AN APPROXIMATION FOR COMPUTING THE SAG OF INVOLUTE TEETH

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To facilitate the computation of sag caused by bending and shear forces the involute curve is usually approximated by various simpler curves such as piecewise linear sections. This paper describes the application of an approximate curve, the use of which makes it possible to compute the sag in a closed form.

I. Principles of computing the sag

The inflection of the tooth is computed on the basis of Castigliano's Theorem. If a force is applied to a body at equilibrium, the displacement of the point of application is given by the partial derivative of the strain work in the direction of, and with respect to, the applied force.

Let us examine the tooth in a rectangular coordinate system as shown in Fig. 1. The origin of the system coincides with the center of the gear, the axis of symmetry of the tooth lies on the x axis, the y axis is perpendicular to the x axis. r_1 and r_2 denote the radii of the root circle and of the addendum circle, respectively. $v(x)$ denotes the half-thickness of the tooth at x.

Let us study the sag caused by a force F, when it is applied to a surface point on the addendum circle, parallel to the y axis. The tooth is taken as a beam, fixed to the dedendum circle. If one allows L equal the strain work and f equal the deflection of the midline of the tooth at the addendum circle, then by Castigliano's theorem,

$$f = \frac{\partial L}{\partial F} . \tag{1}$$

II. Stresses

The stress σ on the tooth flank can only be a tangential stress σ_1. This stress can be expressed as the vector sum of two components σ_e and τ_e, where σ_e is perpendicular to the plane of the cross section, while τ_e is in this plane, parallel to the y axis (see Fig. 2).

The stresses acting at the point $P(x, y)$ can be computed by an approximate method [1] as follows:

Let us imagine that the tooth is put together by fibres so that the tangents to each and every fibre at points of the same abscissa meet at the

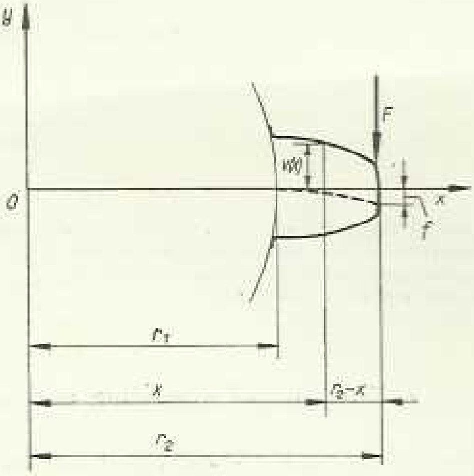

Fig. 1

point S on the x axis. (The imaginary fibre through the point P is shown by a broken line in Fig. 2.) Let the distance between point S and the examined cross section be $d = d(x)$ (a function of x). If the angle enclosed by the tangent

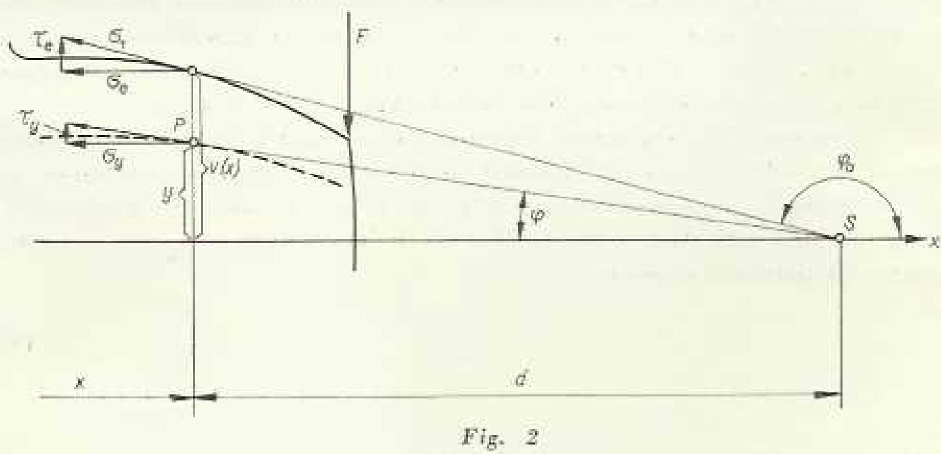

Fig. 2

drawn to the extreme fibre at a point of abscissa x, and by the positive x direction is φ_0 then, for the given cross section, from the equation of the curve of the tooth flank,

