

$$(e_0): \quad y + b = \frac{a}{\varrho + b} x . \tag{6}$$

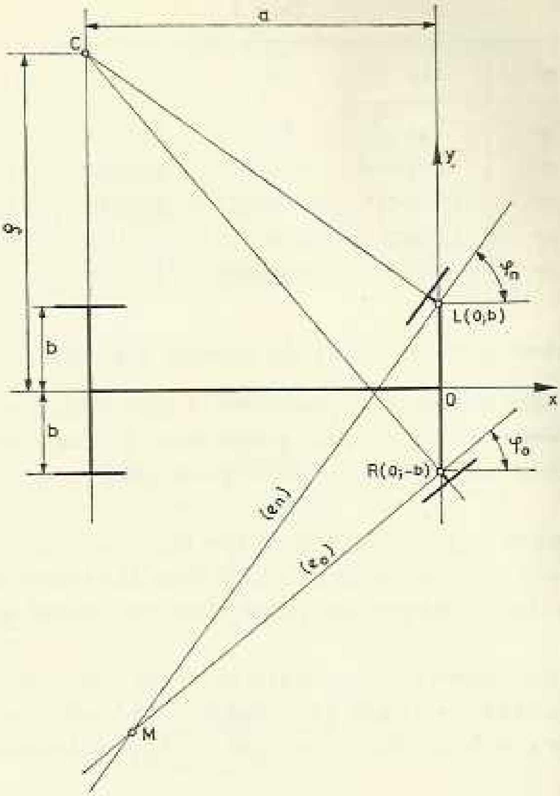

Fig. 4

After solving this set of equations the trace curve of the point M (by using the parameter ϱ) is, in nearside cornering

$$x_M = \frac{b^2 - \varrho^2}{a}, \qquad y_M = -\varrho \tag{7}$$

and in offside cornering

$$x_M = \frac{b^2 - \varrho^2}{a}, \qquad y_M = +\varrho . \tag{8}$$

As the radius of curvature varies, point M describes a second-order curve; by making use of equations (7) and (8) and by eliminating the parameter, the equation of this curve will be

$$y^2 = b^2 - ax. \tag{9}$$

This is the equation of a parabola; its diagram is shown in Fig. 5. The curve intersects the x axis at the value $x_0 = b^2/a$ and the y axis at $y_0 = \pm b$. From equations (7) and (8) it may be seen that the radius of curvature $\varrho_0 = 0$ is associated with the zero axis (which does not actually occur; with the increase of ϱ, the absolute value of y is also increasing, thus, in the case of $\varrho \to \infty$, point M tends to infinity along the parabola. Practically, the branches of the parabola starting only from the points corresponding to the minimum parameter

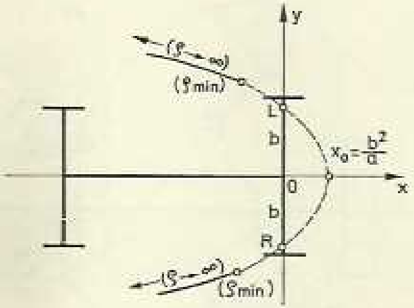

Fig. 5

ϱ_{min} depending on the structural characteristics of the car, may be taken into account (in Fig. 5, these branches of the parabola are represented by continuous lines). Consequently, the possibility that the accurate turning of the wheels could be controlled by moving, for example, the intersecting point of the arms mounted parallel to the wheel planes should be rejected (this would imply a number of structural difficulties, not to mentioned that, in this case, just the straight travel would be impossible because point M will be removed to infinity).

b) One of the feasible structural solutions (at least in principle) is as follows:

Let us consider the intersection point M of the lines (e_n) drawn through point L and parallel with the plane of the nearside front wheel and (e_0) drawn through the point R normal to the plane of the offside front wheel at different positions of the wheels.

According to Fig 6, the equation of the line (e_n) is in a nearside cornering

$$y - b = x \tan \varphi_n,$$

that of the line (e_0)

$$y + b = -x \cdot \frac{1}{\tan \varphi_0};$$

by making use of the above conditions of pure rolling

$$(e_n): \qquad y - b = \frac{a}{\varrho - b}\,x,$$
$$(e_\varrho): \qquad y + b = -\frac{\varrho + b}{a}\,x. \tag{10}$$

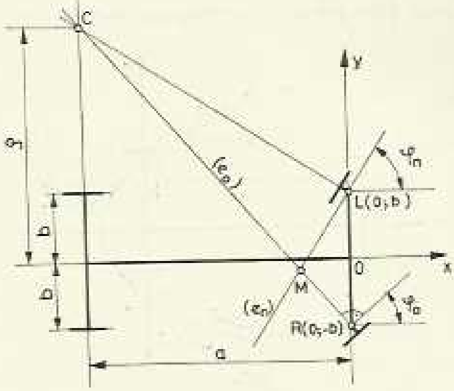

Fig. 6

Solving the set of equations (10) for the co-ordinates of point M

$$\left.\begin{aligned} x_M &= -\frac{2\,ab\,(\varrho - b)}{a^2 - b^2 + \varrho^2} \\ y_M &= b - \frac{2a^2\,b}{a^2 - b^2 + \varrho^2} \end{aligned}\right\} \quad (\varrho^2 \neq b^2 - a^2) \tag{11}$$

will be obtained. Elimination of the parameter ϱ from Eqs (11) yields the trace curve of point M

$$(a^2 - b^2)\,(y - b)^2 + (ax + by - b)^2 = 2a^2\,b(b - y). \tag{12}$$

It can be demonstrated that the trace of point M is the same as that above also in the case of offside cornering. After arranging, Eq. (12) takes the simplified form

$$ax^2 + 2bxy + ay^2 - 2b^2x - ab^2 = 0. \tag{13}$$

This equation should be reduced to the normal shape with the aid of the familiar method of transformation of the second-order curves. First, the system of co-ordinates xy should be rotated around the origin with an angle β, to obtain a system of co-ordinates $\xi\eta$, whose axes will be parallel with the principal axes of the curve (13). Substituting the familiar expressions

$$x = \xi \cos \beta - \eta \sin \beta,$$
$$y = \xi \sin \beta + \eta \cos \beta$$

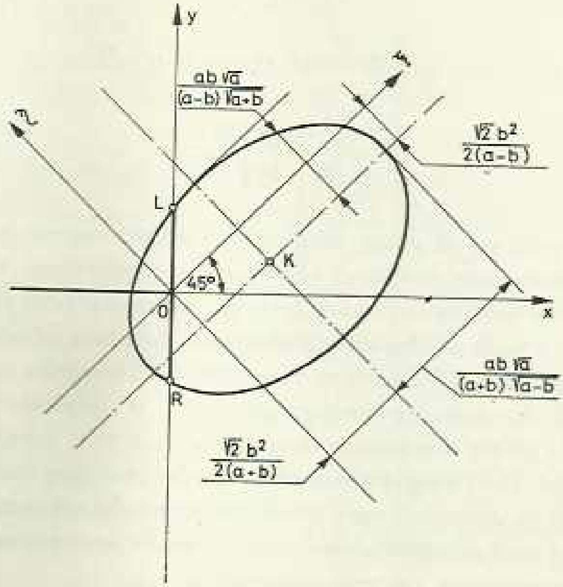

Fig. 7

into Eq. (13), arranging and setting the coefficient of the term ξ, η equal to zero, $\beta = 45°$ will be obtained. In the system of co-ordinates rotated by 45°, the equation of the curve will be

$$(a + b)\xi^2 + (a - b)\eta^2 - \sqrt{2}\, b^2\xi + \sqrt{2}\, b^2\eta - ab^2 = 0. \tag{14}$$

After rearrangement, the equation of the trace curve of point M will be

$$\frac{\left[\xi - \frac{\sqrt{2}\, b^2}{2(a+b)}\right]^2}{\frac{a^3 b^2}{(a-b)(a+b)^2}} + \frac{\left[\eta + \frac{\sqrt{2}\, b^2}{2(a-b)}\right]^2}{\frac{a^3 b^2}{(a+b)(a-b)^2}} = 1 . \tag{15}$$

Thus, the trace of the movement of point M is an ellipse, the principal axes of which are rotated by 45° in comparison to the system of co-ordinates xy; in the new system of co-ordinates $\xi\eta$, the centre of the ellipse, is the point

$$K\left[\frac{\sqrt{2}\, b^2}{2(a+b)};\ -\frac{\sqrt{2}\, b^2}{2(a-b)}\right]$$

(Fig. 7).