$$\frac{v(x)}{d} = -\tan\varphi_0 = -v'(x),$$

or

$$d = -\frac{v(x)}{v'(x)}. \tag{2}$$

The normal stress acting at the point $P(x, y)$ can be computed from the formula

$$\sigma_y = \frac{M}{I} y,$$

where $M = M(x)$ is the bending moment, while $I = I(x)$ is the inertia moment of the cross section. The y-component of the tangential stress acting on the fibre and originating at point P is

$$\tau_y = \sigma_y \cdot \tan \varphi = \frac{M}{I} y \cdot \frac{y}{d} = \frac{M}{I \cdot d} y^2.$$

The sum of the stresses τ_y in the y direction of the examined cross section is

$$V_1 = \int_{(A)} \tau_y dA,$$

where A denotes the entire area of the cross section, since M, I and d are constants for any given cross section. Substituting τ_y we obtain

$$V_1 = \frac{M}{I \cdot d} \int_{(A)} y^2 dA,$$

and

$$\int_{(A)} y^2 dA = I,$$

thus,

$$V_1 = \frac{M}{d}.$$

The force V_1 is balanced by the sum of the stresses τ_y. A shear force of magnitude $F - V_1$ remains which generates stresses τ_x. It is known [1] that the distribution of these τ_x in the cross section under study is [1]

$$\tau_x = \frac{3}{2} \cdot \frac{F - V_1}{A} \left(1 - \frac{y^2}{v^2}\right).$$

Hence, the magnitude of shear stress at the point P in the y direction is:

$$\tau = \tau_x + \tau_y = \frac{3}{2} \cdot \frac{F - V_1}{A} \left(1 - \frac{y^2}{v^2}\right) + \frac{M}{Id} y^2. \tag{3}$$

III. The strain work

Part of the strain work caused by the bending moment $M(x) = F(r_2 - x)$ is:

$$L_b = \frac{1}{2E}\int_{r_1}^{r_2} \frac{[M(x)]^2}{I(x)}\, dx\,,$$

where E is Young's modulus of elasticity for the material of the tooth.

Denoting the face width (the dimension of the tooth normal to the xy plane) by b, one can write in a cross section which is perpendicular to the x axis:

$$I(x) = \frac{b[2\,v(x)]^3}{12} = \frac{2\,b[v(x)]^3}{3}\,,$$

thus,

$$L_b = \frac{3\,F^2}{4\,bE}\int_{r_1}^{r_2} \frac{(r - x)^2}{[v(x)]^3}\, dx\,. \tag{4}$$

The strain work caused by the shear force in a volume element dK at point P is:

$$dL_s = \frac{1}{2G}\,\tau^2\, dK\,,$$

where G is the modulus of torsional shear for the material of the tooth. The strain work created by the shear force in the entire tooth is obtained by integrating the above expression.

$$L_s = \frac{1}{2\,G}\int_{(K)} \tau^2\, dK\,,$$

where K denotes the entire volume of the tooth. Substituting τ from (3):

$$L_s = \frac{1}{2\,G}\int_{x=r_1}^{r_2}\int_{y=-v(x)}^{v(x)}\int_{z=0}^{b}\left[\frac{3}{2}\cdot\frac{F - V_1}{A}\left(1 - \frac{y^2}{v^2}\right) + \right.$$
$$\left. + \frac{M}{Id}\,y^2\right]^2 dz\,dy\,dx = \frac{b}{G}\int_{x=r_1}^{r_2}\int_{y=0}^{v(x)}\left[\frac{9}{4}\left(\frac{F - V_1}{A}\right)^2\left(1 - \right.\right.$$
$$\left.\left. - \frac{2\,y^2}{v^2} + \frac{y^4}{v^4}\right) + 3\cdot\frac{F - V_1}{A}\cdot\frac{M}{Id}\left(y^2 - \frac{y^4}{v^2}\right) + \frac{M^2}{I^2 d^2}\,y^4\right] dy\,dx\,,$$

where, for the sake of brevity, V_1, A, v, M, I and d are not shown as functions of x. It has been taken into consideration that the shape of the tooth is symmetrical around the x axis.

Continuing our integration we obtain:

$$L_s = \frac{b}{G}\int_{x=r_1}^{r_2}\left[\frac{9}{4}\left(\frac{F-V_1}{A}\right)^2\left(y-\frac{2y^3}{3v^2}+\frac{y^5}{5v^4}\right)+\right.$$

$$\left.+\,3\,\frac{F-V_1}{A}\cdot\frac{M}{Id}\left(\frac{y^3}{3}-\frac{y^5}{5v^2}\right)+\frac{M^2}{I^2d^2}\cdot\frac{y^5}{5}\right]_{y=0}^{v}dx=$$

$$=\frac{b}{G}\int_{x=r_1}^{r_2}\left[\frac{9}{4}\left(\frac{F-V_1}{A}\right)^2\left(v-\frac{2v}{3}+\frac{v}{5}\right)+\right.$$

$$\left.+\,3\,\frac{F-V_1}{A}\cdot\frac{M}{Id}\left(\frac{v^3}{3}-\frac{v^3}{5}\right)+\frac{M^2}{I^2d^2}\cdot\frac{v^5}{5}\right]dx=$$

$$=\frac{b}{G}\int_{x=r_1}^{r_2}\left[\frac{9}{4}\left(\frac{F-V_1}{A}\right)^2\frac{8}{15}v+\right.$$

$$\left.+\,3\,\frac{F-V_1}{A}\cdot\frac{M}{Id}\cdot\frac{2}{15}v^3+\frac{M^2}{I^2d^2}\cdot\frac{v^5}{5}\right]dx=$$

$$=\frac{b}{G}\int_{x=r_1}^{r_2}\left[\frac{6}{5}\left(\frac{F-V_1}{A}\right)^2v+\right.$$

$$\left.+\,\frac{2}{5}\cdot\frac{F-V_1}{A}\cdot\frac{M}{Id}v^3+\frac{M^2}{I^2d^2}\cdot\frac{v^5}{5}\right]dx=$$

$$=\frac{b}{5G}\int_{x=r_1}^{r_2}\left[6\left(\frac{F-V_1}{A}\right)^2\cdot v+\right.$$

$$\left.+\,\frac{2(F-V_1)M}{AId}\cdot v^3+\frac{M^2}{I^2d^2}\cdot v^5\right]dx.$$

Considering that

$$M = F(r_2 - x)$$

and

$$V_1=\frac{M}{d}=\frac{F(r_2-x)}{d},$$

simplification and factoring yields

$$L_s=\frac{bF^2}{5G}\int_{x=r_1}^{r_2}\left\{6\,\frac{[d-(r_2-x)]^2}{A^2d^2}\cdot v+2\,\frac{[d-(r_2-x)](r_2-x)}{AId^2}v^3+\right.$$

$$\left.+\,\frac{(r_2-x)^2}{I^2d^2}v^5\right\}dx.$$

Substituting the expressions

$$d = -\frac{v}{v'}, \quad A = 2\,bv, \quad I = \frac{2\,bv^3}{3}$$

the following expression is obtained for the strain work generated by the shear force:

$$L_s = \frac{3\,F^2}{20\,bG} \int_{x=r_1}^{r_2} \frac{2\,v^2 + 2\,vv'(r_2 - x) + 3\,(r_2 - x)^2 \cdot v'^2}{v^3}\,dx. \tag{5}$$