The length of the semi-axis parallel with the axis ξ is

$$A = \frac{ab\sqrt{a}}{(a+b)\sqrt{a-b}},$$

and that of the semi-axis parallel with the axis η will be

$$B = \frac{ab\sqrt{a}}{(a-b)\sqrt{a+b}}.$$

Although guiding of point M along an elliptic curve might be realized through the appropriate design of the steering mechanism, however, this requires straight-line motion slide bars, besides, in the course of steering, the point M should slide on both of the arms parallel to the plane of one wheel and perpendicular to the plane of the other, respectively. The great number of the slip joints built into the steering mechanism make it intricate and presumably would result in a larger inaccuracy in steering than that occurring in the usual steering trapezoid; the straight line motion guides realizing the movement along an ellipse should be arranged very carefully in a very accurate position, parallel to the axles ξ and η. All of these circumstances are against the building up of a mechanism realizing an accurate rolling.

1.3. *Approximate determination of the proportions of the steering trapezoid*

A steering trapezoid having entirely arbitrary dimensions — apart from the turning angles of the wheels $\varphi_n = \varphi_0 = 0$ — can approach the condition (1) of pure rolling only with a very significant error.

The problem is to determine the proportions of the steering mechanism, with the reservation that also in the case of the practicably possible turning angle of the wheels, the condition (1) should, as far as possible, be perfectly satisfied.

Assuming that the arms of the steering trapezoid (in the position represented in dashed line in Fig. 8), compared to the basic position, have been turned away by angles φ_n and φ_0 corresponding to the skid-free rolling. Let now φ_n and φ_0 be the wheel rotations associated with the cornering of minimum radius of curvature, and the value of 2b given.

The length l of the wheel-track bar is constant also during the movement of the mechanism, thus, the length l in the turned position must equal the length readable from the mid-position

$$l = 2b - 2\,r \sin\alpha = 2(b - r\sin\alpha). \tag{16}$$

According to Fig. 8, the co-ordinates of the point L_1 of the left arm of the steering trapezoid are

$$x_{L_1} = -r\cos(\varphi_n + \alpha),$$
$$y_{L_1} = b - r\sin(\varphi_n + \alpha). \tag{17}$$

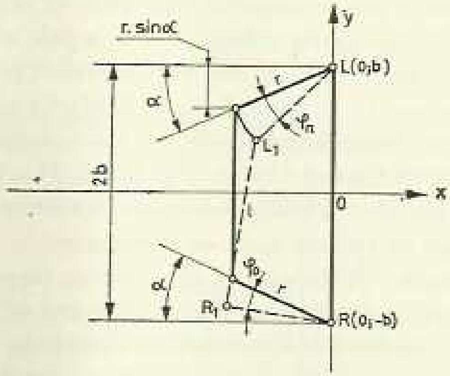

Fig. 8

Those of the point R_1 of the right arm:

$$x_{R_1} = -r\cos(\alpha - \varphi_0),$$
$$y_{R_1} = -b + r\sin(\alpha - \varphi_0) \tag{18}$$

The distance between the two points L_1 and R_1 is

$$l = \sqrt{(x_{L_1} - x_{R_1})^2 + (y_{L_1} - y_{R_1})^2}\,;$$

and by replacing the values (17) and (18) of the co-ordinates one obtains

$$l^2 = [-r\cos(\varphi_n + \alpha) + r\cos(\alpha - \varphi_0)]^2 +$$
$$+ [b - r\sin(\varphi_n + \alpha) + b - r\sin(\alpha - \varphi_0)]^2.$$

Accordingly, the constancy of the length l may be expressed by the equation

$$\sqrt{b^2 - br[\sin(\alpha - \varphi_0) + \sin(\alpha + \varphi_n)] + \frac{1}{2}r^2[1 - \cos(2\alpha + \varphi_n - \varphi_0)]} = b - r\sin\alpha,$$

remembering that

$$\cot\varphi_0 - \cot\varphi_n = \frac{2b}{a}\,. \tag{19}$$

In order to realize a skid-free cornering, the equality (19) must be valid in respect to each pair of angles φ_n, φ_0, between which the relation (1) exists, in the case of certain length values. The deviation from condition (1) occurs in the case of the maximum values of angles φ_n and φ_0 associated with the minimum radius of curvature; one should be satisfied by adjusting the steering trapezoid to an accurate performance at this pair of angles.

Accordingly, with the given values of a, b, a pair of angles φ_n, φ_0 should be assigned which satisfies Eq. (1) These were replaced into Eq. (19). Then, in Eq. (19), either the value of r or that of α may freely be chosen. The solution of the equation is simpler, from the viewpoint of the calculation technique if one adopts the value of α and that of r to be calculated. This being done, the value of l also is yielded by Eq. (16). Then, the angular deviation of steering should be checked for corners of greater radii of curvature.

(Usually the length r of the arm of the steering trapezoid is assigned and, on this basis, the angle α determined by making use of Causant's method of construction [1]. This method of construction consisting of a number of steps which meanwhile also requires the measurement of a length defined by calculation, gives the angle α as a final result. In measuring the angle α, inaccuracy may occur; from the angle α the measured length l of the wheel-track arm may be obtained by a further calculation; here again another error may be committed). In the procedure suggested above, every step is made by calculation — with the desired accuracy — and the value of α does not even occur in the procedure of the calculation.

Example

Let

$$a = 2 \text{ m}, \; b = 0{,}5 \text{ m},$$

be and in the nearside cornering the minimum radius of curvature

$$\varphi_n = 45° \text{ (Fig. 9)}.$$

If one adopts the value of the angle α in such a way that the lengthenings of the arms of the steering trapezoid intersect the rear axle in the midpoint, then

$$\tan\alpha = \frac{b}{a} = \frac{1}{4},$$

$$\tan\varphi_n = \tan 45° = 1,$$

from

$$\frac{a}{\tan\varphi_0} - \frac{a}{\tan\varphi_n} = 2b:$$

$$\tan\varphi_0 = \frac{2}{3}.$$

The trigonometric functions in Eq. (19) may be expressed as tangent functions:

$$\sin\alpha = \frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}} = \frac{1}{\sqrt{17}},$$

$$\sin(\alpha-\varphi_0)=\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}\cdot\frac{1}{\sqrt{1+\tan^2\varphi_0}}-\frac{1}{\sqrt{1+\tan^2\alpha}}\cdot\frac{\tan\varphi_0}{\sqrt{1+\tan^2\varphi_0}}=-\frac{5}{\sqrt{221}},$$

$$\sin(\alpha+\varphi_n)=\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}\cdot\frac{1}{\sqrt{1+\tan^2\varphi_n}}+\frac{1}{\sqrt{1+\tan^2\alpha}}\cdot\frac{\tan\varphi_n}{\sqrt{1+\tan^2\varphi_n}}=\frac{5}{\sqrt{34}},$$

$$\cos(2\alpha+\varphi_n-\varphi_0)=\cos[2\alpha+(\varphi_n-\varphi_0)]=$$

$$=\frac{1}{1+\tan^2\alpha}\cdot\frac{1}{\sqrt{1+\tan^2\varphi_n}\sqrt{1+\tan^2\varphi_0}}[(1-\tan^2\alpha)(1+\tan\varphi_n\cdot\tan\varphi_0)-$$

$$-2\tan\alpha(\tan\varphi_n-\tan\varphi_0)]=\frac{67}{17\sqrt{26}}.$$

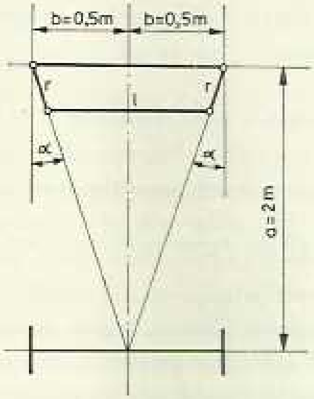

Fig. 9

Replacement of these values into Eq. (19) and arrangement of the equation yields:

$$\left(\frac{1}{2}\right)^2-\frac{1}{2}r\left[-\frac{5}{\sqrt{221}}+\frac{5}{\sqrt{34}}\right]+\frac{1}{2}r^2\left[1-\frac{67}{17\sqrt{26}}\right]=\left(\frac{1}{2}-\frac{1}{\sqrt{17}}r\right).$$