IV. The sag

Partially differentiating the sum of L_b from (4) and L_S from (5) with respect to F one obtains for the sag sought for

$$f = \frac{\partial}{\partial F}(L_b + L_s) = \frac{3\,F}{2\,b}\left[\frac{1}{E}\int_{x=r_1}^{r_2}\frac{(r_2 - x)^2}{v^3}\,dx + \right.$$
$$\left. + \frac{1}{5\,G}\int_{x=r_1}^{r_2}\frac{2\,v^2 + 2\,vv'(r_2 - x) + 3\,(r_2 - x)^2\,v'^2}{v^3}\,dx\right]. \tag{6}$$

V. Approximate curve of the tooth flank

Let us use the equation

$$v(x) = y = \sqrt[3]{\alpha x + \beta}$$

to approximate the equation of the actual curve. The parameters α and β are determined by the required conditions that the approximating curve must pass through the points P_1 and P_2 on the dedendum circle and on the addendum circle, respectively (Fig. 3).

Let v_i be the segmental arch of a circle between the two sides of the tooth drawn through an arbitrary point P on the involute tooth flank. Let the centre of the circle be at the origin of the coordinate system. Let r be the length of the radial vector to the point P. Let the corresponding evolvent generating angle be ε. Let the angle between the axis of symmetry and the radius of the generating base circle drawn to the point of intersection between the upper tooth flank and the base circle be γ. Then, as seen in Fig. 3,

$$\frac{v_i}{2\,r} + ev\varepsilon = \gamma. \tag{7}$$

Any combination of the values v_i, r and ε determines the angle γ. Thus, the rectangular coordinates of an arbitrary involute point P are:

$$x = r\cos(\gamma - ev\varepsilon), \qquad y = r\sin(\gamma - ev\varepsilon); \tag{8}$$

where

$$\varepsilon = \text{arc}\cos\frac{r_a}{r}.$$

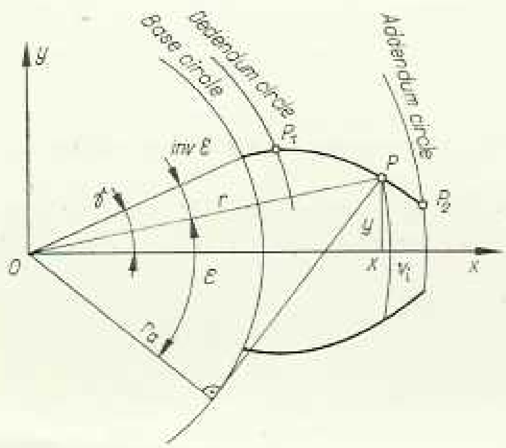

Fig. 3

The involute generating angles corresponding with P_1 on the dedendum circle and P_2 on the addendum circle are, respectively:

$$\varepsilon_1 = \text{arc}\ \cos\frac{r_a}{r_1}, \qquad \varepsilon_2 = \text{arc}\ \cos\frac{r_a}{r_2}. \tag{9}$$

Hence the coordinates of the two selected involute points are

$$P_1 : \begin{cases} x_1 = r_1\cos(\gamma - ev\varepsilon_1), \\ y_1 = r_1\sin(\gamma - ev\varepsilon_1); \end{cases} \qquad P_2 : \begin{cases} x_2 = r_2\cos(\gamma - ev\varepsilon_2), \\ y_2 = r_2\sin(\gamma - ev\varepsilon_2). \end{cases} \tag{10}$$

From the equation of the approximate curve of the tooth flank

$$\alpha x + \beta = y^3.$$

Substituting the coordinates of points P_1 and P_2 one obtains the following system of equations:

$$x_1 \alpha + \beta = y_1^3$$
$$x_2 \alpha + \beta = y_2^3 . \tag{11}$$

From (11) one can compute α and β. This system of equations always has a unique solution, since the determinant

$$\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} \neq 0 .$$

VI. Approximate computation of the sag

Knowing α and β an approximate value of the sag can be computed from (6) if the following relationships are taken into consideration:

$$y = v(x) = \sqrt[3]{\alpha x + \beta}$$

and

$$v'(x) = \frac{\alpha}{3\sqrt[3]{(\alpha x + \beta)^2}} = \frac{\alpha}{3 v^2} .$$

For the two integrals let us first solve, ignoring the scale factors.

$$\int_{r_1}^{r_2} \frac{(r_2 - x)^2}{v^3} dx = \int_{r_1}^{r_2} \frac{(r_2 - x)^2}{\alpha x + \beta} dx = \int_{r_1}^{r_2} \frac{r_2^2 - 2 r_2 x + x^2}{\alpha x + \beta} dx .$$

Carrying out the division in the integrand one obtains

$$\frac{r_2^2 - 2 r_2 x + x^2}{\alpha x + \beta} = \frac{x}{\alpha} - \frac{\frac{\beta}{\alpha} + 2 r_2}{\alpha} + \frac{r_2^2 + \frac{\frac{\beta}{\alpha} + 2 r_2}{\alpha} \beta}{\alpha x + \beta} .$$

Let

$$\frac{\frac{\beta}{\alpha} + 2 r_2}{\alpha} = C ,$$

and

$$r_2^2 + \beta C = D .$$

Then the integral expressing the sag caused by bending action is written as

$$\int_{r_1}^{r_2}\left(\frac{x}{\alpha}-C+\frac{D}{\alpha x+\beta}\right)dx=\left[\frac{1}{\alpha}\cdot\frac{x^2}{2}-Cx+\frac{D}{\alpha}\ln\left|x+\frac{\beta}{\alpha}\right|\right]_{r_1}^{r_2}= \tag{12}$$
$$=\frac{1}{2\alpha}(r_2^2-r_1^2)-C(r_2-r_1)+\frac{D}{\alpha}\ln\frac{\alpha r_2+\beta}{\alpha r_1+\beta}.$$

The other integral to be solved is:

$$\int_{r_1}^{r_2}\frac{2v^2+2vv'(r_2-x)+3(r_2-x)^2v'^2}{v^3}dx.$$

Substituting the expression

$$v'=\frac{\alpha}{3v^2}$$

one obtains the following integral:

$$\int_{r_1}^{r_2}\frac{6v^6+2\alpha(r_2-x)v^3+\alpha^2(r_2-x)^2}{3v^7}dx.$$

After the substitution of the ordinates of the approximate curve of the tooth flank we obtain

$$\int_{r_1}^{r_2}\frac{6(\alpha x+\beta)^2+2\alpha(r_2-x)(\alpha x+\beta)+\alpha^2(r_2-x)^2}{3\sqrt[3]{(\alpha x+\beta)^7}}dx.$$

Introducing the substitution

$$\alpha x+\beta=u^3,$$

one obtains

$$x=\frac{u^3-\beta}{\alpha}$$

and

$$dx=\frac{3u^2\,du}{\alpha}.$$

The limits $x=r_1$ and $x=r_2$ must, of course, also be transformed into

$$u_1=\sqrt[3]{\alpha r_1+\beta}$$

and

$$u_2=\sqrt[3]{\alpha r_2+\beta},$$

respectively. This change of variables and some rearrangement gives

$$\frac{1}{\alpha}\int_{u_1}^{u_2}\frac{5u^6+(r_2\alpha+\beta)^2}{u^5}du=\frac{1}{\alpha}\left[\frac{5u^2}{2}-\frac{(r_2\alpha+\beta)^2}{6u^6}\right]_{u_1}^{u_2}=$$
$$=\frac{1}{\alpha}\left[\frac{5}{2}(u_2^2-u_1^2)-\frac{(r_2\alpha+\beta)^2}{6}\left(\frac{1}{u_2^6}-\frac{1}{u_1^6}\right)\right]. \tag{13}$$

Substituting the results from (12) and (13), into (6), an approximate value is obtained for the sag.