From this, it is found that

$$r=0{,}326 \text{ m},$$

further

$$l=2b-2r\sin\alpha=0{,}842\text{ m}.$$

With these values the mean error of the steering angle may be calculated from Eq. (5) by way of proving.

Accordingly, the numerical data are as follows:

$$a=2\text{ m},\ b=0{,}5\text{ m},\quad r=0{,}326\text{ m},\ l=0{,}842\text{ m},$$

$$\tan\alpha=\frac{1}{4},\qquad \sin\alpha=\frac{\sqrt{17}}{17},\qquad \cos\alpha=\frac{4\sqrt{17}}{17},$$

$$\alpha=14°02'10'',\qquad \alpha-90°=-75°57'50''.$$

$$f=\frac{l^2-4b^2}{2lr}=-0{,}542,$$

$$g=\frac{2b}{l}=1{,}188.$$

With these values the exact value of the angles φ_0 have been calculated with the aid of Eq. (1), to the assigned values of φ_n, and the effective angles φ_{0e}, by making use of Eq. (5), formed by the steering trapezoid constructed with the above dimensions and angles.

Table II

φ_n	φ_0	φ_{0e}	$\varphi_{0e}-\varphi_0$	ϱ
0°	0°	0°	0°	∞
10°	9° 12′56″	8° 56′44″	—0° 16′07″	11,84 m
20°	17° 06′55″	16° 45′49″	—0° 21′06″	5,99 m
30°	24° 08′21″	24° 44′19″	+0° 35′58″	3,96 m
40°	30° 35′51″	30° 41′14″	+0° 05′23″	2,88 m
45°	34° 39′36″	34° 39′36″	0°	2,50 m

The distribution of the errors is more favourable than that in Table I.

2. A more exact investigation of rolling

2.1. *Approximation of the tyres with circular ring surfaces*

In cornering, the front wheels of a vehicle do not roll on theoretically desirable curves. As an approximation, let us assume the tyres to be circular rings coming into contact with the ground surface. Further, let us take into account the actual position of the planes of wheels in the medium position as well as the obliquity of the steering knuckle pin.

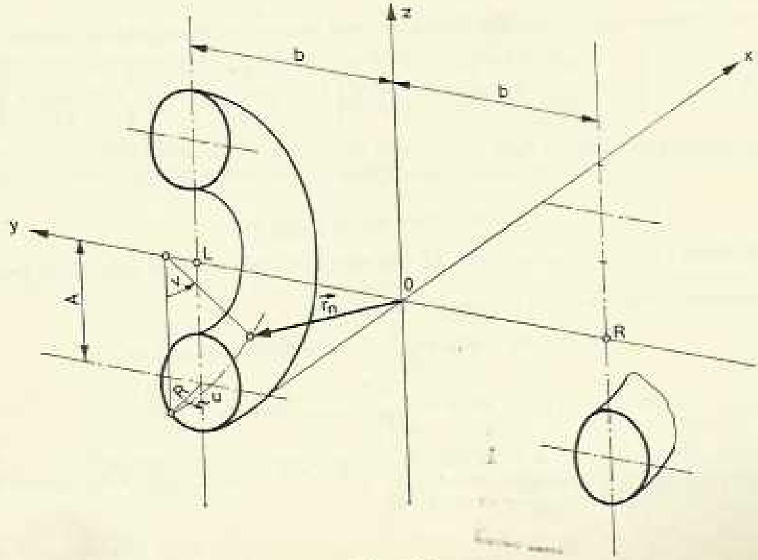

Fig. 10

Let us complete the plane orthogonal co-ordinate system with a z axis (Fig. 10). The tyres should be considered with a reasonable approximation to be circular ring surfaces coming into existence by rotation of circles in the plane yz, with a radius R and of centres $(O; +b; -A)$, and $(O; -b; -A)$, about the y axis.

Accordingly, the position vector describing the surface of the nearside front wheel (Fig. 10) will be

$$\begin{aligned} \vec{r} = \vec{b} + \vec{K} = \{o;\ b;\ o\} + \\ + \{(A + R\cos u)\sin v;\ R\sin u;\ -(A + R\cos u)\cos v\}, \end{aligned} \tag{20}$$

and that describing the surface of the offside front wheel

$$\begin{aligned} \vec{r}_0 = -\vec{b} + \vec{K} = \{o;\ -b;\ o\} + \\ + \{(A+R\cos u)\sin v;\ R\sin u;\ -(A+R\cos u)\cos v\}. \end{aligned} \tag{21}$$

The medium plane being initially normal to the y axis of the circular ring surfaces, should be tilted according to the angle of inclination δ of the front wheels (Fig. 11). The vector $\vec{K}$ included in Eqs (20), (21) of the surfaces, should be rotated — by displacing its origin from the system of coordinates — with an angle δ at the nearside wheel and with $-\delta$ at the offside one about the x axis (the sign being assigned to the angle of inclination δ is established by looking in the direction of the positive x axis). According to Fig. 11 — if the x axis is directed inwards, normal to the plane of the figure — in the case of rotation with a positive δ angle, the rotated axis unit-vectors are

$$\begin{aligned} \vec{e}_1 &= \{1;\ 0;\ 0\} = \vec{i}, \\ \vec{e}_2 &= \{0;\ \cos\delta;\ -\sin\delta\}, \\ \vec{e}_3 &= \{0;\ \sin\delta;\ \cos\delta\}. \end{aligned}$$

Accordingly, the matrix rotating about the x axis

$$F_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{bmatrix}. \tag{22}$$

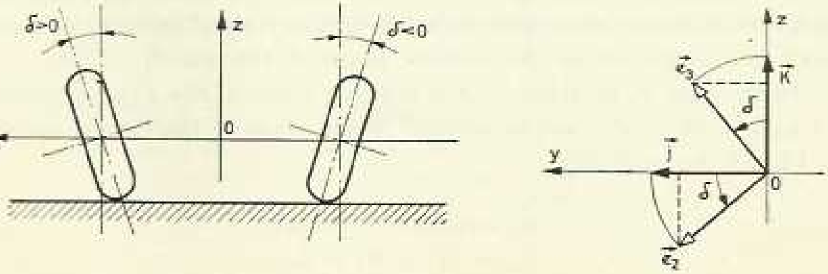

Fig. 11

The angle of convergence of the front wheels may be given in such a way as to rotate the vector $\vec{K}$, already rotated about the x axis, about the z axis — by displacing its starting point into the origin of the system of co-ordinates — in the case of the nearside wheel with an angle $-\varepsilon$ and in the case of an offside wheel, with an angle ε (the sign of ε being established by setting oneself opposite to the direction of the positive z axis). According to Fig. 12, if the z axis is