VII. Example

Given a gear of the following dimensions: $m = 1$ cm, $z = 39$, $\varepsilon_0 = 20°$, $b = 10$ cm, $E = 2 \cdot 10^6$ kp/cm².

Compute the sag at a load $F = 500$ kp.

1. *Pitch circle radius:* $r_0 = mz/2 = 1 \cdot 39/2 = 19{,}5$ cm;
2. *Base circle radius:* $r_a = r_0 \cos \varepsilon_0 = 19{,}5 \cdot \cos 20° = 19{,}5 \cdot 0{,}93969 = 18{,}3239$ cm;
3. *Dedendum circle radius:* $r_1 = r_0 - 7/6\ \text{m} = 19{,}5 - 1{,}1667 = 18{,}3333$ cm;
4. *Addendum circle radius:* $r_2 = r_0 + m = 19{,}5 + 1 = 20{,}5$ cm;
5. *Computation of the angle* γ.

Considering the pitch circle radius, the pressure angle and that

$$v_t = t/2 = m\,\pi/2 = 1 \cdot 3{,}14159/2 = 1{,}5708 \text{ cm.}$$

From (7) we get

$$\gamma = \frac{t/2}{2r_0} + ev\varepsilon_0 = \frac{1{,}5708}{2 \cdot 19{,}5} + ev\ 20° = \frac{1{,}5708}{39} + 0{,}0149 =$$
$$= 0{,}0552 \text{ rad} = 3°09'43''.$$

and from (9):

$$\varepsilon_1 = \text{arc cos}\frac{r_a}{r_1} = \text{arc cos}\frac{18{,}3239}{18{,}3333} = 1°50';$$

$$\varepsilon_2 = \text{arc cos}\frac{r_a}{r_2} = \text{arc cos}\frac{18{,}3239}{20{,}5000} = 26°38'19'';$$

$$ev\ \varepsilon_1 = \tan\varepsilon_1 - \varepsilon_1 = 0{,}03201 - 0{,}03199 = 0{,}00002 = 04'';$$

$$ev\ \varepsilon_2 = \tan\varepsilon_2 - \varepsilon_2 = 0.50161 - 0{,}46493 = 0{,}03668 = 2°06'06''.$$

6. *From (10) the coordinates of the point* P_1 *on the dedendum circle are:*

$$x_1 = r_1 \cos(\gamma - ev\ \varepsilon_1) = 18{,}3333 \cos(3°09'43'' - 04'') =$$
$$= 18{,}3333 \cos 3°09'39'' = 18{,}3333 \cdot 0{,}99848 = 18{,}3054 \text{ cm;}$$
$$y_1 = r_1 \sin(\gamma - ev\ \varepsilon_1) = 18{,}3333 \sin 3°09'39'' = 18{,}3333 \cdot 0{,}05512 = 1{,}0105 \text{ cm.}$$

7. *Also from (10) the coordinates of the point P_2 on the addendum circle are:*

$$x_2 = r_2 \cos(\gamma - \varepsilon v\, \varepsilon_2) = 20{,}5 \cos(3°09'43'' - 2°06'06'') =$$

$$= 20{,}5 \cos 1°03'37'' = 20{,}5 \cdot 0{,}99983 = 20{,}4965 \text{ cm};$$

$$y_2 = r_2 \sin(\gamma - \varepsilon v\, \varepsilon_2) = 20{,}5 \cdot \sin 1°03'37'' =$$