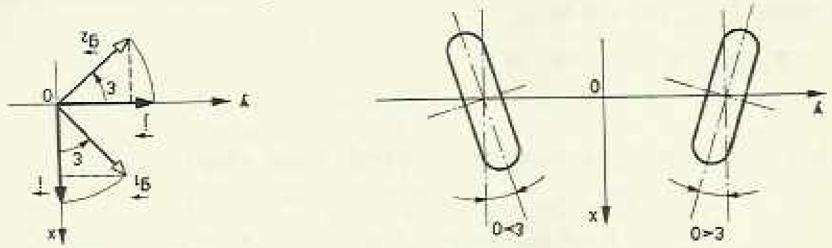

Fig. 12

directed outwards, normal to the plane of the figure, in the case of rotation by an angle $+\varepsilon$, the axis unit-vectors rotated are

$$\begin{aligned} \vec{g}_1 &= \{\cos\varepsilon;\ \sin\varepsilon;\ 0,\} \\ \vec{g}_2 &= \{-\sin\varepsilon;\ \cos\varepsilon;\ 0,\} \\ \vec{g}_3 &= \{0;\ 0;\ 1\} = \vec{k}. \end{aligned}$$

Accordingly, the matrix rotating about the z axis, is as follows

$$F_z = \begin{bmatrix} \cos\varepsilon & -\sin\varepsilon & 0 \\ \sin\varepsilon & \cos\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

When steering, the wheels are turned around the steering knuckle pins with angles φ_n and φ_0, respectively. The steering knuckle pin is not parallel with the z axis but is inclined at an angle $\varkappa$ to it in a plane parallel with the plane xy. (Here, the "side spread" of the steering knuckle pins will be disregarded which, in certain types of cars is actually equal to zero, if the steering knuckle pin is placed in the median plane of the wheel).

In the case of rotation with a positive angle $\varkappa$, the axis unity-vectors if the y axis is directed inwards, normal to the plane of the figure, according to Fig. 13 will be as follows:

$$\begin{aligned} \vec{h}_1 &= \{\cos\varkappa;\ 0;\ \sin\varkappa\}, \\ \vec{h}_2 &= \{0;\ 1;\ 0\} = \vec{j}, \\ \vec{h}_3 &= \{-\sin\varkappa;\ 0;\ \cos\varkappa\}. \end{aligned} \tag{23a}$$

Accordingly, the matrix of rotation is

$$G_y = \begin{bmatrix} \cos \varkappa & 0 & -\sin \varkappa \\ 0 & 1 & 0 \\ \sin \varkappa & 0 & \cos \varkappa \end{bmatrix}. \tag{24}$$

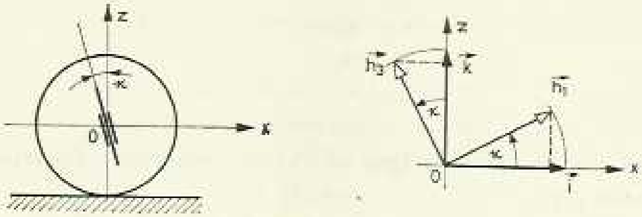

Fig. 13

Similarly, the matrix of the rotation by an angle φ about the z axis:

$$G_z = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{25}$$

The equation of the wheel surfaces running in cornering

$$\begin{aligned} \vec{R}_n &= \vec{b} + G_y G_z F_z F_x \vec{K}\,, \qquad \text{(nearside wheel)} \\ \vec{R}_0 &= \vec{b} + G_y G_z F_z F_x \vec{K}\,, \qquad \text{(offside wheel).} \end{aligned} \tag{26}$$

In order to shorten the calculations to be carried out, the product of the four matrices being before the vector $\vec{K}$ should be formed:

$$M = G_y G_z F_z F_x =$$

$$= \begin{bmatrix} \cos \varkappa & 0 & -\sin \varkappa \\ 0 & 1 & 0 \\ \sin \varkappa & 0 & \cos \varkappa \end{bmatrix} \begin{bmatrix} \cos\varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \varepsilon & -\sin \varepsilon & 0 \\ \sin \varepsilon & \cos \varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & \sin \delta \\ 0 & -\sin \delta & \cos \delta \end{bmatrix}. \tag{26a}$$

Carrying out the multiplication and certain useful trigonometric transformations, we have:

$$M = \begin{bmatrix} \cos \varkappa \cos (\varphi + \varepsilon) & -\cos \varkappa \cos \delta \sin (\varphi + \varepsilon) + \sin \varkappa \sin \delta \\ \sin (\varphi + \varepsilon) & \cos \delta \cos (\varphi + \varepsilon) \\ \sin \varkappa \cos (\varphi + \varepsilon) & -\sin \varkappa \cos \delta \sin (\varphi + \varepsilon) - \cos \varkappa \sin \delta \end{bmatrix} \times$$

$$\times \begin{bmatrix} -\cos \varkappa \sin \delta \sin (\varphi + \varepsilon) - \sin \varkappa \cos \delta \\ \sin \delta \cos (\varphi + \varepsilon) \\ -\sin \varkappa \sin \delta \sin (\varphi + \varepsilon) + \cos \varkappa \cos \delta \end{bmatrix}.$$

In nearside cornering, in respect to the nearside wheel

$$\varphi = \varphi_n > 0; \quad \varepsilon < 0, \quad \delta > 0, \tag{27a}$$

and for the case of the offside wheel:

$$\varphi = \varphi_0 > 0, \quad \varepsilon > 0, \quad \delta < 0 \tag{27b}$$

in the product matrix M the signs of the trigonometric functions should be assigned accordingly.

Determine now the contact points of the front wheels assumed to be circular rings and the ground, in the case of turning angles φ_n and φ_0 of the wheels. Due to the different turning angles of the wheel planes, the horizontal tangent planes of the front wheels in the given system of co-ordinates do not coincide with the initial plane $z = -(A + R)$, but these tangent planes will slide somewhat upwards, owing to which, the car will be slightly tilted as compared to its normal position, however, this tilting will be disregarded in the following. In order to determine the contact points of the surfaces of tyre surfaces and ground, the normal vector

$$\vec{n} = (M\vec{K})'_u \times (M\vec{K})'_v \tag{27c}$$

of the tyre surfaces

$$\vec{R} = \pm\vec{b} + M\vec{K} \tag{27d}$$

should be calculated, taking into account that $\vec{b}$ is a constant vector.

Since the matrix M does not involve the variables u, v, therefore, the normal vector might also be calculated in the form

$$\vec{n} = M(\vec{K}'_u \times \vec{K}'_v) . \tag{27e}$$

$$\vec{K}'_u \times \vec{K}'_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -R \sin u \sin v & R \cos u & R \sin u \cos v \\ (A + R \cos u)\cos v & 0 & (A + R \cos u)\sin v \end{vmatrix} = \tag{28}$$

$$= R(A + R \cos u)\{\cos u \sin v \sin u; \ -\cos u \cos v\}.$$

The components in directions x and y of the normal vector

$$\vec{n} = M(\vec{K}'_u \times \vec{K}'_v) = \{n_x; n_y; n_z\} \tag{28a}$$

associated with the horizontal tangent planes are equal to zero

$$n_x = 0, \ n_y = 0 . \tag{29}$$

In the multiplication

$$\vec{n} = M\begin{bmatrix} R(A + R\cos u)\cos u \sin v \\ R(A + R\cos u)\sin u \\ -R(A + R\cos u)\cos u \cos v \end{bmatrix} \tag{29a}$$

every component of the resultant vector includes the factor $R(A + R\cos u)$, which may be factored out. This having been done, the set of Eqs (29) will take the form

$$\begin{aligned} R(A + R\cos u)(\ldots) &= 0, \\ R(A + R\cos u)(\ldots) &= 0. \end{aligned} \tag{29b}$$

From

$$A + R\cos u = 0,$$

$$\cos u = -\frac{A}{R}$$

would be obtained, the absolute value of which, considering that the wheel dimensions, are greater than 1, thus, from the viewpoint of the solution of the problem $A + R\cos u \neq 0$, it is sufficient to carry out the multiplication

$$M\begin{bmatrix} \cos u \ \sin v \\ \sin u \\ -\cos u \ \cos v \end{bmatrix} \tag{29d}$$

and to set the first two components of the resultant vector equal to zero. After performing these operations, the following set of equations will be obtained:

$$\begin{aligned} &[\cos\varkappa \cos(\varphi + \varepsilon)]\cos u \sin v + [-\cos\varkappa \cos\delta \sin(\varphi + \varepsilon) + \sin\varkappa \sin\delta]\sin u + \\ &+ [-\cos\varkappa \sin\delta \sin(\varphi + \varepsilon) - \sin\varkappa \cos\delta](-\cos u \cos v) = 0, \\ &[\sin(\varphi + \varepsilon)]\cos u \sin v + [\cos\delta \cos(\varphi + \varepsilon)]\sin u + \\ &+ [\sin\delta \cos(\varphi + \varepsilon)](-\cos u \cos v) = 0. \end{aligned} \tag{30}$$

Replacing from the values of the parameters u_0, v_0 found from the set of equations (30) those associated with $z < O$ into Eq. (26), the points M_n and M_0 may be obtained at which the nearside and offside wheels, respectively, come into contact with the ground.

Also the radii of the effective curves of rolling of the two front wheels should be defined. Let the $\vec{j}$ and $-\vec{j}$ be axis unit-vectors, the normal vectors of the medium plane of the front wheels having vertical medium planes. Taking into account the turns applied to the wheels in cornering, the effective normal vectors of the medium planes of the wheels will be the vectors

$$\begin{aligned} \vec{N}_n &= M\vec{j}, \quad \text{(at the nearside wheel)} \\ \vec{N}_0 &= M(-\vec{j}), \quad \text{(at the offside wheel).} \end{aligned} \tag{30a}$$

If the column vectors of matrix M are designated with the symbols $\vec{m}_1$, $\vec{m}_2$, $\vec{m}_3$, then

$$\vec{N}_n = [\vec{m}_1\ \vec{m}_2\ \vec{m}_3] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \vec{m}_2,$$

$$\vec{N}_0 = [\vec{m}_1\ \vec{m}_2\ \vec{m}_3] \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = -\vec{m}_2. \tag{30b}$$

By making use of matrix M given in (27), on the basis of the above results

$$\vec{N}_n = \{-\cos\varkappa\cos\delta\sin(\varphi+\varepsilon) + \sin\varkappa\sin\delta;\ \cos\delta\cos(\varphi+\varepsilon);\ -\sin\varkappa\cos\delta\sin(\varphi+\varepsilon) - \cos\varkappa\sin\delta\}, \quad \left.\begin{matrix}\varphi=\varphi_n\\ \varepsilon<0\\ \delta>0\end{matrix}\right] \tag{31a}$$

and

$$\vec{N}_0 = \{\cos\varkappa\cos\delta\sin(\varphi+\varepsilon) - \sin\varkappa\sin\delta;\ -\cos\delta\cos(\varphi+\varepsilon);\ \sin\varkappa\cos\delta\sin(\varphi+\varepsilon) + \cos\varkappa\sin\delta\}, \quad \left.\begin{matrix}\varphi=\varphi_0\\ \varepsilon>0\\ \delta<0\end{matrix}\right] \tag{31b}$$

Hereafter, the piercing points C_n and C_0 of the normal lines

$$\vec{r}_{nn} = \vec{b} + \lambda\vec{N}_n,\quad \vec{r}_{n0} = -\vec{b} + \lambda\vec{N}_0 \tag{31c}$$

and the associated contact planes of the wheels should be determined; the segments $\overline{M_nC_n} = \varrho_n'$, $\overline{M_0C_0} = \varrho_0'$ are the effective rolling radii of the front wheels.

Example

Let us use the data of the first example, then we have

$$\varphi_n = 45^\circ,\quad \varphi_0 = 34^\circ\,39'36'' \approx 34^\circ\,40',\quad \text{further be } A = 20 \text{ cm},\quad R = 7 \text{ cm},$$
$$\delta = 1^\circ,\quad \varepsilon = 2^\circ,\quad \varkappa = 3^\circ.$$

For the nearside wheel $\varphi_n > 0,\quad \varepsilon < 0,\quad \delta > 0,$

$$\sin(\varphi_n+\varepsilon) = \sin(45^\circ - 2^\circ) = \sin 43^\circ = 0{,}68200,$$
$$\cos(\varphi_n+\varepsilon) = \cos(45^\circ - 2^\circ) = \cos 43^\circ = 0{,}73135,$$
$$\sin\delta = \sin 1^\circ = 0{,}01745,\quad \cos\delta = \cos 1^\circ = 0{,}99985,$$
$$\sin\varkappa = \sin 3^\circ = 0{,}05234,\quad \cos\varkappa = \cos 3^\circ = 0{,}99863.$$

To the first equation of (30):

$$\cos\varkappa\cdot\cos(\varphi_n+\varepsilon) = 0{,}99863\cdot 0{,}73135 = 0{,}73035,$$
$$\cos\varkappa\cdot\cos\delta\cdot\sin(\varphi_n+\varepsilon) = 0{,}99863\cdot 0{,}99985\cdot 0{,}68200 = 0{,}68096,$$
$$\sin\varkappa\cdot\sin\delta = 0{,}05234\cdot 0{,}01745 = 0{,}00091,$$
$$\cos\varkappa\cdot\sin\delta\cdot\sin(\varphi_n+\varepsilon) = 0{,}99863\cdot 0{,}01745\cdot 0{,}68200 = 0{,}01188,$$
$$\sin\varkappa\cdot\cos\delta = 0{,}05234\cdot 0{,}99985 = 0{,}05233.$$

To the second equation of (30):

$$\cos\delta \cdot \cos(\varphi_n + \varepsilon) = 0{,}99985 \cdot 0{,}73135 = 0{,}73124\,,$$
$$\sin\delta \cdot \cos(\varphi_n + \varepsilon) = 0{,}01745 \cdot 0{,}73135 = 0{,}01276\,.$$

With these values Eq. (30) will be as follows:

$$0{,}73035 \cos u \cdot \sin v - 0{,}68005 \sin u + 0{,}06422 \cos u \cdot \cos v = 0\,,$$
$$0{,}68200 \cos u \cdot \sin v + 0{,}73124 \sin u - 0{,}01276 \cos u \cdot \sin v = 0\,.$$

The solution of this set of equations is:

$$u_0 = 3°03'\,, \quad v_0 \approx 0°\,.$$

After substituting these values, the vector $\vec{K}$ associated with the horizontal tangent-plane of the nearside front wheel will be:

$$\vec{K}_n = \{(A + R\cos u_0)\sin v_0: \quad R\sin u_0; \quad -(A + R\cos u_0)\cos v_0\} = $$
$$= \{0;\ 0{,}0037;\ -0{,}2699\}\,.$$

The M matrix according to (27) with the numerical elements:

$$M = \begin{bmatrix} 0{,}73035 & -0{,}68005 & -0{,}06422 \\ 0{,}68200 & 0{,}73124 & 0{,}01276 \\ 0{,}03828 & -0{,}05312 & 0{,}99786 \end{bmatrix}.$$

The position vector associated with the point of contact of the nearside wheel and ground

$$\vec{R}_n = \vec{b} + M\vec{K}\,(u_0;\ v_0) =$$
$$= \begin{bmatrix} 0 \\ 0{,}5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0{,}73035 & -0{,}68005 & -0{,}06422 \\ 0{,}68200 & 0{,}73124 & 0{,}01276 \\ 0{,}03828 & -0{,}05312 & 0{,}99786 \end{bmatrix} \begin{bmatrix} 0 \\ 0{,}0037 \\ -0{,}2699 \end{bmatrix} =$$
$$= \{0{,}01481; \quad 0{,}49982; \quad -0{,}26951\}\,.$$