$$= 20{,}5 \cdot 0{,}01850 = 0{,}3793 \text{ cm}.$$

8. *Computation of the parameters of the curve*

$$y = \sqrt[3]{\alpha x + \beta}$$

passing through the points P_1 (18,3054; 1,0105) and P_2 (20,4965; 0,3793) from (11):

$$18{,}3054\, \alpha + \beta = 1{,}0105^3$$

$$20{,}4965\, \alpha + \beta = 0{,}3793^3.$$

Solving the system of equations one obtains:

$$\alpha = -0{,}4459,$$

$$\beta = 9{,}1942.$$

To compute the portion of the sag caused by bending action one needs to obtain

$$C = \frac{\frac{\beta}{\alpha} + 2 r_2}{\alpha} = \frac{\frac{9{,}1942}{-0{,}4459} + 2 \cdot 20{,}5}{-0{,}4459} = -45{,}7066;$$

$$D = r_2^2 + \beta C = 20{,}5^2 + 9{,}1942\,(-45{,}7066) = 0{,}0144.$$

From (6), ignoring the multiplier $3F/2b$ one obtains for the sag by bending:

$$\frac{1}{E}\int_{r_1}^{r_2} \frac{(r_2 - x)^2}{v^3}\, dx = \frac{1}{E}\left[\frac{1}{2\alpha}(r_2^2 - r_1^2) - C(r_2 - r_1) + \frac{D}{\alpha} \ln \frac{\alpha r_2 + \beta}{\alpha r_1 + \beta}\right] =$$

$$= \frac{1}{2 \cdot 10^6}\left[\frac{1}{2(-0{,}4459)}(20{,}5^2 - 18{,}3333^2) + 45{,}7066\,(20{,}5 - 18{,}3333) + \right.$$

$$\left. + \frac{0{,}0144}{-0{,}4459} \ln \frac{-0{,}4459 \cdot 20{,}5 + 9{,}1942}{-0{,}4459 \cdot 18{,}3333 + 9{,}1942}\right] = 0{,}000\,002\,389\,.$$

For (13) u_1 and u_2 can be computed as follows:

$$u_1 = \sqrt[3]{\alpha r_1 + \beta} = \sqrt[3]{-0{,}4459 \cdot 18{,}3333 + 9{,}1942} = 1{,}0064;$$

$$u_2 = \sqrt[3]{\alpha r_2 + \beta} = \sqrt[3]{-0{,}4459 \cdot 20{,}5 + 9{,}1942} = 0{,}3763\,.$$

That portion of the sag which is due to shearing action is obtained from (6) as follows, ignoring the multiplier $3F/2b$:

$$\frac{1}{5\,G}\int_{r_1}^{r_2}\frac{2\,v^2+2\,vv'\,(r_2-x)+3\,(r_2-x)^2\,v'^2}{v^3}\,dx=$$

$$=\frac{1}{5\,G}\cdot\frac{1}{\alpha}\left[\frac{5}{2}\,(u_2^2-u_1^2)-\frac{(r_2\alpha+\beta)^2}{6}\left(\frac{1}{u_2^6}-\frac{1}{u_1^6}\right)\right]=$$

$$=\frac{1}{5\cdot 8\cdot 10^5}\cdot\frac{1}{-0{,}4459}\left[\frac{5}{2}\,(0{,}3763^2-1{,}0064^2)-\right.$$

$$\left.-\frac{(-20{,}5\cdot 0{,}4459+9{,}1942)^2}{6}\left(\frac{1}{0{,}3763^6}-\frac{1}{1{,}0064^6}\right)\right]=0{,}000\,001\,315\,.$$

Hence the total sag from both bending and shearing action is

$$f=\frac{3\,F}{2\,b}\,[0{,}000\,002\,389+0{,}000\,001\,315]=$$

$$=\frac{3\cdot 500}{2\cdot 10}\cdot 0{,}000\,003\,705=0{,}000\,2779\ \text{cm}\approx 2{,}77\ \mu\,.$$