Thus, the point of contact of the nearside front wheel and ground

$$M_n\,(0{,}01481; \quad 0{,}49982; \quad -0{,}26951)\,.$$

According to (31a), the normal vector of the plane of the nearside front wheel turned by an angle φ_n is

$$\vec{N}_n = \vec{m}_2 = \{-0{,}68005; \quad 0{,}73124; \quad -0{,}05312\}\,.$$

Equation of the normal of the plane of the nearside wheel

$$\vec{r}_{nn} = \vec{b} + \lambda\,\vec{N}_n = \{0;\ 0{,}5;\ 0\} + \lambda\{-0{,}68005; \quad 0{,}73124; \quad -0{,}05312\} =$$
$$= \{-0{,}68005\,\lambda;\ 0{,}5 + 0{,}73124\,\lambda; \quad -0{,}05312\,\lambda\}\,.$$

With this normal the tangent-plane characterized by the equation

$$z = -0{,}26951$$

of the nearside front wheel should be pierced through.
From

$$-0{,}05312\,\lambda = -0{,}26951$$
$$\lambda = 5{,}074\,.$$

Substituting this value into the components of the vector $\vec{r}_{nn}$, the piercing point (vertex of the rolling cone) is:

$$C_n\,(-3{,}45057; \quad 4{,}21031; \quad -0{,}26951)\,.$$

The radius of rolling of the nearside front wheel

$$\varrho'_n = \overline{M_n C_n} = \sqrt{(x_{M_n} - x_{C_n})^2 + (y_{M_n} - y_{C_n})^2} \approx 5{,}06 \text{ m} .$$

The theoretically exact value being

$$\varrho_n = \sqrt{2^2 + 2^2} \approx 2{,}82 \text{ m}.$$

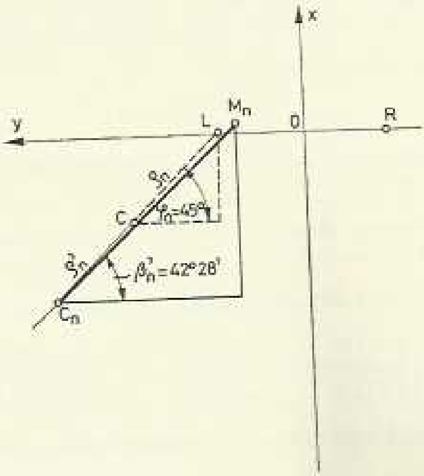

Fig. 14

The value of the angle β'_n to which the actual radius of rolling is inclined in the direction of the y axis, is as follows:

$$\tan \beta'_n = \left| \frac{x_{M_n} - x_{C_n}}{y_{C_n} - y_{M_n}} \right| = 0{,}92596 ;$$

from this:

$$\beta'_n \approx 42°48' ,$$

the theoretically exact value being

$$\varphi_n = \beta_n = 45° .$$

For the offside wheel:

$$\varphi_0 = 34°40'$$

$$\delta = -1°, \quad \varepsilon = 2°, \quad \varkappa = 3° ,$$

$$\sin \delta = -0{,}01745, \qquad \cos \delta = 0{,}99985$$

$\sin \varkappa$ and $\cos \varkappa$, as given above, further

$$\sin (\varphi_0 + \varepsilon) = \sin 36°40' = 0{,}59716 ,$$

$$\cos (\varphi_0 + \varepsilon) = \cos 36°40' = 0{,}80212 .$$

With these numerical data matrix (27) will be:

$$M = \begin{bmatrix} 0{,}80102 & -0{,}59716 & -0{,}04192 \\ 0{,}59716 & 0{,}80200 & -0{,}01400 \\ 0{,}04198 & -0{,}01382 & 0{,}99903 \end{bmatrix} .$$

And the set of equations (30) will have the form:

$$0{,}80102 \cos u \cdot \sin v - 0{,}59716 \sin u + 0{,}04192 \cos u \cdot \cos v = 0 ,$$

$$0{,}59716 \cos u \cdot \sin v + 0{,}80200 \sin u \cdot + 0{,}01400 \cos u \cdot \cos v = 0 .$$

Solutions of this set of equations are

$$u_0 = -0°48', \quad v_0 \approx 0° .$$

With the values obtained

$$\vec{K}_0 = \vec{K}(u_0;\ v_0) =$$
$$= \{ (A + R \cos u_0) \sin v_0;\quad R \sin u_0;\quad -(A + R \cos u_0) \cos v_0 \} =$$
$$= \{0;\ -0{,}00098;\quad -0{,}26999\} .$$

The position vector associated with the point of contact of the offside front wheel and ground:

$$\vec{R}_0 = -\vec{b} + M\vec{K}(u_0;\ v_0) =$$

$$= \begin{bmatrix} 0 \\ -0{,}5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0{,}80102 & -0{,}59716 & -0{,}04192 \\ 0{,}59716 & 0{,}80200 & -0{,}01400 \\ 0{,}04198 & -0{,}01382 & 0{,}99903 \end{bmatrix} \begin{bmatrix} 0 \\ -0{,}00098 \\ -0{,}26999 \end{bmatrix} =$$

$$= \{ 0{,}01191;\quad -0{,}50408;\quad -0{,}26972 \} .$$

The point of contact of the offside front wheel and ground will be

$$M_0(0{,}01191;\quad -0{,}50408;\quad -0{,}26972) .$$

By comparing the z co-ordinates of M_n and M_0, it is to be seen that the deviation is not noticeable, as has been stated above.

According to (31b), the normal vector of the offside front wheel turned away by an angle φ_0, is

$$\vec{N}_0 = -\vec{m}_2 = \{ 0{,}59716;\quad -0{,}80200;\quad 0{,}01382 \} .$$

The equation of the normal of the offside wheel-plane:

$$\vec{r}_{n0} = -\vec{b} + \lambda \vec{N}_0 = \{ 0;\ -0{,}5;\ 0 \} + \lambda \{ 0{,}59716;\ -0{,}80200;\ 0{,}01382 \} =$$
$$= \{ 0{,}59716\ \lambda;\quad -0{,}5-0{,}80200\ \lambda;\ 0{,}01382\ \lambda \} .$$

With this line the tangent-plane characterized by the equation

$$z = -0{,}26792$$

should be pierced through.
From

$$0{,}01382\ \lambda = -0{,}26972 ,$$
$$\lambda = -19{,}51.$$

By substituting this value into the components of the vector $\vec{r}_{n0}$, one obtains the piercing point (vertex of the rolling cone):

$$C_0\,(-11{,}65060;\quad 15{,}14702;\quad -0{,}26972) .$$

The radius of rolling of the offside front wheel:

$$\varrho_0' = \overline{M_0 C_0} = \sqrt{(x_{M_0} - x_{C_0})^2 + (y_{M_0} - y_{C_0})^2} \approx 19{,}52\ \text{m} .$$

The theoretically exact value is

$$\varrho_0 = \sqrt{2^2 + 3^2} \approx 3{,}61\ \text{m} .$$

The angle β_0' at which the actual radius of rolling is inclined in the direction of the y axis, may be found as follows:

$$\tan\ \beta_0' = \left| \frac{x_{M_0} - x_{C_0}}{y_{M_0} - y_{C_0}} \right| = 0{,}74515 ,$$

from this

$$\beta_0' = 36°42' ,$$

while the theoretically exact value is

$$\varphi_0 = \beta_0 = 34°40'$$

The conditions are shown in Fig. 15.

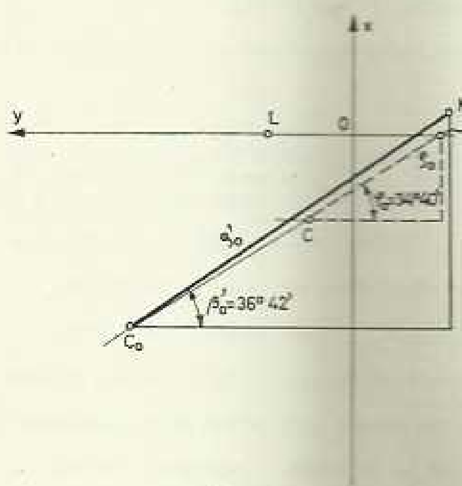

Fig. 15

From the above example it might be concluded that it is no use to define the proportions of the steering trapezoid in such a way as to try to approximately fulfil the conditions (1), even at large steering angles of the front wheels; in the case of application of the actual wheels, in a cornering of a small radius of curvature, the radii of rolling of the front wheels significantly differ from the values theoretically required.

2.2. *Construction of a trapezoidal steering linkage permitting only reduced skidding*

The following parameters of a vehicle should be given:

$$a, \ b, \ \delta, \ \varepsilon, \ \varkappa.$$

Let us choose a cornering having an average radius of curvature ϱ_a which frequently occurs in driving and within which the car runs at a relatively high speed (cornering with a minimum radius of curvature should not be taken for basis because — although being associated with large skids — they do not occur frequently and the driving speed in them is very low).

Referring to the above example, the fact that the car is slightly tilted in comparison to its initial position owing to the rather complicated turnings of the front wheels, will be disregarded.

First, the co-ordinates of the centre C of the cornering having a radius of curvature ϱ_a in the common lower tangent-plane of the wheels should be

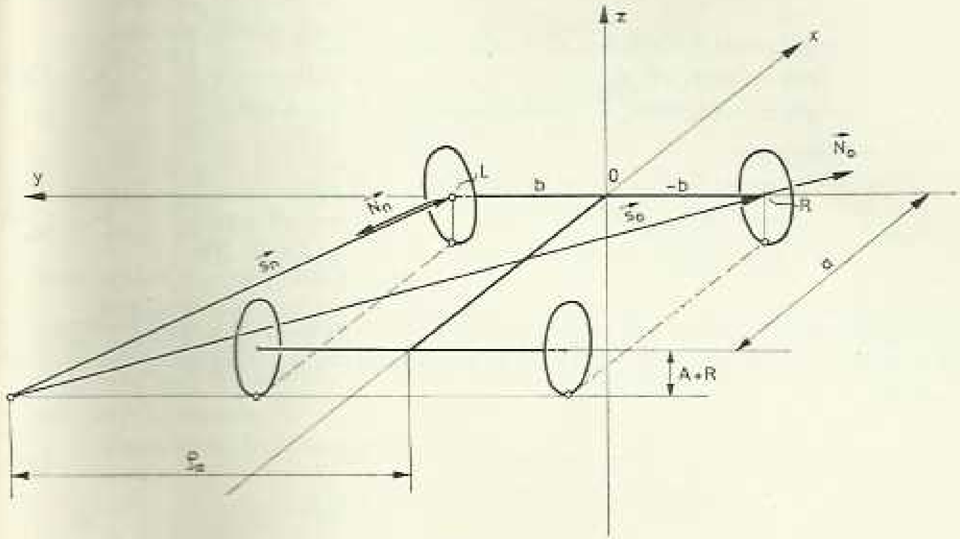

Fig. 16

calculated. Equation of this plane is given (with a very reasonable approximation) by

$$z = -(A + R)\,. \tag{32}$$

The co-ordinates of point C, according to Fig. (16) are

$$C(-a;\ \varrho_a;\ -(A + R))\,. \tag{33}$$

Let us now determine the vectors $\overrightarrow{CL} = \vec{s}_n$ and $\overrightarrow{CR} = s_0$:

$$\vec{s}_n = \{a;\ b - \varrho_a;\ A + R\}, \tag{34a}$$

$$\vec{s}_0 = \{a;\ -b-\varrho_a;\ A + R\}\,. \tag{34b}$$

In the cornering of the average radius of curvature the front wheels are turned to an extent that the normal $\overrightarrow{N}_n$ of the plane of the nearside front wheel should be parallel with $\vec{s}_n$, and the normal $\overrightarrow{N}_0$ of the plane of the offside front wheel should be parallel with $\vec{s}_0$. The axes of the rolling cones of the front wheels intersect point C. From the parallelism of the vectors it follows:

$$\vec{N}_n = \mu_n \vec{s}_n,\ \mu_n < 0, \qquad (35)$$
$$\vec{N}_0 = \mu_0 \vec{s}_0,\ \mu_0 > 0.$$

By reducing the vector equalities to sets of equations consisting of equalities of components of the vectors it is found that:

$$\left.\begin{array}{ll} -\cos\varkappa\cos\delta\sin(\varphi_n+\varepsilon)+\sin\varkappa\sin\delta & = \mu_n a \\ \cos\delta\cos(\varphi_n+\varepsilon) & = \mu_n(b-\varrho_a) \\ -\sin\varkappa\cos\delta\sin(\varphi_n+\varepsilon)-\cos\varkappa\sin\delta & = \mu_n(A+R) \end{array}\right\} \left.\begin{array}{l}\delta > 0\\ \varepsilon < 0\end{array}\right\}. \qquad (36a)$$

$$\left.\begin{array}{ll} \cos\varkappa\cos\delta\sin(\varphi_0+\varepsilon)-\sin\varkappa\sin\delta & = \mu_0 a \\ -\cos\delta\cos(\varphi_0+\varepsilon) & = -\mu_0(b+\varrho_a) \\ \sin\varkappa\cos\delta\sin(\varphi_0+\varepsilon)+\cos\varkappa\cos\delta & = \mu_0(A+R) \end{array}\right\} \left.\begin{array}{l}\delta < 0\\ \varepsilon > 0\end{array}\right\}. \qquad (36b)$$

From the above two sets of equations, the values of φ_n and φ_0 may be obtained. From the second equation of the set of equations (36a), concerning the nearside wheel

$$\cos(\varphi_n+\varepsilon) = \frac{\mu_n(b-\varrho_a)}{\cos\delta}, \qquad (36c)$$

by making use of this equation and the relation

$$\sin(\varphi_n+\varepsilon) = \sqrt{1-\cos^2(\varphi_n+\varepsilon)}, \qquad (36d)$$

the first and third equation of (36a) will take the form:

$$-\cos\varkappa\cos\delta\sqrt{1-\left[\frac{\mu_n(b-\varrho_a)}{\cos\delta}\right]^2}+\sin\varkappa\sin\delta = \mu_n a, \qquad (36e/1)$$

$$-\sin\varkappa\cdot\cos\delta\sqrt{1-\left[\frac{\mu_n(b-\varrho_a)}{\cos\delta}\right]^2}-\cos\varkappa\cdot\sin\delta = \mu_n(A+R). \qquad (36e/2)$$

Dividing the two equations with each other and arranging yields

$$\mu_n = \frac{+\sqrt{\cos^2\delta-\left[\dfrac{a\cos\varkappa+(A+R)\sin\varkappa}{a\sin\varkappa-(A+R)\cos\varkappa}\right]^2\cdot\sin^2\delta}}{b-\varrho_a} < 0,$$

and by replacing this into the first equation of the set of equations (36a)

$$\sin(\varphi_n+\varepsilon)=\frac{\sin\varkappa\sin\delta-a\dfrac{\sqrt{\cos^2\delta-\left[\dfrac{a\cos\varkappa+(A+R)\sin\varkappa}{a\sin\varkappa-(A+R)\cos\varkappa}\right]^2\sin^2\delta}}{b-\varrho_a}}{\cos\varkappa\cos\delta}, \qquad (37a)$$

$$\delta>0,\ \varepsilon<0,$$

from which φ_n may be found.

Similarly, from the set of equations (36b)

$$\sin(\varphi_0+\varepsilon)=\frac{\dfrac{\sqrt{\cos^2\delta-\left[\dfrac{a\cos\varkappa+(A+R)\sin\varkappa}{(A+R)\cos\varkappa-a\sin\varkappa}\right]^2\cdot\sin^2\delta}}{b+\varrho_a}\cdot a+\sin\varkappa\sin\delta}{\cos\varkappa\cos\delta}, \qquad (37b)$$

$$\delta<0,\ \varepsilon>0,$$

from which φ_0 may be obtained.