VIII. Estimating the error of approximation

In the example most of the sag comes from bending action. Assuming that the tooth thickness computed from the equation for the approximate curve is

$$\lambda v(x)\,,$$

where $v(x)$ is the actual tooth thickness and λ constant, then, ignoring the constant multiplier, in lieu of the integral

$$\int_{r_1}^{r_2}\frac{[M(x)]^2}{[v(x)]^3}\,dx=J$$

the integral has been solved by means of the approximation:

$$\frac{1}{\lambda^3}\int_{r_1}^{r_2}\frac{[M(x)]^2}{[v(x)]^3}\,dx=\frac{J}{\lambda^3}\,.$$

Since λ is actually not a constant the estimated error is greatly increased. (λ varies from zero at the points P_1 and P_2 to a maximum at the pitch point.)

Thus a conservative estimate of the relative error is:

$$h=\frac{J-\dfrac{J}{\lambda^3}}{J}=1-\frac{1}{\lambda^3}\,.$$

From (10) the actual y_0 and y_{0k} ordinates are computed of a surface point on the pitch circle:

$$y_0 = r_0 \sin(\gamma - ev\,\varepsilon_0) = 19{,}5 \cdot \sin(3°09'43'' - 0°51'13'') =$$
$$= 19{,}5 \cdot \sin 2°18'30'' = 19{,}5 \cdot 0{,}04027 = 0{,}7853 \text{ cm.}$$

The corresponding abscissa is

$$x_0 = r_0 \cos(\gamma - ev\,\varepsilon_0) = 19{,}5 \cdot \cos 2°18'30'' = 19{,}5 \cdot 0{,}99919 = 19{,}4842 \text{ cm.}$$

From the equation of the approximate curve:

$$y_{0k} = \sqrt[3]{\alpha x_0 + \beta} = \sqrt[3]{-0{,}4459 \cdot 19{,}4842 + 9{,}1942} = 0{,}7969 \text{ cm.}$$

For the pitch point:

$$\lambda = \frac{y_{0k}}{y_0} = \frac{0{,}7969}{0{,}7853} \approx 1{,}015\,.$$

Assuming the value of λ to be constant along the entire length of the tooth flank, the estimated error of the sag is

$$h = 1 - \frac{1}{\lambda^3} = 1 - \frac{1}{1{,}015^3} \cong 0{,}05\,.$$

Thus the error is about 5%.

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ANNÄHERNDE BERECHNUNG DER DURCHBIEGUNG VON EVOLVENTENZAHNEN

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ZUSAMMENFASSUNG

Bei der Berechnung der durch die Biege- und die Scherkraft verursachten Zahnbiegung ist es üblich, den Evolventenbogen durch verschiedene einfachere Kurven (teilweise durch gerade Strecken) zu ersetzen. Das hier gezeigte Verfahren beruht auf der Anwendung einer Näherungskurve, die es ermöglicht, die Durchbiegung in geschlossener Form zu berechnen.

CALCUL APPROCHÉ DU FLÉCHISSEMENT DES DENTURES A DÉVELOPPANTE

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RÉSUMÉ

Lors du calcul du fléchissement de la dent, dû aux forces de flexion et de cisaillement, la courbe développante est remplacée généralement par des courbes plus simples (et en partie par des sections de droite). Le procédé présenté par l'auteur consiste dans l'application d'une courbe approchée permettant le calcul du fléchissement sous une forme finie.

ПРИБЛИЖЕННЫЙ РАСЧЕТ ПРОГИБА ЭВОЛЬВЕНТНЫХ ЗУБЬЕВ

Л. ХУСТИ

РЕЗЮМЕ

При вычислении прогиба зубьев от изгибающих и срезающих усилий обычно принято заменять эвольвентную дугу различными более простыми кривыми (частично прямолинейными отрезками). Приведенный в статье метод состоит в применении такой приближенной кривой, которая позволяет произвести вычисление прогиба в закрытой форме.