Substituting the values of φ_n and φ_0 calculated from formulas (37a) and (37b) into Eq. (19) and adopting the value of α, the length r of the rods of the trapezoid may be calculated. From Eq. (16) also the length of the wheel-track rod may be obtained.

From (19), by arranging

$$r=\frac{b\,[\sin(\alpha-\varphi_0)+\sin(\alpha+\varphi_n)-2\sin\alpha]}{\dfrac{1}{2}\,[1-\cos(2\alpha+\varphi_n-\varphi_0)]-\sin^2\alpha}. \qquad (38)$$

Example

Be

$$a=2\ \text{m}\quad b=0{,}5\ \text{m},\quad A=0{,}2\ \text{m}\quad R=0{,}07\ \text{m}$$
$$\delta=1^\circ\quad \varepsilon=2^\circ\quad \varkappa=3^\circ\quad \varrho_a=30\text{m}, \qquad (\alpha)$$

then

$$\sin\delta=0{,}01745,\quad \cos\delta=0{,}99985\qquad \sin\varkappa=0{,}05234,\quad \cos\varkappa=0{,}99863 \qquad (\beta)$$

$\sin\varkappa\cdot\sin\delta=0{,}00091$ (at the nearside wheel positive, at the offside wheel negative)

$\cos\varkappa\cdot\cos\delta=0{,}99848$ (at both wheels positive) (γ)

$$a\sqrt{\cos^2\delta-\left[\frac{a\cos\varkappa+(A+R)\sin\varkappa}{(A+R)\cos\varkappa-a\sin\varkappa}\right]^2\sin^2\delta}= \qquad (\delta)/1$$

$$=2\sqrt{0{,}99985^2-\left[\frac{2\cdot 0{,}99863+0{,}27\cdot 0{,}05234}{0{,}27\cdot 0{,}99863-2\cdot 0{,}05234}\right]^2\cdot 0{,}01745^2}=1{,}95382. \qquad (\delta)/2$$

Replacing these values into formulas (37a) and (37b)

$$\sin(\varphi_n+\varepsilon)=\frac{0{,}00091-\dfrac{1{,}95382}{0{,}5-30}}{0{,}99848}=0{,}06724,$$

$$\sin(\varphi_0+\varepsilon)=\frac{\dfrac{1{,}95382}{0{,}5+30}-0{,}00091}{0{,}99848}=0{,}06324. \qquad (\varepsilon)$$

Thus,

$$\varphi_n = \arcsin 0{,}06724 + 2^\circ = 3^\circ 51' + 2^\circ = 5^\circ 51' ,$$
$$\varphi_0 = \arcsin 0{,}06324 - 2^\circ = 3^\circ 38' - 2^\circ = 1^\circ 38' . \qquad (\zeta)$$

Let us first assume that the lengthening lines of the rods of the trapezoid intersect in the midpoint of the rear axle of the vehicle, then

$$\tan\alpha = 1/4 , \quad \alpha = 14^\circ 03' , \qquad (\zeta a)$$

in this case

$$\alpha - \varphi_0 = 14^\circ 03' - 1^\circ 38' = 12^\circ 25' ,$$
$$\alpha + \varphi_n = 14^\circ 03' + 5^\circ 51' = 19^\circ 54' , \qquad (\eta)$$
$$2\alpha + \varphi_n - \varphi_0 = 32^\circ 19' .$$

The length of the rod of the trapezoid, according to (38)

$$r = \frac{0{,}5\,(\sin 12^\circ 25' + \sin 19^\circ 54' - 2 \sin 14^\circ 03')}{0{,}5(1 - \cos 32^\circ 19') - \sin^2 14^\circ 03'} =$$

$$= \frac{0{,}5(0{,}21205 + 0{,}34038 - 2 \cdot 0{,}24277)}{0{,}5(1 - 0{,}84511) - 0{,}24277^2} \approx 1{,}8 \text{ m} .$$

This is a dimension which cannot be realized! Assuming $\alpha = 40^\circ$, from the formula (38)

$$r = 0{,}5 \text{ m}$$

is found, and finally, the length of the wheel-track rod:

$$l = 2\,(b - r \sin\alpha) = 2\,(0{,}5 - 0{,}5 \cdot 0{,}64279) = 0{,}36 \text{ m} .$$

REFERENCES

1. JUREK, A.: Automobiles. Műszaki könyvkiadó, Budapest 1963.
2. TERPLÁN, Z.: Mechanisms. Műszaki Könyvkiadó, Budapest 1958.
3. FAROBIN, YA. E.: Teoriya povorota transportnih machine (Izhdatelstvo "Machinostroenie", Moscov 1970).

Bemerkungen zur Lenkgeometrie von Personenkraftwagen mit steifer Vorderachse. Die rutschfreie Rollbewegung der gesteuerten Vorderräder eines Personenkraftwagens in Kurven kann mit guter Näherung mit der bekannten Lenktrapez-Vorrichtung, durch geeignete Wahl der Längen und Winkeleinstellungen der einzelnen Lenkstangen, gesichert werden. Jedoch kann die Abweichung zwischen den zum reinen Rollen theoretisch erforderlichen und durch das Lenktrapez gesicherten Drehwinkeln der Räder, hinsichtlich des Rutschens der Räder, nur dann vernachlässigt werden, wenn die Räder auf die Flächenebene der Straße senkrecht stehende, im geometrischen Sinne genommen ebene Kreisflächen sind und die Achsschenkelbolzen, um die die Räder gedreht werden können, sich auch in einer auf die Straßenfläche senkrechten Stelle befinden. Infolge der wirklichen Stellung der Mittelebenen der Vorderräder und der begrenzten Ausdehnung der Radreifen, können in den Kurven zwischen den theoretischen und tatsächlichen Rollhalbmessern ziemlich große Unterschiede auftreten. Diese Abhandlung weist auf die Möglichkeit der Schaffung eines solchen Lenktrapezes hin, das, unter der Voraussetzung, daß die Radreifen kreisringförmig und die Vorderräder in einem der Wirklichkeit entsprechenden Winkel eingestellt sind, die Rollbewegung der Vorderräder in den Kurven — wenigstens in einer solchen mit einem häufig vorkommenden durchschnittlichen Krümmungshalbmesser — theoretisch vollkommen, jedoch auch praktisch mit guter Näherung genau gestaltet. Die Überlegungen beziehen sich auf einen Wagen mit starrer Vorderachse.

Замечания к геометрии управления автомобилей с жесткой передней осью *(Л. Хусти).* Качение без скольжения передних управляемых колес автомобилей на поворотах может быть решето с хорошим приближением с помощью известного механизма рулевой

трапеции, а именно при подходящем выборе длины и устанавливаемого угла отдельных рычагов. Однако, отклонением между углами смещения колес, требуемым теоретически для чистого качения и практически обеспечиваемым рулевой трапецией, при таком рулевом механизме можно пренебречь с точки зрения скольжения колес только в том случае, если колеса представляют собою перпендикулярные к плоскости дороги плоские круглые пластины (в геометрическом отношении) и если рулевые шкворни, вокруг которых можно повернуть колеса, перпендикулярны к плоскости дороги. Из-за действительного положения нейтральной плоскости передних колес и конечных размеров резиновых шин между теоретическими и действительными радиусами качения при повороте могут иметь место очень большие отклонения. В данной статье показана возможность оформления такой рулевой трапеции, в случае которой достигается теоретически совершенно точное, а практически с хорошим приближением точное качение передних колес на поворотах (хотя бы на поворотах с радиусом поворота, встречающимся со средней частотой), исходя при этом из шины с формой в виде круглого кольца и принимая передние колеса с углом наклона, соответствующим действительности. Теоретические рассуждения действительны для автомобилей с жесткой передней осью.